Second-Order Statistics of System with Microdiversity and Macrodiversity Reception in Gamma-Shadowed Rician Fading Channels

Milos Bandjur, Nikola Sekulovic, Mihajlo Stefanovic, Aleksandra Golubovic, Petar Spalevic, and Dejan Milic

In this letter, a wireless communication system with microdiversity and macrodiversity reception in gamma-shadowed Rician fading channels is considered. Exact and rapidly converging infinite-series expressions for the average level crossing rate and average fade duration at the output of the system are provided. Numerical results are presented graphically to illustrate the proposed mathematical analysis and to examine the effects of the system's parameters on the quantities considered.

Keywords: Rician fading, gamma shadowing, microdiversity, macrodiversity, level crossing rate, average fade duration.

I. Introduction

In many real life propagation links, short-term fading and long-term fading (shadowing) occur simultaneously, yielding a composite fading environment [1]. Short-term fading is the result of multipath propagation due to reflection, diffraction, and scattering effects. Shadowing is the result of large obstacles and large deviations in the terrain profile between the transmitter and receiver. Various models can be used to describe the fading envelope of the received signal. In the open technical literature, the most frequently used models are the Rayleigh, Rician, Nakagami-*m*, and Weibull. The average power of the received signal is also random when shadowing is concurrently present. A lognormal distribution is generally used to characterize the average power fluctuations. Unfortunately, the use of lognormal distribution to account for shadowing does not lead to a closed-form solution for the probability density function (PDF) of the signal-to-noise ratio (SNR) [2]-[6]. The absence of a closed-form solution for the PDF makes the system analysis very cumbersome. Papers [7], [8] show that, based on theoretical results and measured data, gamma and lognormal distribution match reasonably well. An approach that assumes gamma distribution in modeling of the average signal power leads to a closed-form expression for the PDF of the SNR.

Space diversity, based on using multiple antennas, is a powerful processing technique used to improve the reliability of communication over wireless channels. The microdiversity technique is a technique of signal combining in a single base station, which helps reduce the effects of short-term fading. Diversity that mitigates the effects of shadowing from buildings and objects is called macrodiversity. Macrodiversity is implemented by combining signals received by several base stations or access points [9]. Having in mind that maximal-ratio combining (MRC) is an optimal combining algorithm in the sense that it gives the best performance and that selection combining (SC) is basically a fast response hand-off mechanism that instantaneously or with minimal delay chooses the best base station to provide service to the user [10], microdiversity and macrodiversity reception can be realized using MRC and SC, respectively. Independent fading channels at the micro level can be ensured when the separation between antennas is on the order of one half of a wavelength [9].

Manuscript received Aug. 23, 2012; revised Jan. 29, 2013; accepted Feb. 18, 2013.

Milos Bandjur (phone: +38 128 425 320, milos.bandjur@yahoo.com) and Petar Spalevic (petarspalevic@gmail.com) are with the Department of Electrical and Computer Engineering, Faculty of Technical Sciences, University of Pristina, Kosovska Mitrovica, Serbia.

Nikola Sekulovic (corresponding author, sekulani@gmail.com) is with the Department of Communication Technologies, School of Higher Technical Professional Education, Nis, Serbia.

Mihajlo Stefanovic (mihajlo.stefanovic@elfak.ni.ac.rs), Aleksandra Golubovic (aleksandra321@gmail.com), and Dejan Milic (dejan.milic@elfak.ni.ac.rs) are with the Department of Telecommunications, Faculty of Electrical Engineering, University of Nis, Nis, Serbia.

http://dx.doi.org/10.4218/etrij.13.0212.0370

Unfortunately, shadowing has a larger correlation distance (base stations are likely to be shadowed by the same obstacles), and it is difficult to ensure that base stations operate independently, especially in microcellular systems [11].

Motivated by the results of propagation measurements in microcellular and picocellular systems, the outage probability and average bit error probability (ABEP) of a wireless communication system with microdiversity and dual macrodiversity reception in a composite Rician-gamma environment were analyzed in [8], [12]. However, in certain applications, such as in adaptive transmission, the outage probability and ABEP do not provide enough information for the overall system design and configuration [13]. In that case, the system's second-order statistics should be obtained to reflect the correlation properties of the fading channels and to provide a dynamic representation of the system's performance. The level crossing rate (LCR) and average fade duration (AFD) represent the system's second-order statistics, and they can be used as important performance measures for a proper selection of the adaptive symbol rates, interleaver depth, packet length, and time slot duration. Motivated by the above, we analyze the second-order statistics and present a continuation of the work on first-order statistics described in [8], [12].

