Stochastic Differential Equations for Modeling of High Maneuvering Target Tracking

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In this paper, we propose a new adaptive single model to track a maneuvering target with abrupt accelerations. We utilize the stochastic differential equation to model acceleration of a maneuvering target with stochastic volatility (SV). We assume the generalized autoregressive conditional heteroscedasticity (GARCH) process as the model for the tracking procedure of the SV. In the proposed scheme, to track a high maneuvering target, we modify the Kalman filtering by introducing a new GARCH model for estimating SV. The proposed tracking algorithm operates in both the non-maneuvering and maneuvering modes, and, unlike the traditional decision-based model, the maneuver detection procedure is eliminated. Furthermore, we stress that the improved performance using the GARCH acceleration model is due to properties inherent in GARCH modeling itself that comply with maneuvering target trajectory. Moreover, the computational complexity of this model is more efficient than that of traditional methods. Finally, the effectiveness and capabilities of our proposed strategy are demonstrated and validated through Monte Carlo simulation studies.

Keywords: Maneuvering target, stochastic differential equations, GARCH process, modified Kalman filtering.

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I. Introduction

Target tracking is a basic problem due to its significance in many applications, such as cellular radio network, air traffic control, and radar [1]. For example, one of the important issues in the cellular radio network is mobility tracking, which provides efficient network control. Similar issues appear in many other civilian applications. In air surveillance systems, which may manage a number of airplanes, a tradeoff between tracking accuracy performance and real-time computation is required. Maneuvering target tracking (MTT) is one of the important issues in target tracking because acceleration may not be directly observable or measurable. Additionally, apparent accelerations are induced by human control, an autonomous guidance system, or atmospheric disturbances. In this paper, the proposed target tracking algorithm assumes that the target motion can be formulated by stochastic differential equation (SDE) modeling of acceleration, which provides a sufficiently accurate model for acceleration tracking. In most MTT systems, which may control a number of targets (or objects), a tradeoff between tracking accuracy performance and real-time computation is a major issue. Therefore, we propose a new real-time algorithm based on adaptive state space model corresponding for SDE, which provides desirable performance for the tracking of a trajectory.

In the past decades, various methods have been proposed to address the problem of MTT. The simplest useful model for target tracking is Kalman filtering based on a single model [2]. However, its performance is often seriously degraded when the target maneuver occurs. One of the major challenges in the tracking of a maneuvering target arises from the abrupt change of its acceleration. A conventional solution is to model the noisy kinematic components; that is, acceleration, as a random process with a known exponential autocorrelation function. This model is known as the Singer model [2]. The Singer model is the basis model for the development of many other effective models, such as the mean-adaptive acceleration model [3]. These types of models are basically *a priori* models, which cannot suitably perform for the various accelerations of practical scenarios [4]. So, the tracking performance is seriously affected by the inappropriate value of the *a priori* parameters. Although, the Singer model performs well in low maneuvering situations, its performance degrades during constant-velocity and high maneuvering scenarios.

In recent studies, decision-based methods and multiple-model algorithms were the two main approaches used to improve MTT performance. In a decision-based method, a maneuver is detected and then a model is matched with that maneuver. Input estimation (IE) techniques [5]-[8], the variable dimension (VD) filter [9], and the two-stage Kalman estimator [10] are the most familiar decision-based techniques. In addition to the basic filtering computation, these techniques require additional effort, such as the estimation and detection of acceleration and the compensation of the state estimate or the transition between the non-maneuvering filter and the maneuvering filter to deal with the unknown target maneuvers [11]. In particular, the IE algorithm is widely accepted as one of the most effective decision-based methods. This tracking algorithm has many notable qualities, especially its ability to perform well in the case of a high maneuvering trajectory.

Multiple-model (MM) algorithms, which describe the motion of a target using multiple subfilters, are also commonly used. These approaches include the generalized pseudo-Bayesian (GPB) method [12], the interacting MM (IMM) method [13]-[16], the adaptive IMM method [17]-[19], and so on. However, the improvement is gained at the price of increasing the computational complexity and incurring a high risk of failure in the decision making process. Our main focus in this paper is on the single motion model.

In our prior research on target tracking, we proposed a single model based on SDE and stochastic volatility (SV) [20], [21]. In these papers, the original state and the SV are estimated simultaneously with a particle filter. However, the particle filter approach requires significantly higher computation than a Kalman filter. The proposed state equation given by the generalized autoregressive conditional heteroscedasticity (GARCH) has a structure in which the linear Kalman filter approach fails because of the non-Gaussianity of the noise process. Therefore, in this paper, we extend our approaches in [20], [21] by proposing a modified Kalman filtering based on the adaptive single GARCH modeling of acceleration that illustrates suitable accuracy in comparison with the traditional single model.

The variance and the time constant of the target acceleration are two parameters that specify the maneuver capability. In previous research, time invariant parameters were used for these parameters; in particular, it was assumed that the underlying variance of the acceleration model is constant over the entire trajectory and is unaffected by the changes in the acceleration level of the target. So, the success of such models relies on a correct determination of the design parameters. For a maneuvering target, if there is an abrupt change in acceleration, the variance of time history of acceleration changes suddenly as well. The GARCH modeling provides a statistical tool to model the time series whose variance is a stochastic process. Furthermore, the GARCH process in our application has two main advantages: it has a heavy-tailed probability density function and has heteroscedasticity (time varying variance). So, GARCH is an appropriate modeling tool for MTT since the acceleration of the target can increase or decrease quite abruptly. In this paper, we utilize these features of GARCH for the maneuvering target modeling. However, the GARCH-based model, which is a nonlinear time process, is used to accommodate the characteristics of target tracking, such as heavy-tailed distributions of the innovation process and the bursty nature of the target acceleration.

