

# Hardware Implementation on the Weight Calculation of Iterative Algorithm for CT Image Reconstruction

Xixin Cao, Kaisheng Ma, Renchun Lian, and Qihui Zhang

*The weight calculation in an iterative algorithm is the most computationally costly task in computed tomography image reconstruction. In this letter, a fast algorithm to speed up the weight calculation is proposed. The classic square pixel rotation approximate calculation method for computing the weights in the iterative algorithm is first analyzed and then improved by replacing the square pixel model with a circular pixel model and the square rotation approximation with a segmentation method of a circular area. Software simulation and hardware implementation results show that our proposed scheme can not only improve the definition of the reconstructed image but also accelerate the reconstruction.*

*Keywords:* Weight calculation, circular pixel model, segmentation method, simulation, FPGA.

## I. Introduction

The electronic X-ray computed tomography (CT) technique is one of the essential inventions of the 1970s [1] and is now widely used in industrial nondestructive examination and medical image processing. The key technology of CT is image reconstruction from the projection data, for which there are two main methods: mathematical transformation method [2] and iterative method. Although the former has relatively low computational complexity, it produces degraded reconstructed pictures when the projection data is incomplete [3], [4]. The latter method requires a significant amount of calculation, the calculation of weight accounting for the majority of that

calculation; thus, bottlenecking results during the reconstruction period in real-time applications. As a suitable multiple instruction, multiple data device, a field programmable gate array (FPGA) can provide a much higher speed than a single instruction, single data device or a single instruction, multiple data (SIMD) device. Algorithms using FPGAs [5]-[8] were proposed to accelerate the reconstruction of the mathematical transformation method. However, the FPGA technique is not always used to accelerate the iteration reconstruction algorithm.

In this letter, we present a new weight calculation method by replacing the traditional square pixel model with a realistic circular pixel model and by using a segmentation method of a circular area instead of the square rotation approximation. Also, we improve the algorithm to adapt to the unique SIMD characteristics of the FPGA and transform the multiplication and division operations into addition and subtraction operations by removing redundancy, judging symmetry, and seeking internal recursion relations. The simulation and implementation results prove that our proposed weight calculation algorithm can adapt well to hardware and improve the reconstruction quality and speed.

## II. Weight Calculation Algorithm

The parallel-beam scan mode includes an X-ray source and a detector array, which allows only two kinds of movement. In one kind of movement, the X-ray source and the detector array vertically move in parallel at the connection between them. In this condition, the projection data is collected from a group of parallel X-rays. In the other kind of movement, either the CT or the object slightly rotates. With these two movements, the detector array can collect a huge amount of data.

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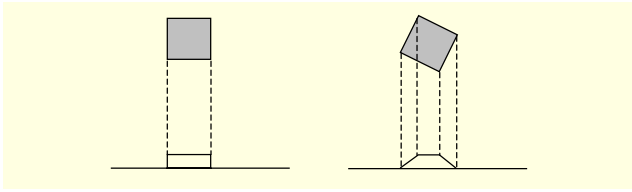


Fig. 1. Original pixel and pixel after rotation.

Every pixel can be regarded as a square in the traditional square pixel model. The approximation of projection calculation is based on the hypothesis that one pixel can only locate in two detectors. Figure 1 shows the pixel model, and the approximate weight can be expressed as

$$g_{ij}(u) = \begin{cases} 1 & |u - P_{(i,j)}| = 0, \\ 1 - |1 - u - P_{(i,j)}| & \text{when } 0 < |u - P_{(i,j)}| \leq 1, \\ 0 & |u - P_{(i,j)}| \geq 1, \end{cases} \quad (1)$$

where  $P(i, j)$  stands for the projection of the pixel on the detector array and  $u$  is the middle coordinate of the detector array.

To create a more realistic pixel model, we improve the pixel model shape from square to circular and replace the square rotation approximation with a segmentation method of a circular area. As shown in Fig. 2, the weight is the sum of the areas of the different color columns. The three processing steps are as follows.

1) Calculate the projection of the center of the circular pixel on the detector array.

2) As shown in Fig. 2, the donation to each X-ray according to the proposed area allocation method is calculated. Each pixel can donate weight to no more than two X-rays.

3) After the traversal of each pixel, the weight is calculated.

Several methods are considered to reduce the amount of calculation. These methods can be described as follows.