II. System and Channel Model

This letter considers a wireless communication system with two L-branch MRC receivers at the micro level and a dual-branch SC receiver at the macro level. The resulting signal at the MRC output of the i-th (i=1, 2) base station is $R_i = \sum_{j=1}^{L} r_{ij}^2$, where r_{ij} is the envelope of the faded signal at the j-th diversity branch of the i-th base station. Assuming that envelopes are statistically independent, in a propagation environment with a dominant line-of-sight component, random variable R_i follows the Rician distribution given by [5]

$$f_{R_i}\left(R_i \middle| \Omega_i\right) = \frac{K+1}{\Omega_i} \exp\left(-\frac{(K+1)R_i}{\Omega_i} - KL \right) \times \left(\frac{(K+1)R_i}{KL\Omega_i} \right)^{\frac{L-1}{2}} I_{L-1}\left(2\sqrt{\frac{KL(K+1)R_i}{\Omega_i}} \right), \quad i = 1,2, \tag{1}$$

where K is the Rician factor defined as the ratio of the signal power in the dominant component over the scattered power, Ω_i represents the average power of the signal per base station branch, and $I_n(\cdot)$ is the modified Bessel function of the first kind and n-th order. The conditional nature of the PDF in (1) reflects the existence of shadowing with Ω_i being a random variable. In this letter, Ω_1 and Ω_2 are correlated and identically gamma distributed with the joint PDF given as [14]

$$f_{\Omega_1\Omega_2}\left(\Omega_1,\Omega_2\right) = \frac{\rho^{-\frac{c-1}{2}}}{\Gamma(c)(1-\rho)\Omega_0^{c+1}} \left(\Omega_1\Omega_2\right)^{\frac{c-1}{2}} \times \exp\left(-\frac{\Omega_1+\Omega_2}{\Omega_0(1-\rho)} \right) I_{c-1}\left(\frac{2\sqrt{\rho\Omega_1\Omega_2}}{\Omega_0(1-\rho)} \right), \tag{2}$$

where Ω_0 is related to the average power of Ω_1 and Ω_2, ρ is the correlation between Ω_1 and Ω_2, and $\Gamma(\cdot)$ is the Gamma function. Variable c represents a measurement of shadowing severity (as the value of c decreases, the shadowing increases). The relationship between the parameter c and standard deviation σ of shadowing in dB in the lognormal shadowing exists through $\sigma(\text{dB}) = 4.3429\sqrt{\psi'(c)}$, where $\psi'(\cdot)$ is the trigamma function [15]. In practice, the typical values of σ are between 2 dB and 12 dB.

The joint PDF of the signal at the output of the i-th base station and its time derivative is [16]

$$f_{R_i\dot{R}_i}\left(R_i,\dot{R}_i \middle| \Omega_i\right) = f_{R_i}\left(R_i \middle| \Omega_i\right) f_{\dot{R}_i}\left(\dot{R}_i\right). \tag{3}$$

The derivative of R_i with respect to time is $\dot{R}_i = 2\sum_{j=1}^{L} r_{ij}\dot{r}_{ij}$, where $\dot{r}_{ij}$ is the time derivative of r_{ij}. For isotropic scattering, $\dot{r}_{ij}$ is a Gaussian distributed random variable with zero mean and variance $\sigma_{\dot{r}_{ij}}^2 = \pi^2 f_m^2 \Omega_i/(K+1)$, with f_m defined as the maximum Doppler frequency [17]. In that case, $\dot{R}_i$ is also a Gaussian distributed random variable with zero mean and variance $\sigma_{\dot{R}_i}^2 = 4R_i\pi^2 f_m^2\Omega_i/(1+K)$.

