Reducing the PAPR of OFDM Systems by Random Variable Transformation

Montadar Abas Taher, Mandeep Jit Singh, Mahamod Bin Ismail, Salina Abdul Samad, and Mohammad Tariqul Islam

Peak power reduction techniques in orthogonal frequency division multiplexing (OFDM) has been an important subject for many researchers for over 20 years. In this letter, we propose a side-information-free technique that is based on the concept of random variable (RV) transformation. The suggested method transforms RVs into other RVs, aiming to reshape the constellation that will consequently produce OFDM symbols with a reduced peak-to-average power ratio. The proposed method has no limitation on the mapping type or the mapping order and has no significant effect on the bit error rate performance compared to other methods presented in the literature. Additionally, the computational complexity does not increase.

Keywords: Peak-to-average power ratio, OFDM, sum of two random variables, transformation of random variables, PAPR, computational complexity reduction.

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is one of the most important techniques in multicarrier modulation systems. It is simple to implement through an inverse/discrete Fourier transform (I/DFT) or its fast version, inverse/fast Fourier transform (I/FFT). OFDM supports high data rates, is almost free of intersymbol interference, and combats multipath fading and frequency-selective fading channels [1]. However, signals generated by multicarrier systems suffer from high output peaks compared with the average amount of power. This problem, referred to as the peak-to-average power ratio (PAPR) or the crest factor, results in the high power amplifier operating in the nonlinear operating region (saturation region), which leads to intermodulation distortion.

To overcome the problem, researchers developed two classes of technique, namely, blind (no side information) and non-blind (transmits some bits of side information). Amplitude clipping and filtering (ACF), tone reservation (TR), tone injection (TI), and active constellation extension (ACE) are all methods belonging to the blind category [1]. Non-blind techniques include selected mapping (SLM) and partial transmit sequences (PTS) [1]. The ACF method is the simplest method but it produces high in-band and out-of-band distortion. The TR, TI, and ACE techniques require more transmission power. The PTS and SLM methods are distortionless, but complex computations are involved, making computational complexity very high.

The constellation reshaping methods were used recently, but they can only be used with M-QAM mapping [2], [3]. The TR [4] needs to increase the transmit power. Dalakas and others introduced a scheme to reduce the PAPR that did not depend on the constellation type (M-QAM/PSK) [5]. Hence, the scheme in [5] reshapes the constellation mapping, but in the time domain (time-domain constellation reshaping [TD-CR]), based on shifting the samples of the OFDM symbol such that the PAPR can be reduced, but the bit error rate (BER) suffers degradation. The study that was recently introduced by Li and others [3] reshaped the constellation points in the frequency domain, but this work can be implemented for the M-QAM mapping only. However, the BER was the cost, as in [5]. Moreover, the computational complexity was increased due to the reshaped M-QAM mapping (R-M-QAM) being combined

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with the SLM scheme or the PTS scheme, that is, more cost was incurred. The R-M-QAM will change the constellation points such that the phase rotation vectors that are needed with the PTS or the SLM will not be sent to the receiver; hence, the technique is completely blind. Motivated by the BER degradation due to the time-domain operations [5] and the increment in the computational complexity [3], in this letter, we propose a blind approach capable of reducing the PAPR without affecting the BER performance. The proposed method, frequency-domain random variable transformation (FD-RVT), likewise supports more reduction in the computational complexity compared to the TD-CR [5] and the rotated SLM (R-SLM) [3] methods. Furthermore, the limitation of the R-SLM is that it can only be used with M-QAM mapping and is therefore not applicable to M-PSK mapping. The key point of the suggested method is that it utilizes the transformation of RVs in the frequency domain to change the constellation points of the M-ary QAM/PSK and thereby reduces the PAPR without limitation with respect to the mapping type (M-ary QAM/PSK), number of subcarriers (N), and the mapping order (M).

II. PAPR Reduction

An OFDM signal consists of N subcarriers modulated by random complex independent symbols drawn from M-ary QAM/PSK constellations and can be written as

$$x(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}X(n)e^{j2\pi\frac{nk}{N}}, \tag{1}$$

where $X(n)$ represents the modulating frequency-domain symbols and $x(k)$ is the output OFDM time-domain symbols. Hence, the sum of statistically independent identically distributed sinusoids generates the OFDM signal. Random symbols are denoted by $X(n)$, among which some may have the same phases. Thus, the summation process in (1) may bring high peaks with respect to the average of the signal. This phenomenon is referred to as the PAPR problem. The mathematical formulation of this value may be written as

$$\xi=\frac{\|x(k)\|_{\infty}}{\frac{1}{uN}\sum_{n=0}^{uN-1}|X(n)|^{2}}, \tag{2}$$

where u is the upsampling factor (u=4 [5]). PAPR performance measurement is handled using the complementary cumulative distribution function (CCDF) of the PAPR. In this work, we propose making use of the principle of an RV transformation to reduce the PAPR. Since $X(n)$ is an RV of mean μ and variance σ^2 and since n=0, 1, … N–1, where N>64, the central limit theorem indicates that $X(n)$ tends to be normally distributed (Gaussian distribution) as μ=0 and unity variance with a probability density function (PDF) expressed as in (3):

$$f=\frac{1}{\sqrt{2\pi\sigma^{2}}}e^{-\frac{(X-\mu)^{2}}{2\sigma^{2}}}. \tag{3}$$

