A New Solution for Projective Reconstruction Based on Coupled Line Cameras

Joo-Haeng Lee

We provide a new solution for the projective reconstruction problem based on coupled line cameras (CLCs) and their geometric properties. The proposed solution is composed of a series of optimized steps, and each step is more efficient than those of the initial solution proposed in [1]. We also give a new determinant condition for rectangle determination, which leads to less ambiguity in implementation. The key steps of the proposed solution can be represented with more compact analytic equations due to the intuitive geometric interpretations of the projective reconstruction problem based on CLCs: the center of projection corresponds to the intersection point of the two solution circles of each line camera involved.

Keywords: Geometric computer vision, projective reconstruction, coupled line cameras, camera calibration, diagonal parameterization.

I. Introduction

Recently, it was shown that a projective structure including a scene rectangle can be reconstructed to scale from a single image quadrilateral using a novel solution based on coupled line cameras (CLCs) [1]. The solution also provides a simple equation to determine if an image quadrilateral is the projection of any scene rectangle. The computational efficiency of this method also leads to the extension to handle the camera calibration problem. However, many useful properties of CLCs were not well analyzed in [1], which hinders optimizing the computational efficiency of the existing solution.

In this letter, we investigate various aspects of CLCs and present an accelerated solution based on additional analytic and geometric properties. For example, we show that the reconstructed center of projection (or the apex of the projective structure) corresponds to the intersection point of two solution circles of each line camera involved, which contributes to optimizing the solution steps. Notably, we give new analytic expressions of the length of the common principal axis, the diagonal angle of the scene rectangle, and the position of the center of projection of the CLC. In addition, we give a new determinant condition, which results in faster computation as well as less ambiguity in implementation.

Notably, using the CLC-based solution, the projective reconstruction and the rectangle determination can be performed without prior knowledge of correspondences or camera parameters. According to [2], we can reconstruct a rectangle without explicit correspondence information. However, we need to first find intrinsic camera parameters using absolute conics. The other feature is that a scene rectangle can be analytically reconstructed from a single image, which is not the case in previous works [3]-[5].

Following a brief review of CLCs in section II, we present an optimized solution in sections III and IV for projective reconstruction based on analytic and geometric properties. We conclude this letter in section V with remarks on future works.

II. Review of Coupled Line Cameras

1. Single Line Cameras

A line camera is an imaginary 1D pinhole camera in which the world line segment $\overline{v_0v_2}$ is projected to an image line segment $\overline{u_0u_2}$. See Fig. 1(a). The position p_c and the angle θ_0 denote the center of projection and the orientation of a line camera, respectively. For the simplicity in derivation, we assume a canonical configuration of a line camera where the principal axis $\overline{p_cv_m}$ always passes through the center of a

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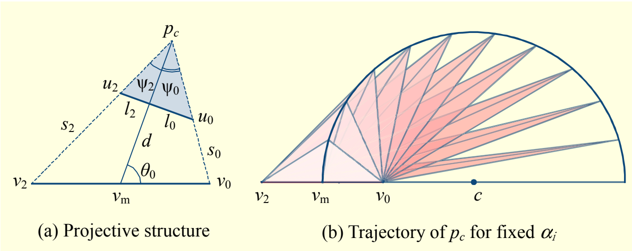


Fig. 1. Geometric configuration of line camera.

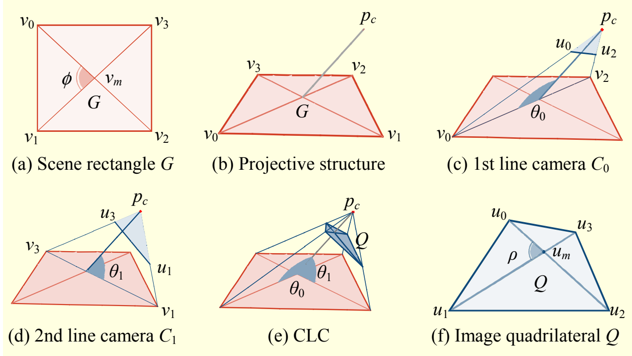


Fig. 2. Geometric configuration of CLCs.

world line: $v_m = (v_0 + v_2)/2$, where $v_m = (0,0,0)$ and $|\overline{v_m v_i}| = 1$. Note that v_i denotes a vector to the i-th vertex from the world origin.