Build the iteration formula based on the correlation between adjacent pixels; the following equations can be obtained:

$$P(i+1, j) = P(i, j) - 2R * \cos \theta, \quad (2)$$

$$P(i, j+1) = P(i, j) + 2R * \sin \theta, \quad (3)$$

where  $P(i, j)$  stands for the projection of the pixel on the detector array,  $\theta$  is the turning angle between the object and the detector array, and  $R$  is the radius of the pixel model. If  $P(i, j)$ , then any of the projections on the detector array of the circular pixel center can be calculated, and we can easily compute  $P(i+1, j)$  and  $P(i, j+1)$  by addition and subtraction operations but not complex multiplication and division operations.

Clearly, the center of the object is always projected onto the center of the detector array. By using symmetry, if we have already calculated one side of the projection location, we can

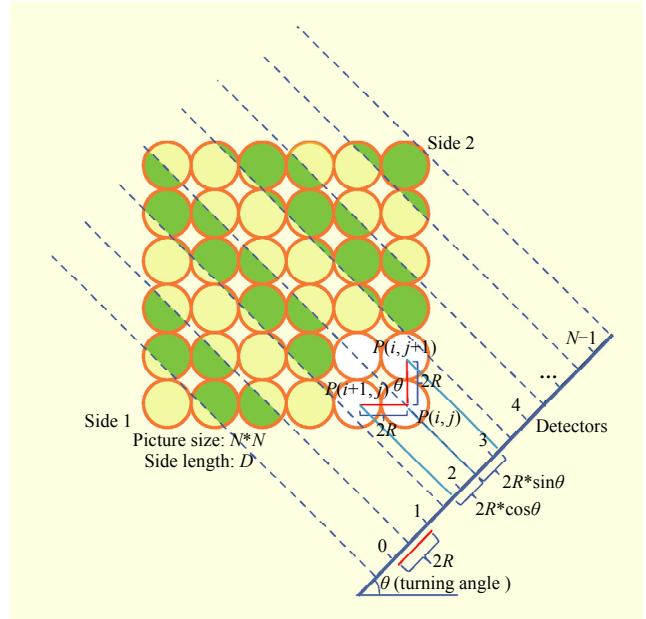


Fig. 2. CPMSMCA.

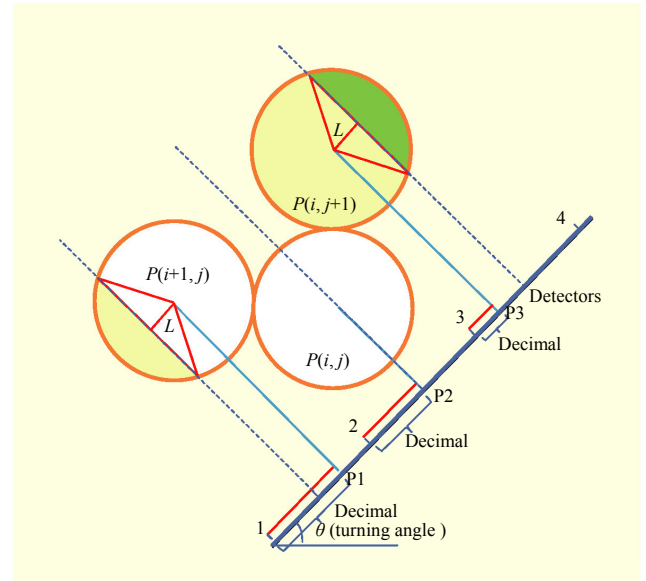


Fig. 3. Details for segmentation method of circular area.

obtain the projection location of the other side through the following expression.

$$P(N-1-i, N-1-j) = D - P(i, j), \quad (4)$$

where  $D$  stands for the side length of the object, and  $D = N * 2 * R$ .

When  $\theta$  is a special turning angle, for example, 0 degrees, 45 degrees, or 90 degrees, the calculation is relatively easy. In the detailed implementation of circular pixel model and segmentation method of circular area (CPMSMCA), the area of the different parts can be obtained by adding or subtracting

the area of the triangle to or from the area of the arc. The distance between the center of the circular pixel and the middle of the two detectors is represented by  $L$  and has two cases:

$$L = R - P_d, \quad (5)$$

when  $P_d < R$ ; see  $P(i+1, j)$  in Fig. 3.

$$L = P_d - R, \quad (6)$$

when  $P_d \geq R$ ; see  $P(i, j+1)$  in Fig. 3.