The paper is organized as follows. First, the basic theory of the GARCH process and the proposed model are introduced in section II. Section III addresses the modified Kalman filtering as a suitable solution for the proposed dynamic equation of motion. In section IV, the proposed algorithm is compared with the Singer model [2], Wang and Varshney's IE method [8], and the modified IE (MIE) algorithm in [7] for four synthetic problems extracted from current literature on tracking trajectories.

II. Problem Formulation

1. GARCH Model

The autoregressive conditional heteroscedasticity (ARCH) process introduced by Engle in [22] allows the conditional variance to change over time as a function of past errors leaving the unconditional variance constant. A more general class of processes, GARCH, was introduced by Bollerslev in [23]. GARCH processes have much more flexible lag structures. Time series typically exhibit time-varying conditional standard deviations and correlations. Regarding the GARCH process, various applications have been proposed, including image denoising [24], target tracking [20], and blind dependent source separation [25].

The conditional standard deviation is also called the volatility.

Higher volatilities increase the risk of motions. Therefore, models of time varying volatilities and correlations are essential for maneuver management. GARCH processes are dynamic models of conditional standard deviations and correlations. The autoregressive GARCH (AR-GARCH) process follows a pure AR(P)-GARCH(p, q) model if

$$x_i = \sum_{k=1}^{P} a_k x_{k-i} + z_i, \quad z_i = \sqrt{h_i}\varepsilon_i \ , \tag{1}$$

$$h_i = \alpha_0 + \alpha_1 h_{i-1} + \beta_1 z_{i-1}^2 \ , \tag{2}$$

where $\alpha_0 > 0$ is a constant value and is the ARCH parameter and $\alpha_1, \beta_1 \geq 0$ are the GARCH parameters, i is the time index, a_k and P in (1) are the parameter and the order of the AR part, respectively, ε_i is a sequence of zero-mean independent and identically distributed random variables with unity variance, and h_i is the one sample ahead conditional variance of the z_i. In practice, ε_i are often assumed to be independent Gaussian random variables. From (2), it is obvious that at each sample, both the neighboring sample variance and the neighboring conditional variance play a role in the current conditional variance. Using the results from [23], it is possible to derive the kurtosis of GARCH(1, 1) to determine that it is greater than 3, which is greater than the kurtosis of Gaussian distribution.

Hence, GARCH(1, 1) can successfully model a heavy-tailed random process. A characteristic feature of many time series is volatility clustering, where periods of high and low volatility occur in the data. On the other hand, we can observe the same characteristic for target motion. Therefore, a GARCH process has a constant unconditional variance but a non-constant conditional variance. We say that it is homoscedastic but conditionally heteroscedastic. Traditionally, GARCH modeling has been proposed to estimate volatilities. In many realistic settings, the simplest GARCH(p, q) modeling, $p, q = 1, 2$, is adequate to track volatilities, even over long periods [26]. Since abrupt changes in acceleration are translated into heavy-tailed volatilities of acceleration, we model the target acceleration according to the GARCH(1, 1) process.

2. Dynamic Equations of Target Motion

The target model selected for tracking applications must be adequately simple to permit easy implementation in practical systems. On the other hand, accuracy is an important issue. Therefore, a tradeoff between computational complexity and tracking accuracy must be considered. Moreover, because of the continuous nature of the actual target motion, the target model offered here accounts for this objective in a way that is simple and provides a proper representation of the maneuver target behavior. The target model presented here is a totally different approach that detects the existence of target maneuvers and directly estimates the magnitude of the unknown SV. The model is based on the fact that the target acceleration, a_t, can henceforth be considered the target maneuver variable. The presented model assumes that the acceleration volatility is correlated in time and, hence, so is the amount of target maneuvering. If a target is accelerating with large (small) variance at time t, it is likely to be accelerating with large (small) variance at time $t + \tau$ [20]. A typical representative model of the acceleration function is the statistical Markov process that can be described by the Ito SDE [27] as follows:

$$da_t = \mu a_t \, dt + \sqrt{h_t} \, dW_t, \tag{3}$$

where μ is the constant drift of acceleration a_t, h_t is the stochastic acceleration volatility (stochastic variance of acceleration), and dW_t denotes a white Brownian motion. The power spectral density is defined as

$$E((\sqrt{h_t} \, dw_t)^2) = q_a dt \ , \tag{4}$$

where $q_a = 2\mu h_t$. Due to the fundamental suitability of GARCH modeling for heteroscedastic processes, the non-constant variance of the maneuvering target acceleration can be modeled by the GARCH process. In addition, GARCH models account for the volatility clustering; in other words, large changes tend to follow large changes and small changes tend to follow small changes, which is generally compatible with the target maneuvering. Allowing volatility to be generated according to an autonomous SDE generally translates into approximating schemes based on conditionally heteroscedastic autoregressive models; the typical reference model assumes that the acceleration of the target follows (3), with h_t being the solution of another SDE:

$$dh_t = \theta(\omega - h_t) dt + \xi h_t dB_t, \tag{5}$$

where ω is the long-term mean volatility, θ is the rate at which the volatility reverts toward its long-term mean, ξ is the conditional standard deviation of the volatility process, and dB_t is another Brownian motion that is correlated with dW_t of (3) with constant correlation factor ρ. The GARCH model assumes that the randomness of the variance process varies with the variance. In this model, a large value of $|a_{t-1}|$, which is an indication of high volatility of a_{t-1}, increases h_t, the volatility of a_t. This model has been utilized in tracking simulations and has been shown to give a suitable representation of the target's maneuver trajectory [20], [21].