The division ratio coefficient α_i of a line camera C_i is defined by the partial lengths, $l_i = |\overline{u_m u_i}|$ and $l_{i+2} = |\overline{u_m u_{i+2}}|$, of the image line $\overline{u_i u_{i+2}}$, as follows:

$$\alpha_i = (l_i - l_{i+2})/(l_i + l_{i+2}). \tag{1}$$

Now, the most fundamental geometric property of a line camera can be expressed as

$$\cos\theta_i = d\ \alpha_i, \tag{2}$$

where $d = |\overline{p_c v_m}|$ is the length of the principal axis. Equation (2) states that there exists a simple relation between the configuration (that is, d and θ_i) of a line camera and its image line (that is, α_i). Figure 1(b) shows a circular trajectory of p_c for a given division ratio α_i.

2. Coupled Line Cameras

Two canonical line cameras can be coupled to share the common principal axis and the center of projection. This configuration of a CLC is a novel representation of a pinhole camera model, which is especially efficient in representing a projective structure with a centered base rectangle [1] (Fig. 2).

When the principal axis $\overline{p_c v_m}$ passes through the center of a rectangle G, we can assign a line camera for each diagonal, $\overline{v_0 v_2}$ and $\overline{v_1 v_3}$, of G to get a configuration of the CLC. The projection image of such CLCs can be represented with two image lines, $\overline{u_0 u_2}$ and $\overline{u_1 u_3}$, which represent diagonals of an image quadrilateral Q. Note that CLC also follows a canonical configuration for simpler derivation. When the diagonal angle of $G = \{v_0, \ldots, v_3\}$ is ϕ, the vertices are defined as follows: $v_0 = (1,0,0)$, $v_1 = (\cos\phi, \sin\phi, 0)$, $v_2 = -v_0$, $v_3 = -v_1$, and $|v_i| = 1$.

For each line camera, the property of (2) holds individually. A geometric relation between two line cameras is encoded in the relative length coefficient β, which is defined as the ratio of partial lengths $l_0 = |\overline{u_m u_i}|$ and $l_1 = |\overline{u_m u_{i+1}}|$ of image lines $\overline{u_i u_{i+2}}$ and $\overline{u_{i+1} u_{i+3}}$:

$$\beta = l_1 / l_0. \tag{3}$$

3. Projective Reconstruction Based on CLCs

When an image quadrilateral Q is given (Fig. 2(f)), the problem of projective reconstruction based on CLCs is to find the frustum geometry of a projective structure (Fig. 2(b)) that is defined by two elements: the center of projection p_c of an unknown camera and the centered base rectangle G (Fig. 2(a)) that is projected to Q (Fig. 2(e)).

The solution presented in [1] is composed of a sequence of steps, which starts by computing the orientation angle θ_0 of the first line camera on $\overline{v_0 v_2}$ from Q:

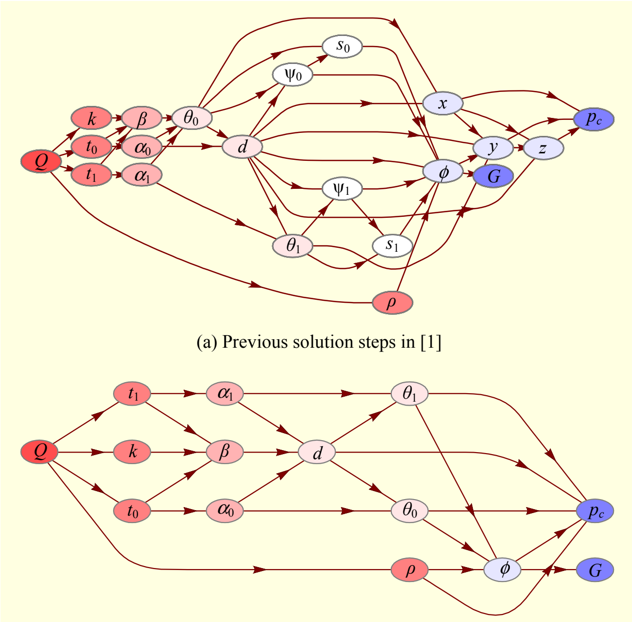
$$\tan\frac{\theta_0}{2} = \sqrt{\frac{A_0 + A_1 \pm \sqrt{A_0 A_1}}{A_1 - A_0}}, \tag{4}$$