Selection diversity based on the total input average power is applied at the macro level. Namely, the base station with the larger total input average power is selected to provide service to the user. The joint PDF of the signal and its derivation after diversity combining at the micro and macro levels can be derived as

$$\begin{aligned} f_{R\dot{R}}\left(R,\dot{R}\right) &= \int_0^\infty d\Omega_1 \int_0^{\Omega_1} f_{R_1\dot{R}_1}\left(R,\dot{R} \middle| \Omega_1\right) f_{\Omega_1\Omega_2}\left(\Omega_1,\Omega_2\right)\, d\Omega_2 \\ &\quad + \int_0^\infty d\Omega_2 \int_0^{\Omega_2} f_{R_2\dot{R}_2}\left(R,\dot{R} \middle| \Omega_2\right) f_{\Omega_1\Omega_2}\left(\Omega_1,\Omega_2\right)\, d\Omega_1 \\ &= 2\int_0^\infty f_{R_1\dot{R}_1}\left(R,\dot{R} \middle| \Omega_1\right) d\Omega_1 \int_0^{\Omega_1} f_{\Omega_1\Omega_2}\left(\Omega_1,\Omega_2\right)\, d\Omega_2. \end{aligned} \tag{4}$$

Substituting the adequate joint PDFs and after integrations, analytical expression for the joint PDF of the signal and its derivation can be obtained but is omitted here for brevity.

III. Average LCR and AFD

The average LCR is defined as the number of times per unit duration that the fading process $R(t)$ crosses a given value in a positive (or negative) going direction. This quantity for a given threshold R is given by [18]

$$N_R(R)=\int_0^\infty \dot{R} f_{R\dot{R}}(R,\dot{R})\,d\dot{R}. \quad (5)$$

Introducing the joint PDF of signal and its derivation into (5) and after integration, the final infinite-series expression for the average LCR becomes

$$N_R(R)=2^{\frac{L}{2}-c+\frac{11}{4}}\frac{\exp(-KL)\pi f_m}{\sqrt{2\pi}\Gamma(c)}$$
$$\times\sum_{i_1,i_2,i_3=0}^{\infty}\frac{(K+1)^{\frac{i_1+2i_2+i_3+2c+L-\frac{1}{2}}{2}}(KL)^{i_1}\rho^{i_2}R^{\frac{i_1+2i_2+i_3+2c+L-\frac{1}{2}}{2}}}{2^{\frac{2i_2+i_3-i_1}{2}}i_1!i_2!\Gamma(i_1+L)\Gamma(i_2+c)\prod_{l=0}^{i_3}(i_2+c+l)}$$
$$\times\frac{1}{\Omega_0^{\frac{i_1+2i_2+i_3+2c+L-\frac{1}{2}}{2}}(1-\rho)^{\frac{i_1+2i_2+i_3+L-\frac{1}{2}}{2}}}$$
$$\times K_{i_1-2i_2-i_3-2c+L-\frac{1}{2}}\left(2\sqrt{\frac{2(K+1)R}{\Omega_0(1-\rho)}}\right), \quad (6)$$

where $K_n(\cdot)$ is the n-th order modified Bessel function of the second kind.

The AFD is defined as the average time that signal remains below a specified level R after crossing that level in a downward direction and is given by [18]

$$AFD=F_R(R)\,/\,N_R(R), \quad (7)$$

where $F_R(R)$ is the cumulative distribution function of R derived in [8].

To the best of the authors' knowledge, the presented analytical expressions for second-order statistical quantities are new and are applicable to finite-state Markov modeling of fading channels, analysis of handoff algorithms, and estimation of packet error rates.

The main problem in the infinite-series expressions can be their convergence. Expressions for the average LCR and AFD converge rapidly, and, thus, they can be efficiently used. As an indicative example, the number of terms needed to be summed in the expression for the average LCR to achieve four-significant-figure accuracy is shown in Table 1.

Table 1. Number of terms of (6) required for four-significant-figure accuracy (K=2.1 dB, L=3, Ω_0=0 dB, f_m=1 Hz).

	c (σ) \ R	–20 dB	0 dB	20 dB
ρ=0.2	0.39 (12 dB)	13	14	32
	1.16 (5 dB)	11	12	28
ρ=0.6	0.39 (12 dB)	13	14	41
	1.16 (5 dB)	11	12	44

IV. Numerical Results

On the basis of previous mathematical analysis, in this section, numerical results describing behavior of the second-order statistical measures for various system parameters are graphically presented.

Figures 1 and 2 depict the normalized average LCR (N_R/f_m) versus the normalized envelope level ($R_{th}=R/\Omega_0$). Clearly, LCR increases as the value of the normalized envelope increases, until it reaches the maximum for $R_{th}= R_{th0}$, at which time it starts decreasing.