From another perspective, the use of Gaussian system noise causes inaccurate estimation to the state when abrupt changes occur. To overcome this problem, we propose the use of a new unimodal heavy-tailed non-Gaussian distribution for the innovation process. The GARCH model is capable of taking into account this characteristic of noise, namely heavy-tailed distribution. Let the target equations of motion be expressed in terms of the GARCH noise q_t as follows:

$$\dot{a}_t = \mu a_t + q_t, \quad q_t = \sqrt{h_t}\, w_t, \tag{6}$$

where $\dot{a}_t$ is the derivative of acceleration a_t, μ is the constant drift of the acceleration a_t, h_t is the stochastic acceleration volatility, and w_t is the white Gaussian noise.

We assume that the target moves in a two-dimensional plane. Dynamic equations of the target motion can now be expressed in terms of the white noise $u(t)$ as follows:

$$\begin{aligned}\dot{\mathbf{X}}(t) &= \mathbf{F}(t, \mathbf{X}(t))\mathbf{X}(t) + \mathbf{G}\mathbf{q}(t),\\ \mathbf{q}(t) &= \sqrt{h(t)}\mathbf{u}(t),\end{aligned} \tag{7}$$

where t shows continuous time index, $\mathbf{X}(t)=\left[\mathbf{x}(t)^T\, \mathbf{v}(t)^T\, \mathbf{a}(t)^T\right]^T$ denotes a six-dimensional position-velocity-acceleration parameter vector wherein $\mathbf{x}(t) = [x(t)\ y(t)]^T$, $\mathbf{v}(t)=[v_x(t)\ v_y(t)]^T$, and $\mathbf{a}(t) = [a_x(t)\ a_y(t)]^T$ are the position, the velocity, and the acceleration of the target, respectively. In (7), $\mathbf{F}$ is a state transition matrix, as below:

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & -\mu & 0\\ 0 & 0 & 0 & 0 & 0 & -\mu \end{bmatrix} \tag{8}$$

and $\mathbf{G} = [0\ 0; 0\ 0; 0\ 0; 0\ 0; 1\ 0; 0\ 1]^T$ is a matrix for addition of system noise and $\mathbf{u}(t) = [u_x(t)\ u_y(t)]^T$ is the system noise vector.

3. Discrete Time Equations of Motion

Since the volatility is independent of states in (7), the SDE of the target is linear in the narrow sense [27], [28]. Therefore, $\mathbf{X}$ can be expressed as the solution of the following stochastic integral equation [27], [28]:

$$\mathbf{X}(t) = \exp(\mathbf{F}t)\mathbf{X}(0) + \int_0^t \exp(\mathbf{F}(t-s))\mathbf{G}\sqrt{h(t)}dW_s, \tag{9}$$

where the integral is called an Ito integral and

$$\exp(\mathbf{F}t) = \sum_{k=0}^{\infty} \frac{\mathbf{F}^k t^k}{k\,!}. \tag{10}$$

These assertions formally follow the standard formulas in ordinary differential equation (ODE) theory if we write $\sqrt{h(t)}dW = \sqrt{h(t)}\nu dt$, consider ν as the usual denoting white noise, and regard $\sqrt{h(t)}\nu$ as an inhomogeneous term driving the ODE:

$$\dot{\mathbf{X}}(t) = \mathbf{F}\mathbf{X}(t) + \mathbf{G}\sqrt{h(t)}\nu. \tag{11}$$

While there are a number of discretization schemes available, we focus on the simplest and perhaps most common scheme, the Euler scheme. Therefore, an approximate explicit solution can be given by the exponential Euler (E-Euler) algorithm, as in [29] and as follows.

Express the exact evolution of $\mathbf{X}$ in terms of $\mathbf{\Phi}(t) = e^{\mathbf{F}t}$:

$$\mathbf{X}(t) = \mathbf{\Phi}(t)\left(\mathbf{X}_0 + \int_0^t \mathbf{\Phi}^{-1}(\tau)\mathbf{f}(\tau)d\tau\right), \tag{12}$$

where $\mathbf{f}(\tau) = \mathbf{G}\sqrt{h(\tau)}\nu$. Change variables $\mathbf{\Phi}^{-1}d\tau = \mathbf{F}^{-1}d\mathbf{\Phi}$:

$$\mathbf{X}(t) = \mathbf{\Phi}(t)\left(\mathbf{X}_0 + \mathbf{F}^{-1}\int_{\mathbf{I}}^{\mathbf{\Phi}(t)} \mathbf{f}(\tau)d\mathbf{\Phi}\right). \tag{13}$$