where A_i is a coefficient defined by division ratio α_i of (1) and relative length β of (3). Once θ_0 is found, the solution sequentially proceeds for the other unknowns: d, θ_1, ψ_i, s_i, ϕ, and p_c. The parameters, including θ_i, d, ψ_i, and s_i, are determined regardless of the diagonal angle ρ of Q. However, two goal parameters, ϕ and p_c, should be determined along with ρ. (See the solution steps of [1] in Fig. 3(a).) Note that the solution steps need not be unique, an aspect elaborated upon in sections III and IV.

III. New Analytic Solution

1. Length of Common Principal Axis

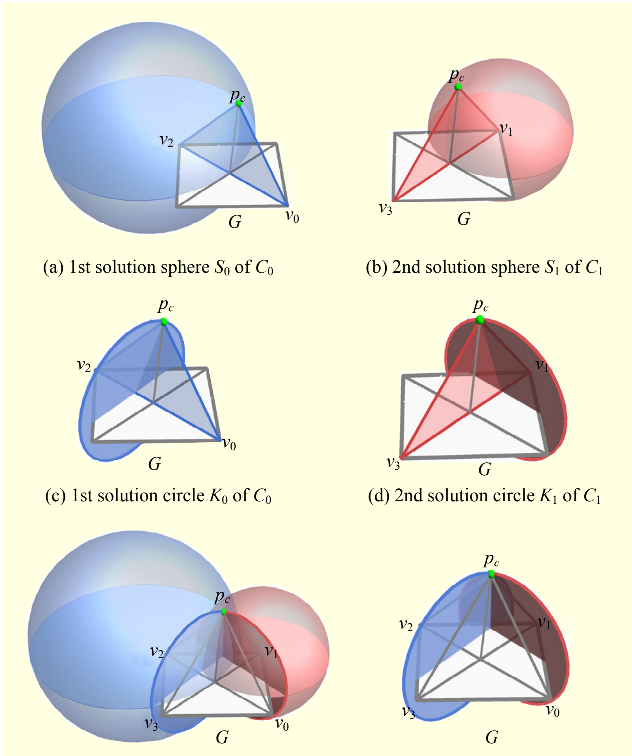
When reconstructing the projective structure, it is possible to start by solving the length d of the common principal axis, which allows for better optimization via the solution steps than in [1], in which orientation angle θ_0 must first be computed with a more complex expression of (4). See Fig. 3(a). An analytic solution for d can be expressed by the coefficients, α_i



(a) Previous solution steps in [1]

(b) New solution steps based on (5), (6) and (8)

Fig. 3. Comparison of solution steps: previous vs. proposed.



(a) 1st solution sphere S_0 of C_0

(b) 2nd solution sphere S_1 of C_1

(c) 1st solution circle K_0 of C_0

(d) 2nd solution circle K_1 of C_1

(e) Solution spheres and circles

(f) Intersection point of K_0 of K_1

Fig. 4. Geometric interpretation of common center of projection using solution spheres and circles of CLCs.

and β, from the image quadrilateral Q:

$$d = \sqrt{A_0 / A_1}, \qquad (5)$$

where $A_0 = (1-\alpha_1)^2 \beta^2 - (1-\alpha_0)^2$ and $A_1 = \alpha_0^2 (1-\alpha_1)^2 \beta^2 - (1-\alpha_0)^2 \alpha_1^2$. Note that diagonal angle ρ is not involved in (5). Once d is computed, each orientation angle, θ_i, can be computed using (2) with α_i.