In Fig. 1, the influence of fading severity and order of microdiversity is represented. The average LCR decreases as the Rician parameter increases. This effect is more visible for lower values of the signal level and a lower number of diversity branches. For the values of $R_{th}<R_{th0}$, the average LCR decreases with an increase in the number of diversity branches at the micro level. Otherwise, the number of crossings increases as the microdiversity order increases.

Figure 2 shows the influence of shadowing severity and the correlation coefficient. For lower values of the signal envelope, the average LCR is higher in an environment under higher shadowing, whereas, for higher values of the signal envelope, the average LCR decreases as shadowing severity increases. The influence of the correlation coefficient on the considered statistical quantity is substantial for small envelope values. Signal fluctuations are more rapid in the case in which separation between base stations is smaller.

Figure 3 presents the normalized AFD ($T_R f_m$) versus the normalized envelope for a different number of diversity branches at the micro level. It is obvious that when an output signal has faded below the determined value, it spends less time below that value if the microdiversity order is higher.

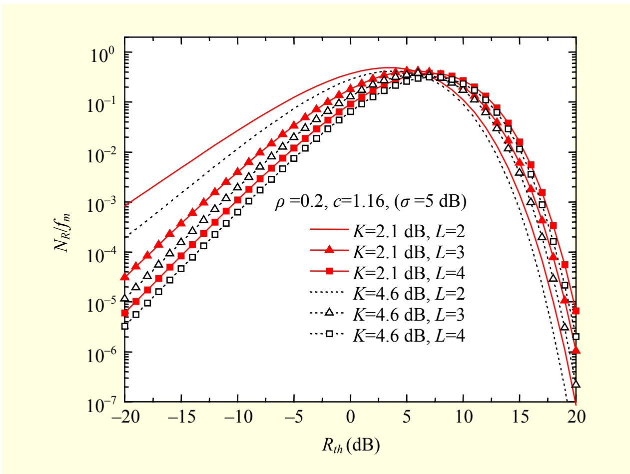

Fig. 1. Normalized average LCR for different values of fading severity and number of diversity branches at micro level.

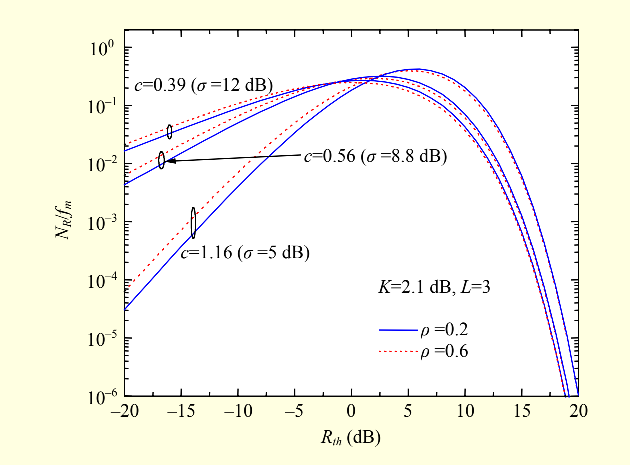

Fig. 2. Normalized average LCR for different values of shadowing severity and correlation coefficient.

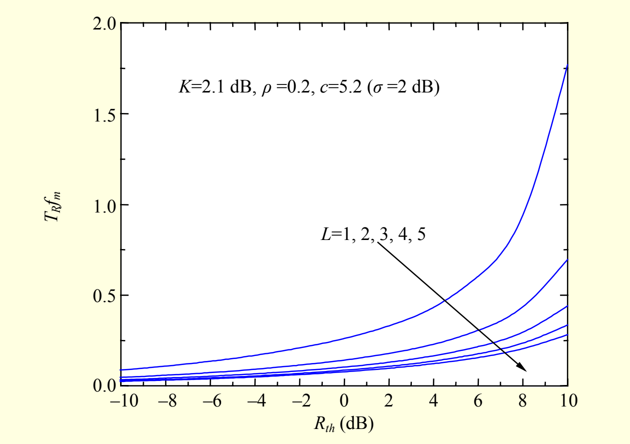

Fig. 3. Normalized AFD for different order of microdiversity.