The discretization is now with respect to $\mathbf{\Phi}$ instead of τ. Using the rectangular approximation of the integral, the E-Euler output is

$$\mathbf{X}_{i+1} = \mathbf{\Phi}_{i+1}\left[\mathbf{X}_i + \mathbf{F}^{-1}\left(\mathbf{\Phi}^{-1}_{i+1} - \mathbf{I}\right)\mathbf{f}_i\right], \tag{14}$$

where $\mathbf{\Phi}_{i+1} = \mathbf{\Phi}(t \mid t = (i+1)T)$ and $\mathbf{f}_i = \mathbf{f}(t \mid t = iT)$. We can simplify (14) as

$$\mathbf{X}_{i+1} = \mathbf{\Phi}_{i+1}\mathbf{X}_i + \frac{\mathbf{I} - \mathbf{\Phi}_{i+1}}{\mathbf{F}}\mathbf{f}_i\,. \tag{15}$$

According to (14), the appropriate discrete time target equations of motions are given by

$$\mathbf{X}_{i+1} = \mathbf{\Phi}(T,\mu)\mathbf{X}_i + \mathbf{\Gamma}(T,\mu)\mathbf{q}_i, \quad \mathbf{q}_i = \sqrt{\mathbf{h}_i}\, z_i, \tag{16}$$

where $z_i \sim \mathcal{N}(0,1)$. The target state transition matrix, $\mathbf{\Phi}$, can be approximated using (9) as follows:

$$\mathbf{\Phi} = \begin{bmatrix} 1 & 0 & T & 0 & \varphi_1 & 0\\ 0 & 1 & 0 & T & 0 & \varphi_1\\ 0 & 0 & 1 & 0 & \varphi_2 & 0\\ 0 & 0 & 0 & 1 & 0 & \varphi_2\\ 0 & 0 & 0 & 0 & e^{-\mu T} & 0\\ 0 & 0 & 0 & 0 & 0 & e^{-\mu T} \end{bmatrix} \tag{17}$$

and, by applying the discretization shown in (14), we have

$$\mathbf{\Gamma} = \left[\gamma_1\mathbf{I} \quad \gamma_2\mathbf{I} \quad \gamma_3\mathbf{I}\right]^T, \tag{18}$$

where $\mathbf{I}$ is a 2×2 identity matrix and

$$\gamma_1 = \frac{1}{\mu}\left(\frac{T^2}{2} - \varphi_1\right),$$

$$\varphi_1 = \gamma_2 = \frac{1}{\mu}\left(T - \varphi_2\right),$$

$$\varphi_2 = \gamma_3 = \frac{1}{\mu}\left(1 - e^{-\mu T}\right).$$

Represented by the downward arrows in Fig. 1, the derived state (15) is directly suitable for modified Kalman filter applications. However, the discretized SV, h_i, is unknown in this adaptive equation. In the proposed method, the SV parameter can be estimated by the GARCH model in (5). Therefore, the forward Euler discretization [29] can be used to approximate the variance differential (5) on a discrete time grid. Let $\left[0 = t_0 < t_1 < \cdots < t_M = T\right]$ be a partition of time interval into M equal segments of length Δt, that is, $t_i = iT/M$. The discretization for the variance process is

$$h_i = \alpha_0 + \alpha_1 h_{i-1} + \beta_1 q_{i-1}^2, \tag{19}$$

where

$$\begin{aligned} \alpha_0 &= \theta\omega\left(T/M\right), \\ \alpha_1 &= 1 - \theta\left(T/M\right), \\ \beta_1 &= \delta\sqrt{T/M} \end{aligned} \tag{20}$$

are constant values, that is, the ARCH parameter and GARCH parameter of the stochastic proposed variance in (2), respectively.

III. Modified Kalman Filter Equations

The tracking sensor measures target location (range, bearing, or elevation) and provides the following output equation:

$$\mathbf{y}(k) = \mathbf{HX}(k) + v(k) \quad , \tag{21}$$

where $\mathbf{H} = [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0]$ and $v(k)$ is additive Gaussian noise with zero mean and variance σ_R^2. The adaptive state equation given by (15) is such that the optimal linear filter is identical to the Kalman filter. While the other filters can be used to estimate the target state vector, the Kalman filter provides the best performance in terms of minimizing the mean square error (MSE). Moreover, it can generally be easily implemented. The variance of motion equation noise, h_i, is an unknown parameter; therefore, an estimate of h_i is required to implement the Kalman filter. So, its performance is sensitive to this estimate. The GARCH estimation is proposed here to deal with this problem. We consider the parameter h_i to be a random parameter and propose one sample of it at each step. Thus, our uncertainties for the state equation in a high maneuvering target can be compensated by adding a flexibility to determine h_i. The algorithm starts with a basic estimate of h_i. Here, this parameter will be modified by each sample. This modification of variance during the algorithm results in a better performance, especially in high maneuvering motions.

The model can also be equivalently expressed in probabilistic terms with distributions:

$$\begin{aligned} p(\mathbf{X}_k \mid \mathbf{X}_{k-1}) &\sim \mathcal{N}(\mathbf{\Phi X}_{k-1}, \mathbf{Q}_{k-1}), \\ p(\mathbf{y}_k \mid \mathbf{X}_k) &\sim \mathcal{N}(\mathbf{HX}_k, \boldsymbol{\sigma}_R^2), \end{aligned} \tag{22}$$

where $\mathbf{Q}_k$ can be given as [30], [31]

$$\mathbf{Q}_k = \int_0^T \mathrm{e}^{\mathbf{F}(T-\tau)}\,\mathbf{G}\mathbf{Q}_c\mathbf{G}^T \left[\mathrm{e}^{\mathbf{F}(T-\tau)}\right]^T d\tau \quad , \tag{23}$$

where $\mathbf{Q}_c = 2\mu\mathbf{h}_k$ is the power spectral density shown in (4). In some cases, the $\mathbf{Q}_k$ can be calculated analytically; however, in this case where it is not possible, the matrix can still be calculated efficiently using the following matrix fraction decomposition [30], [31]:

$$\begin{pmatrix} \mathbf{C}_k \\ \mathbf{D}_k \end{pmatrix} = \exp\left\{\begin{pmatrix} \mathbf{F} & \mathbf{G}\mathbf{Q}_c\mathbf{G}^T \\ 0 & -\mathbf{F}^T \end{pmatrix} T\right\}\begin{pmatrix} 0 \\ \mathbf{I} \end{pmatrix}. \tag{24}$$