The area of the smaller part, which stands for the donation to Detector  $P_b$ , can be obtained by subtracting the area of the triangle from the area of the arc. The area of the larger part, which stands for the donation to Detector  $P_b$ , can be obtained by adding the area of the triangle to the area of the arc.

Although there are two conditions for  $L$ , we can combine them into one formula. Therefore, the donation to Detector  $P_i$  is

$$\begin{aligned} W(P_i) &= 2 * \left( \frac{\arccos\left(\frac{R-P_d}{R}\right)}{2\pi} * \pi * R^2 - \frac{1}{2} * (R-P_d) * \sqrt{R^2 - (R-P_d)^2} \right) \\ &= R^2 * \arccos\left(\frac{R-P_d}{R}\right) - (R-P_d) * \sqrt{R^2 - (R-P_d)^2}. \end{aligned} \quad (7)$$

Assume  $R$  equals 1. Weight is represented by  $W(P_i)$ , which is actually the area, donated to the  $P_i$ -th detector in the detector array from the pixel, whose projection has an integer part  $P_i$ . We also use  $P_i$  to index the detector. As shown in (7),  $P_d$  has a relation with the weight donated to Detector  $P_i$ . The other part of the weight in this pixel is donated to the detector integer +1, and it can be calculated as follows:

$$W(P_i + 1) = \pi R^2 - S(P_i + 1). \quad (8)$$

By using this method, complex calculation can be avoided.

### III. Hardware Structure and Modules for Weight Calculation

Figure 4 shows the hardware structure for the image reconstruction, wherein the upper side is the data flow and the bottom side is the control component. The data flow can be divided into three modules: weight calculation, area addition, and reconstruction. First, the weight calculation module fetches the sin&cos values from ROM. Then, the projection calculation is executed. Finally, the weight is computed and stored in dual-port RAM through one port. The next module obtains the data from the other port of the dual-port RAM and addition is performed. The last module fetches the real projection data, estimates the difference between the estimated projection value and the real projection data, and then updates the image.

As shown in Fig. 5, sin&cos data is stored in the ROM. After being taken out, the control component controls the calculation of the data flow with floating-point IP cores. The most

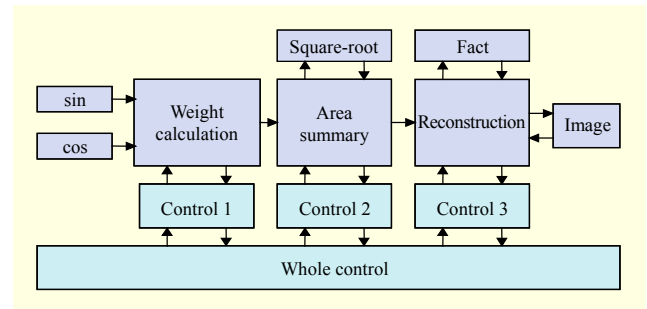


Fig. 4. Hardware structure for image reconstruction.