However, an RV can be transformed into any other form by adding or subtracting constants to or from it or by adding two RVs together without affecting the PDF of the RVs. The present work uses a function that sums each consecutive RV to produce an RV. For example, given the RV Z_1, the second RV can be subtracted from the first RV to give another RV, Z_2. Mathematically, $Z_1=X_1+X_2$ and $Z_2=X_1-X_2$. These two new RVs do not change the PDF of the original RVs because the sum of two Gaussian RVs is another Gaussian RV. Then, Z_1 and Z_2 have PDFs written as $f_{Z_1}=1\big/\sqrt{2\pi\sigma_{Z_1}^{2}}\exp\left[-(Z_1)^2/2\sigma_{Z_1}^{2}\right]$ and $f_{Z_2}=1\big/\sqrt{2\pi\sigma_{Z_2}^{2}}\exp\left[-(Z_2)^2/2\sigma_{Z_2}^{2}\right]$, where $\sigma_{Z_1}^2=\sigma_{Z_2}^2=\sigma_1^2+\sigma_2^2$ are their variances. Hence, such transformation can be used as a method to modify constellation mapping. This method will obviously change constellation mapping, and the minimum distance between the constellation points will be changed randomly. Therefore, the BER performance will certainly undergo degradation. However, extensive simulations are conducted, and the transformation is found to reduce the PAPR without significant degradation in BER performance, compared with the TD-CR in [5]. Let $X=[X_n\ X_{n+1}]^{\mathrm{T}}$, and Z_n and Z_{n+1} will be obtained by

$$Z=\begin{bmatrix}Z_n\\ Z_{n+1}\end{bmatrix}=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}X_n\\ X_{n+1}\end{bmatrix}. \tag{4a}$$

In other words, the new vector can be expressed as

$$Z=\left[(X_0+X_1)(X_0-X_1)(X_2+X_3)\right.$$
$$\left.(X_2-X_3)\cdots(X_{N-2}+X_{N-1})(X_{N-2}-X_{N-1})\right]. \tag{4b}$$

The OFDM symbol can then be computed using the conventional I/FFT process by using (4b) as a substitute in (1), as follows:

$$x(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}Z(n)e^{j2\pi\frac{nk}{N}}. \tag{5}$$

Equation (5) is the modified version of (1), but it includes the transformed RV X as Z. Using this technique, the multiplication operations are avoided, unlike in [5]. Moreover, the requirements for the SLM scheme [3] are dropped. Only N of the addition operations are included in (4), plus that of the conventional I/FFT processes, thereby reducing the computational complexity. Figure 1 represents four points selected randomly from a 16-QAM mapping; it shows the locations of the new points after transformation.

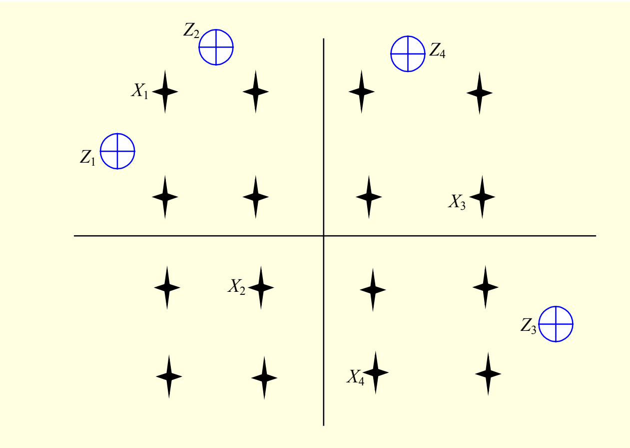

Fig. 1. Constellation transformation sample for four points only selected from 16-QAM.

On the receiver side, the signal can be recovered without side information if we consider perfect channel estimation (for simplicity); the received signal after the FFT operation can be expressed as

$$Z^r = Z + \omega, \tag{6}$$

where ω is the additive white Gaussian noise (AWGN). Then, from (6),

$$Z^r = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} X_n \\ X_{n+1} \end{bmatrix} + \omega. \tag{7}$$

Therefore, the received samples will be

$$X^r = \begin{bmatrix} X_n^r \\ X_{n+1}^r \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} Z_n^r \\ Z_{n+1}^r \end{bmatrix} + \omega. \tag{8}$$

From (8), it is clear that the received signal needs 3 dB more power. That is, the cost for FD-RVT is the additional power required at the transmitter. Moreover, (8) shows that the samples are recovered without side information.