The matrix $\mathbf{Q}_k$ is then given as

$$\mathbf{Q}_k = \mathbf{C}_k \mathbf{D}_k^{-1}. \tag{25}$$

Subsequently, (24) using (4) is further simplified as

$$\begin{pmatrix} \mathbf{C}_k \\ \mathbf{D}_k \end{pmatrix} = \exp\left\{\begin{pmatrix} \mathbf{F} & \mathbf{G}(2\mu\mathbf{h}_k)\mathbf{G}^T \\ 0 & -\mathbf{F}^T \end{pmatrix} T\right\}\begin{pmatrix} 0 \\ \mathbf{I} \end{pmatrix}. \tag{26}$$

The steps of the proposed algorithm translate to equations as follows:

- Prediction:

$$\begin{aligned} \mathbf{m}_k^- &= \mathbf{\Phi m}_{k-1} \\ \mathbf{p}_k^- &= \mathbf{A}_{k-1}\mathbf{P}_{k-1}\mathbf{A}_{k-1}^T + \mathbf{Q}_{k-1}. \end{aligned} \tag{27}$$

- Update:

$$\begin{aligned} \mathbf{v}_k &= \mathbf{y}_k - \mathbf{H}\mathbf{m}_k^-, \\ \mathbf{S}_k &= \mathbf{H}\mathbf{P}_k^-\mathbf{H}^T + \boldsymbol{\sigma}_R^2, \\ \mathbf{K}_k &= \mathbf{P}_k^-\mathbf{H}^T\mathbf{S}_k^{-1}, \\ \mathbf{m}_k &= \mathbf{m}_k^- + \mathbf{K}_k\mathbf{v}_k, \\ \mathbf{P}_k &= \mathbf{P}_k^- - \mathbf{K}_k\mathbf{S}_k\mathbf{K}_k^T. \end{aligned} \tag{28}$$

- Volatility Estimation:

$$\begin{aligned} &\mathbf{h}_k = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1\mathbf{h}_{k-1} + \boldsymbol{\beta}_1\left[\mathbf{m}_k - \mathbf{\Phi X}_{k-1}\right]^{\{5:6\}}, \\ &\begin{pmatrix} \mathbf{C}_{k-1} \\ \mathbf{D}_{k-1} \end{pmatrix} = \exp\left\{\begin{pmatrix} \mathbf{F} & \mathbf{G}(2\mu\mathbf{h}_k)\mathbf{G}^T \\ 0 & -\mathbf{F}^T \end{pmatrix} T\right\}\begin{pmatrix} 0 \\ \mathbf{I} \end{pmatrix}, \\ &\mathbf{Q}_{k-1} = \mathbf{C}_{k-1}\mathbf{D}_{k-1}^{-1}, \end{aligned} \tag{29}$$

where $\left[\mathbf{m}_k - \mathbf{\Phi X}_{k-1}\right]^{\{5:6\}}$ is a 2×1 dimensional vector that contains the fifth and sixth elements of 6×1 dimensional vector $\left[\mathbf{m}_k - \mathbf{\Phi X}_{k-1}\right]$, and, according to (20), we have $\boldsymbol{\alpha}_0 = \alpha_0[1 \quad 1]^T$, $\boldsymbol{\alpha}_1 = \alpha_1[1 \quad 1]^T$, and $\boldsymbol{\beta}_1 = \beta_1[1 \quad 1]^T$. The

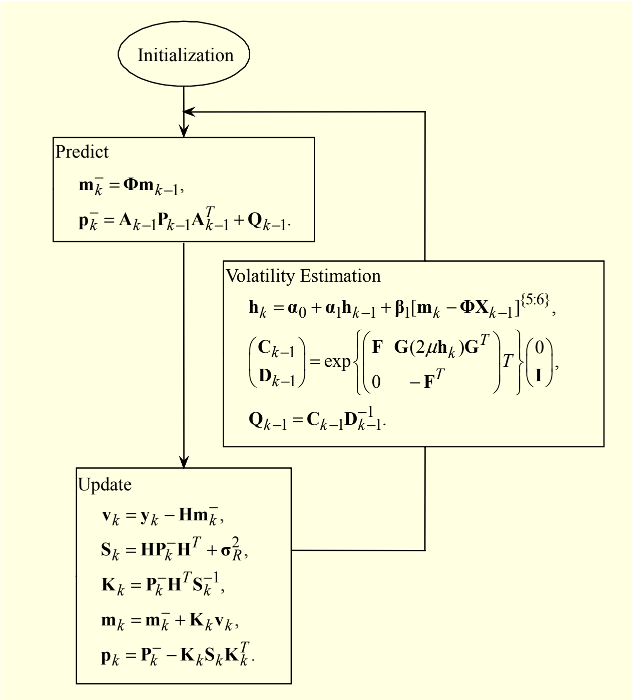

Fig. 1. Functional structure of proposed approach.