2. Rectangle Reconstruction

We also provide a simple analytic solution to compute diagonal angle ϕ of unknown scene rectangle G:

$$\cos\phi = \cos\theta_0 \cos\theta_1 + \sin\theta_0 \sin\theta_1 \cos\rho. \qquad (6)$$

Following [1], diagonal angle ϕ should be computed in a procedural way without a compact analytic expression. Moreover, it requires computing such intermediate parameters as ψ_i and s_i. See Fig. 3(a). Hence, using the new solution based on (5) and (6) significantly accelerates computation, as intermediate parameters are not needed. See Fig. 3(b).

IV. New Solution Based on Geometric Properties

1. Single Line Cameras

Solution Sphere. Assuming the canonical configuration, division ratio α_i of image line $\overline{u_i u_{i+2}}$ of line camera C_i is invariant if the center of projection p_c exists on the surface of solution sphere S_i with the radius being $r_i = 1/|\alpha_i|$ and the center being $c_i = 1/2\alpha_i v_i$. See Figs. 4(a) and 4(b). Note that every solution sphere, S_i, passes through the origin (0, 0, 0), which is the center, v_m, of the world line in the canonical configuration. This property can be derived from (2). The half division, $l_i = l_{i+2}$ or $\alpha_i = 0$, of an image line, $\overline{u_i u_{i+2}}$, is a singular case wherein solution sphere S_i becomes an infinite plane that is normal to world line $\overline{v_i v_{i+2}}$ while passing origin $v_m = (0,0,0)$.

Solution Circle. When division ratio α_i and length d of the principal axis are known, orientation angle θ_i can be computed using (2). Then, solution sphere S_i is confined to solution circle K_i with the radius being $r_{Ki} = d\sin\theta_i$ and the center being $c_{Ki} = d\cos\theta_i v_i$. See Figs. 4(c) and 4(d).

2. Coupled Line Cameras

Solution Circles of CLCs. In a CLC, two solution spheres, S_i and S_{i+1}, are defined for each line camera of image lines $\overline{u_i u_{i+2}}$ and $\overline{u_{i+1} u_{i+3}}$ of Q. When length d of the common principal axis is known using (5), solution spheres S_i and S_{i+1} are confined to solution circles K_i and K_{i+1}. See Figs. 4(c) and 4(d). The radius, r_{Ki}, of each solution circle is independent to the diagonal angles, ρ and ϕ.

Intersection Point of Solution Circles. The intersection point of two solution circles, K_i and K_{i+1}, corresponds to the common center of projection p_c of the CLC. See Figs. 4(e) and 4(f). Let $c_{ki} = s_{ki} v_i$ be the vector to the center of each solution circle, K_i, where $s_{ki} = |c_{ki}| = d\cos\theta_i$. Then, the position of p_c can be expressed as follows:

$$\begin{aligned} p_c &= a\,c_{k0} + b\,c_{k1} + (0,0,h), \\ a &= (s_{k0} - s_{k1}\cos\phi)/(s_{k0}\sin^2\phi), \\ b &= (s_{k1} - s_{k0}\cos\phi)/(s_{k1}\sin^2\phi), \\ h &= \sqrt{d^2 - (s_{k0}^2 + s_{k1}^2 - 2 s_{k0} s_{k1}\cos\phi)/\sin^2\phi}. \end{aligned} \quad (7)$$

More elaboration on (7) generates a compact expression for p_c, which corresponds to the final solution step of Fig. 3(b):

$$p_c = \frac{d}{\sin\phi}(\cos\theta_0 \sin\phi, -\cos\theta_0\cos\phi + \cos\theta_1, \sin\theta_0 \sin\theta_1 \sin\rho). \quad (8)$$