V. Conclusion

In this letter, the dynamics of the received signal in a system with microdiversity and macrodiversity reception in a composite Rician-gamma fading environment was investigated. Numerical results for the average LCR and AFD were presented graphically to illustrate the effects of the number of diversity branches, the severity of fading and shadowing, and the correlation between base stations on the system performance. Expressions obtained in the letter can be used for system parameter optimization in different propagation conditions.

References

[1] M.K. Simon and M.S. Alouini, *Digital Communication over Fading Channels,* New York: John Wiley & Sons, Inc., 2000.

[2] H. Suzuki, "A Statistical Model for Urban Radio Propagation," *IEEE Trans. Commun.*, vol. 25, no. 7, July 1977, pp. 673-680.

[3] F. Hansen and F.I. Mano, "Mobile Fading-Rayleigh and Lognormal Superimposed," *IEEE Trans. Veh. Technol.*, vol. 26, no. 4, 1977, pp. 332-335.

[4] A.A. Abu-Dayya and N.C. Beaulieu, "Micro- and Macrodiversity NCFSK (DPSK) on Shadowed Nakagami-Fading Channels," *IEEE Trans. Commun.*, vol. 42, no. 9, 1994, pp. 2693-2702.

[5] J. Zhang and V. Aalo, "Effect of Macrodiversity on Average-Error Probabilities in a Rician Fading Channel with Correlated Lognormal Shadowing," *IEEE Trans. Commun.*, vol. 49, no. 1, 2001, pp. 14-18.

[6] A.S. Panajotovic, M.C. Stefanovic, and D.Lj. Draca, "Effect of Microdiversity and Macrodiversity on Average Bit Error Probability in Shadowed Fading Channels in the Presence of Interference," *ETRI J.*, vol. 31, no. 5, Oct. 2009, pp. 500-505.

[7] A. Abdi and M. Kaveh, "On the Utility of Gamma PDF in Modeling Shadow Fading (Slow Fading)," *Proc. 49th IEEE Veh. Technol. Conf.*, vol. 3, Houston, TX, USA, 1999, pp. 2308-2312.

[8] N. Sekulovic and M. Stefanovic, "Performance Analysis of System with Micro- and Macrodiversity Reception in Correlated Gamma Shadowed Rician Fading Channels," *Wireless Personal Commun.*, vol. 65, no. 1, 2012, pp. 143-156.

[9] A. Goldsmith, *Wireless Communications*, New York: Cambridge University Press, 2005.

[10] S. Mukherjee and D. Avidor, "Effect of Microdiversity and Correlated Macrodiversity on Outages in a Cellular System," *IEEE Trans. Wireless Commun.*, vol. 2, no. 1, 2003, pp. 50-58.

[11] V. Graziano, "Propagation Correlations at 900 MHz," *IEEE Trans. Veh. Technol.*, vol. 27, no. 4, 1978, pp. 182-189.

[12] V. Milenkovic et al., "Effect of Microdiversity and Macrodiversity on Average Bit Error Probability in Gamma Shadowed Rician Fading Channels," *ETRI J.*, vol. 32, no. 3, June 2010, pp. 464-467.

[13] L. Yang and M.S. Alouini, "Average Outage Duration of Wireless Communication Systems," *Wireless Communications Systems and Networks*, M. Guizani, Ed., New York: Springer US, 2004, ch. 8.

[14] E. Xekalaki, J. Panaretos, and S. Psarakis, "A Predictive Model Evaluation and Selection Approach – The Correlated Gamma Ratio Distribution," *Stochastic Musings: Perspectives from the Pioneers of the Late 20th Century*, J. Panaretos, Ed., Mahwah, NJ: Lawrence Erlbaum Associates, 2003, pp. 188-202.

[15] P.M. Shankar, "Performance Analysis of Diversity Combining Algorithms in Shadowed Fading Channels," *Wireless Personal Commun.*, vol. 37, no. 1-2, 2006, pp. 61-72.

[16] J.D. Parsons, *The Mobile Radio Propagation Channel*, 2nd ed., England: John Wiley & Sons, Inc., 2000.

[17] X. Dong and N.C. Beaulieu, "Average Level Crossing Rate and Average Fade Duration of Selection Diversity," *IEEE Commun. Lett.*, vol. 5, no. 10, Oct. 2001, pp. 396-398.

[18] S.O. Rice, "Statistical Properties of a Sine Wave Plus Random Noise," *Bell Syst. Tech. J.*, vol. 27, Jan. 1948, pp. 109-157.