Table 1. Average of MSEs of Monte Carlo simulation results for 200 runs for $T = 1$ s.

Maneuver level	Method	Position (m)	Velocity (m/s)	Acceleration (m/s^2)
High	GARCH	30.6302	3.4512	2.6905
	MIE	1886.7431	964.9210	21.7034
	IE	2135.2412	1246.12	26.6521
	Singer	diverge	diverge	diverge

Table 2. Average of MSEs of Monte Carlo simulation results for 200 runs for $T = 0.5$ s.

Maneuver level	State elements	Position (m)	Velocity (m/s)	Acceleration (m/s^2)
High	GARCH	72.0329	4.7312	3.9462
	MIE	940.8795	832.5653	24.7437
	IE	1121.8751	878.6752	27.295
	Singer	diverge	diverge	diverge

flowchart in Fig. 1 illustrates the functional structure of the proposed GARCH method described in (27) through (29).

IV. Experiment Results

In this section, the performances of the proposed GARCH model are evaluated by a Monte Carlo simulation over a represented two-dimensional test trajectory.

1. Scenario I

As the first example, to show the effectiveness of the proposed model, a scenario for tracking a maneuvering target with MM behavior is examined. The statistical parameter values of the observation noise vector for our simulation are chosen as $\sigma_R^2 = \text{diag}\{100^2,\ 100^2,\ 5^2,\ 5^2\}$. In this example, we consider a target initial condition with state $\mathbf{X}(0) = [-10\text{ m} \quad 40\text{ m} \quad -5\text{ m/s} \quad 3\text{ m/s} \quad 0\text{ m/s}^2 \quad 0\text{ m/s}^2]^T$ for $0 \le t \le 50$ s, and the target begins to maneuver as $\mathbf{a}(t) = [0 \quad 3.2g]^T$ for $50\text{ s} < t \le 100$ s, where $g = 9.8\text{ m/s}^2$. Moreover, the target acceleration vector for $100\text{ s} < t \le 200$ s is $\mathbf{a}(t) = [4g \quad 3.2g]^T$. In these simulations, the sampling time is $T = 1$ s and $T = 0.5$ s. The average of the MSEs of the Monte Carlo simulation results for 200 runs for $T = 1$ s is shown in Table 1 and for $T = 0.5$ s is shown in Table 2.

Tables 1 and 2 show that the proposed scheme can significantly improve the results of the state estimation and that it is superior to the MIE method, IE method, and Singer scheme. Since target tracking is a real-time problem, timing is a crucial factor, and many other researchers of IE and MIE have reported degraded performances for fast maneuver detection for high maneuvering targets. The conventional IE techniques based on a constant acceleration assumption have not been successful because the actual value of target acceleration during a typical target maneuver such as the generated path in this simulation in the period of $50\text{ s} < t \le 200$ s is not constant.

Although the MIE performs better than IE, it does not perform better than the proposed method in a high maneuvering situation. As a consequence of the volatility clustering feature of the proposed method, we are not concerned with the maneuver detection algorithm; hence, the performance of the proposed method is greatly improved. Moreover, the Singer model is in essence an *a priori* model since it does not use online information about the target maneuver. We cannot reasonably expect any *a priori* model to have a remarkable effectiveness for the diverse acceleration situations of actual target maneuvers such as MM situations simulated in this simulation. As a consequence of its *a priori* nature, the Singer model is also symmetric in that the assumed ternary-uniform mixture distribution of the acceleration is symmetric. One of the main shortcomings of the Singer model stems from this symmetry; that is, the target acceleration has zero mean at any moment. This condition does not correspond to any real situation, such as Scenario I.

On the other hand, in Tables 1 and 2, if we have access to a

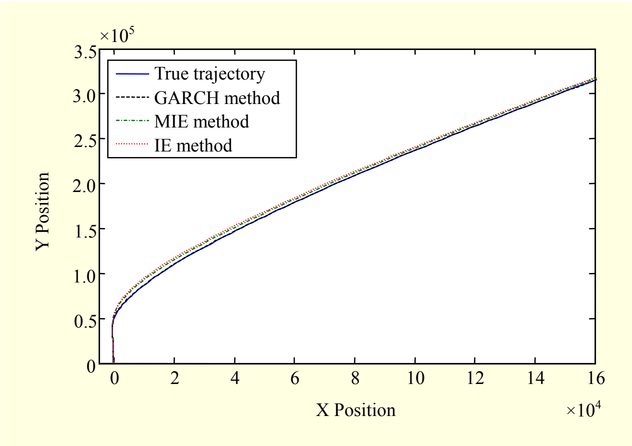

Fig. 2. Trajectory and estimation of high maneuvering target.

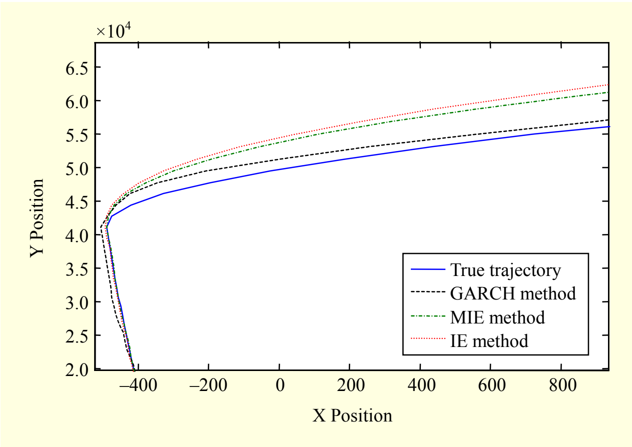

Fig. 3. Selected maneuver interval for target trajectory.