3. Determinant

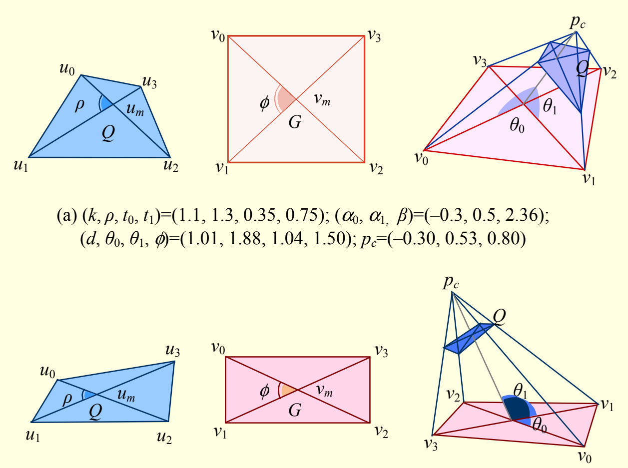
To get a valid value of d in (5), two conditions should be satisfied: 1) A_0 and A_1 should have the same sign; 2) The length of the common principal axis should not exceed the diameter of each solution sphere: $d \le \min(1/|\alpha_0|, 1/|\alpha_1|)$. These conditions and (5) can be compactly expressed as a new determinant, D, in a Boolean form:

$$D = \left(\beta \ge \frac{1-\alpha_0}{1-\alpha_1} \wedge 1 \ge \left|\frac{\alpha_1}{\alpha_0}\right|\right) \vee \left(\beta \le \frac{1-\alpha_0}{1-\alpha_1} \wedge \le \left|\frac{\alpha_1}{\alpha_0}\right|\right). \quad (9)$$

Note that (9) can be evaluated with the coefficients from an image quadrilateral, Q. Hence, before reconstructing the projective structure, we can determine if Q is the projection of any scene rectangle in a canonical configuration. For a general configuration, such as an off-centered scene rectangle, the determinant can also be applied after the preprocessing step based on vanishing points, as proposed in [1].

4. Examples of Projective Reconstruction

Figure 5 shows examples of the projective reconstruction based on the proposed method. Note that an image quadrilateral is represented by diagonal parameterization with four parameters (k, ρ, t_0, t_1): $k = |\overline{u_1 u_3}| / |\overline{u_0 u_2}|$; ρ is the angle between two partial diagonals, $\overline{u_m u_0}$ and $\overline{u_m u_1}$; $t_i = |\overline{u_m u_i}| / |\overline{u_i u_{i+2}}|$. See Fig. 2 for the configuration of Q. Note that D of (9) is evaluated to be true for each example. For Fig. 5, the proposed method is about two times faster than [1] when implemented in *Mathematica*: 0.221 ms vs. 0.446 ms.



(a) (k, ρ, t_0, t_1)=(1.1, 1.3, 0.35, 0.75); $(\alpha_0, \alpha_1, \beta)$=(−0.3, 0.5, 2.36); $(d, \theta_0, \theta_1, \phi)$=(1.01, 1.88, 1.04, 1.50); p_c=(−0.30, 0.53, 0.80)

(b) (k, ρ, t_0, t_1)=(1.3, 0.75, 0.35, 0.45); $(\alpha_0, \alpha_1, \beta)$=(−0.3, −0.1, 1.67); $(d, \theta_0, \theta_1, \phi)$=(2.43, 2.39, 1.82, 0.85); p_c=(−1.76, 0.78, 1.47)

Fig. 5. Examples of projective reconstruction based on CLC.

V. Summary and Future Work

We introduced a new solution for the projective reconstruction problem based on coupled line cameras (CLCs) and a novel geometric interpretation of CLCs. Compared to the method presented in [1], our solution showed optimization without intermediate parameters, and each step of the solution was refined to a compact analytic expression.

Coupled line projectors (CLPs) are similar to CLCs but with reversed projection. Recently, it was shown that CLPs can be used to analytically solve the projector pose estimation problem [6], which is a dual problem of the projective reconstruction based on CLCs. Hence, we believe that the similar geometric properties can be used to accelerate the CLP solution. Moreover, it will be interesting to combine CLC and CLP solutions for a projector-camera system [7].

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