III. Results and Discussion

To conduct a comparison of FD-RVT with the TD-CR of [5], the number of subcarriers is chosen to be 256, with mapping order M=16 for both M-QAM and M-PSK, and the number of randomly generated OFDM symbols is more than 12,000. The signal is passed to the receiver through an AWGN channel. Moreover, the upsampling factor is chosen to be 4 [5], the same value used by Li and others [3]. Furthermore, we choose 1 to be the reduction factor α, which reflects one of the many simulations conducted in [5] and has the same simulation parameters set above. The detection process for the signal is accomplished by the hard decision after the FFT demodulation, as implemented in [3].

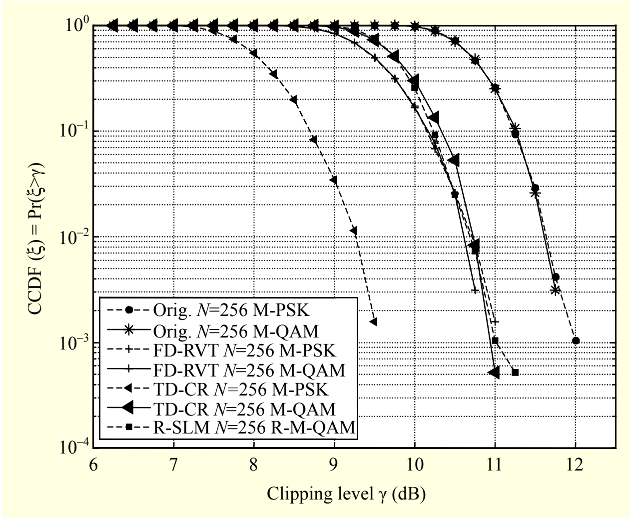

Fig. 2. CCDF of PAPR for conventional OFDM, TD-CR-based OFDM, and FD-RVT-based OFDM systems.

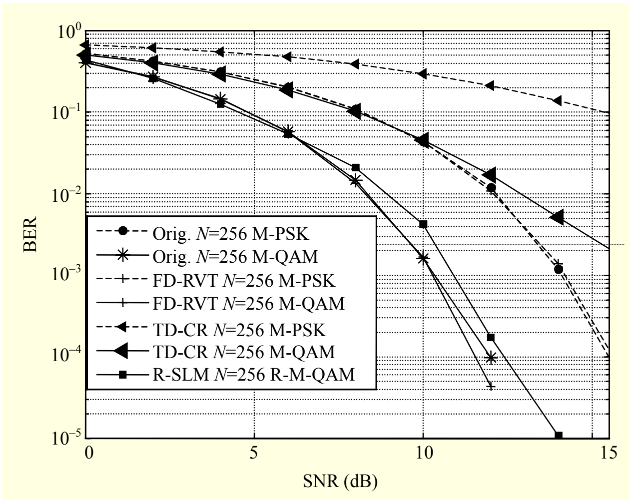

Fig. 3. BER comparison of conventional OFDM system with TD-CR-based OFDM and FD-RVT-based OFDM systems.

Figure 2 shows the CCDF of the PAPR for the three systems: conventional OFDM, TD-CR-based OFDM, and proposed FD-RVT-based OFDM. It is shown that the PAPR of the FD-RVT is reduced around 1 dB for both constellation types, and the PAPR of TD-CR is reduced by 2.25 dB for the 16-PSK modulation mapping and by 1 dB for the 16-QAM mapping. It is clear that the TD-CR outperforms the FD-RVT with respect to the PAPR reduction performance, but the BER performance suffers degradation, as shown in Fig. 3.

Figure 3 shows that the FD-RVT-based OFDM system outperforms the TD-CR-based OFDM system, where the BER of the FD-RVT is barely degraded for both types of constellation mapping, unlike the TD-CR, where the BER is notably degraded. For the 16-QAM mapping, the TD-CR BER performance degradation is less than that for the 16-PSK mapping. Hence, the TD-CR scheme degrades the BER performance of the system, whereas the FD-RVT does not affect the BER performance.

IV. Conclusion

A simple and novel algorithm to reduce PAPR without affecting the BER and with low computational complexity compared with [5] was introduced. FD-RVT is based on the principle of RV transformation, which can easily be implemented in the frequency domain. The PAPR was lowered by approximately 1 dB when N=256 for both families of the constellation mapping (M-QAM/PSK). Compared with TD-CR, the reduction gain was low, but the degradation of the BER performance was negligible. The proposed scheme is seen to be a suitable candidate for PAPR reduction in OFDM systems, without limitations on the number of subcarriers, constellation mapping, and even the constellation type.

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