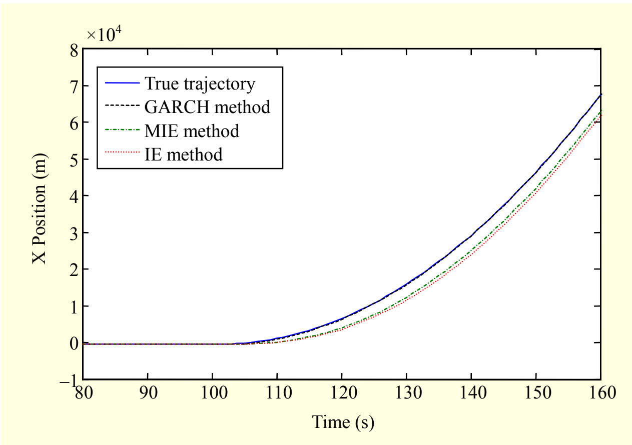

Fig. 4. Selected time interval for target trajectory in X direction.

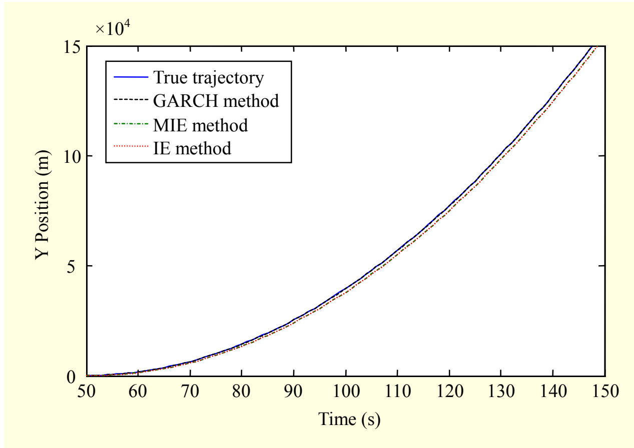

Fig. 5. Selected time interval for target trajectory in Y direction.

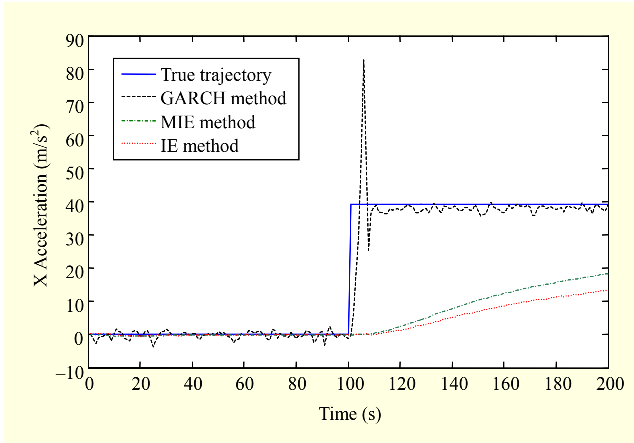

Fig. 6. Actual values and estimated acceleration in X direction.

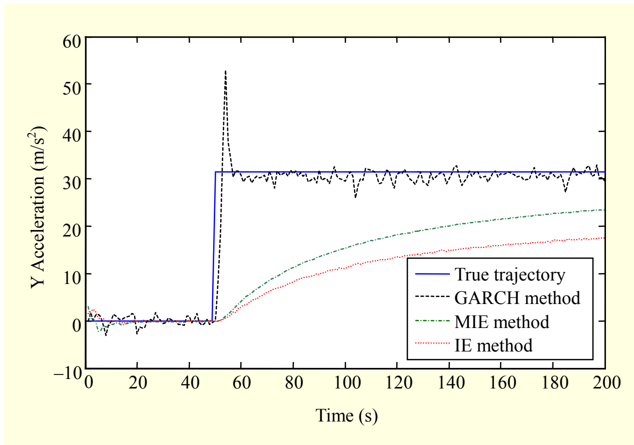

Fig. 7. Actual values and estimated acceleration in Y direction.

large number of samples (that is, sampling time decreases), the trajectory will be in low maneuvering situations. So, in our proposed scheme, by using heavy-tailed distribution modeling, that is, GARCH, instead of traditional Gaussian distribution for the system noise of a state space model, the adaptive model provides fine tracking accuracy for a high maneuvering target (that is, sampling time increases). However, this leads to performance degradation for the traditional methods, such as Singer, IE, and MIE.

In addition, Figs. 2 through 10 are the actual values and the

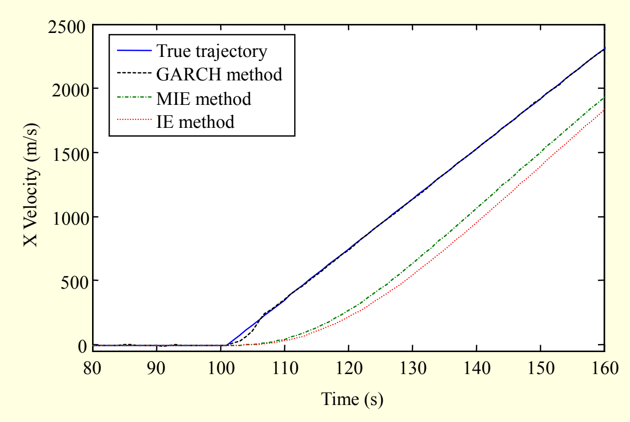

Fig. 8. Selected time interval for actual values and estimated velocity in X direction.