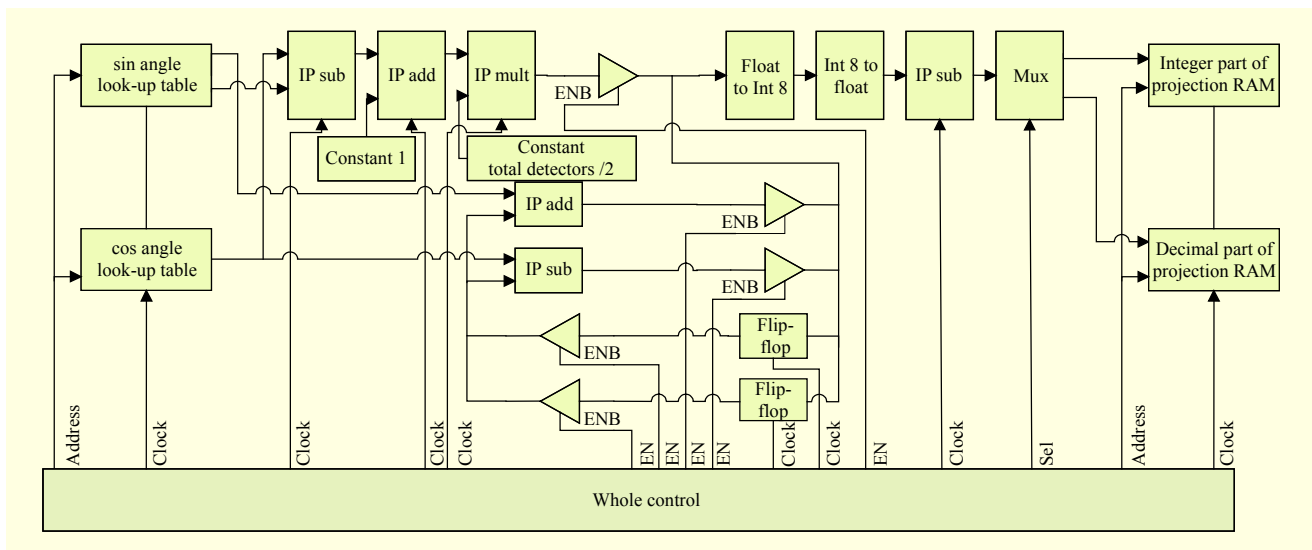


Fig. 5. Details for weight calculation module.

important parts of this module are the D flip-flops and the three-state gates, whose main responsibility is storing the projection location of the circular pixel model of each row or column. After the traversal of each pixel, both the integer and decimal are stored in dual-port RAM to exchange data with the next module.

#### IV. Simulation and Implementation Results

The projection data source comes from the Computer Tomography Laboratory of Capital Normal University, Beijing, China, and it is the result of scanning a turbine in parallel-in mode with a linear detector array. The resolution is  $241 \times 241$ , and the software used to simulate the environment is Microsoft Visual Studio 2008.

The simulation results and image definition evaluation results are shown in Figs. 6 and 7, respectively. From the results, we conclude that the new circular pixel model is better in picture quality than square pixel model and square rotation approximation (SPMSRA), which is the traditional method.

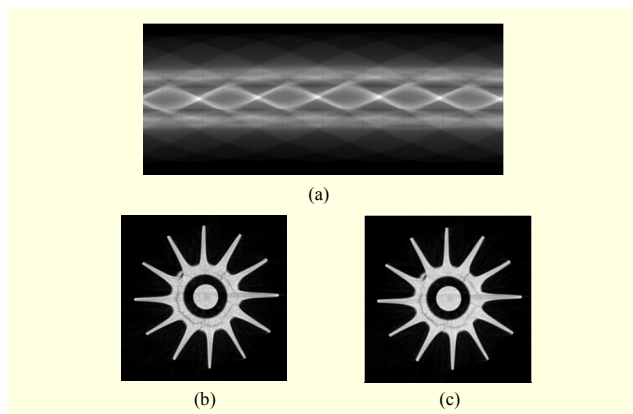


Fig. 6. Original projection data: (a) 576 angles and 241 detectors in detector array, (b) reconstructed image using SPMSRA, and (c) reconstructed image using CPMSMCA.

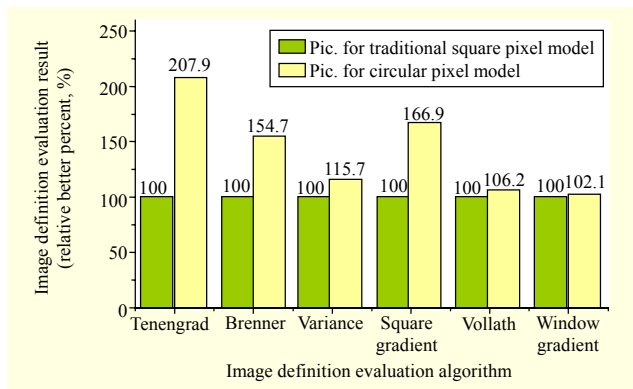


Fig. 7. Comparison of results.

The structure is designed with Verilog HDL, and ModelSim 6.1SE is used for hardware simulation. The FPGA implementation results show that the control logic and data flow cost a total of 5,000 LEs, 590,004 bits of memory, and seven embedded multiplier 9-bit elements. The frequency of our Altera FPGA platform is 50 MHz, and the on-chip frequency is 400 MHz. It costs 0.8275 seconds to achieve an iteration in the Altera FPGA platform, whereas the same data calculation costs 10.1 seconds in the Microsoft Visual Studio 2008 PC platform with C language.

#### V. Conclusion

In this letter, a fast weight calculation algorithm was proposed and successfully implemented in FPGA hardware. By using the circular pixel model instead of the square pixel model and the segmentation method of a circular area instead of the square rotation approximation, the improved algorithm can achieve better reconstruction quality. Additionally, because optimization is achieved according to the features of the hardware, the hardware platform adapts well. The software simulation results and hardware testing results show that our proposed scheme can easily meet the requirements of real-time processing of the weight calculation.

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