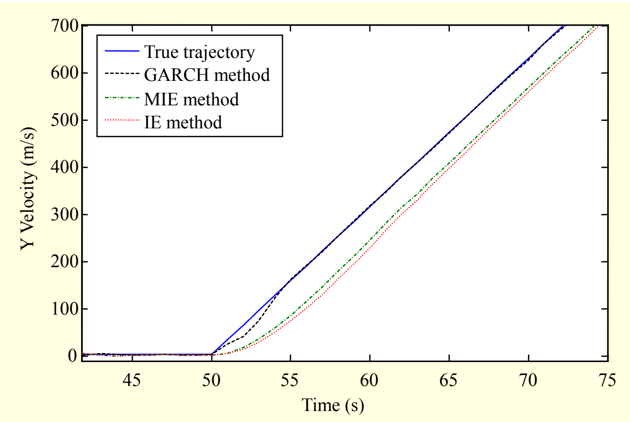

Fig. 9. Selected time interval for actual values and estimated velocity in Y direction.

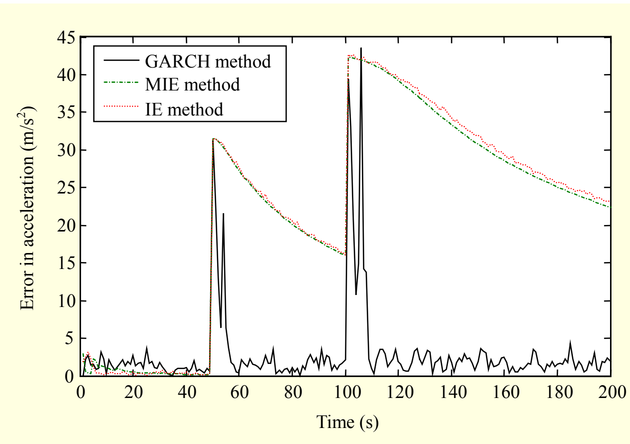

Fig. 10. Error in acceleration.

estimations of trajectory, the actual values and the estimations of $a_x(t)$ and $a_y(t)$, the actual values and the estimations of $v_x(t)$ and $v_y(t)$, and also their corresponding errors by the proposed GARCH method and the MIE method, IE method, and Singer method, respectively.

Table 3. Average of MSEs of Monte Carlo simulation for 200 runs.

Maneuver level	Method	Position (m)	Velocity (m/s)	Acceleration (m/s^2)
Medium	GARCH	28.5563	2.6354	1.1384
	MIE	868.5321	21.3136	2.1755
	IE	1123.742	24.1841	2.6232
	Singer	30. 3957	6.5625	2.7334
Low	GARCH	21.4949	2.5654	1.1047
	MIE	92.4155	3.7368	0.2746
	IE	114.2401	4.0732	1.4131
	Singer	28.0992	5.4113	2.7767

2. Scenario II

Since the performance of the proposed GARCH scheme is presented in a high maneuvering condition as an estimation of the tracking of positions, velocities, and accelerations, a comparison based on the Monte Carlo simulation is also made to evaluate the performances between the proposed GARCH method and the traditional single algorithms, that is, the MIE algorithm, IE method, and Singer method in medium and low maneuvering target conditions. In this scenario, we consider a target with conditions similar to Scenario I with the exception of acceleration. For the medium maneuvering target, the target begins to maneuver in the y direction as $\mathbf{a}(t)=[0 \quad 0.32g]^T$ for $50\text{ s}<t\le 100\text{ s}$. Moreover, the target acceleration vector for $100\text{ s}<t\le 200\text{ s}$ is $\mathbf{a}(t)=[0.4g \quad 0.32g]^T$. Finally, for the low maneuvering target, the target begins to maneuver in the y direction as $\mathbf{a}(t)=[0 \quad 0.032g]^T$ for $50\text{ s}<t\le 100$ s. Moreover, the target acceleration vector for $100\text{ s}<t\le 200$ s is $\mathbf{a}(t)=[0.04g \quad 0.032g]^T$. In these simulations, the sampling time is $T=2$ s. The average of the MSEs of the Monte Carlo simulation results for 200 runs for the medium and low maneuvering target conditions is shown in Table 3.

V. Conclusion

In this paper, we introduced a maneuvering motion model of a target using an SDE whose volatility was subsequently modeled by a GARCH process. This approach allowed us to evaluate the performance of an SDE model based on the GARCH process for tracking a maneuvering target with a significant jump in its acceleration. As shown, by using heavy-tailed distribution modeling, that is, GARCH, instead of traditional Gaussian distribution for system noise of the state space model, the adaptive model provides fine tracking accuracy for a high maneuvering target. Furthermore, we introduced a new type of Kalman filtering to simultaneously estimate the parameters of motion (position, velocity, and acceleration). The simulation results show that the proposed method can model and track abrupt changes in acceleration more accurately. Unlike the previous methods reported in the literature, our proposed strategy does not assume that volatility (variance) is constant. Consequently, the proposed SV has enhanced performance in MTT.

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