

# SER-Based Relay Selection for Two-Way Relaying with Physical Layer Network Coding

Dandan Li, Ke Xiong, Zhengding Qiu, and Guanyao Du

*To enhance the symbol error rate (SER) performance of the two-way relay channels with physical layer network coding, this letter proposes a relay selection scheme, in which the relay with the maximal minimum distance between different points in its constellation among all relays is selected to assist two-way transmissions. We give the closed-form expression of minimum distance for binary phase-shift keying and quadrature phase-shift keying. Additionally, we design a low-complexity method for higher-order modulations based on look-up tables. Simulation results show that the proposed scheme improves the SER performance for two-way relay networks.*

*Keywords:* Two-way relay, relay selection, SER, minimum distance, physical layer network coding (PNC).

## I. Introduction

As for network coding, its use at the network layer was proposed in [1]. Later, the authors in [2] and [3] applied it to the physical layer, which is referred to as analog network coding (ANC) and physical layer network coding (PNC). PNC decodes the signal at the relay node, so the received noise at the relay can be removed. Thus, PNC has better performance than ANC [4]. Therefore, we focus on PNC in this letter.

Recently, works on relay selection for two-way relay networks with PNC were published. In [5], the best relay was selected by modifying the well-known selection cooperation,

and, in [6], the best relay was selected by maximizing the bottleneck of the capacity region of both information flows.

In [7], we proposed a relay selection scheme to improve the end-to-end symbol error rate (SER) with binary phase-shift keying (BPSK). Since no closed-form expression of the SER of two-way relay PNC with higher-order modulation has been derived yet, the scheme in [7] cannot be directly extended to higher-order modulations. Considering the fact that higher-order modulations are widely deployed in practical wireless communication systems, especially in high SNR scenarios, and no work has been done on the relay selection scheme with higher-order modulation to improve the SER performance of two-way relay networks, we propose a relay selection scheme to improve the SER performance for higher-order modulations.

In our scheme, the relay node with the maximal minimum distance between different points in the constellation of relays is selected. In fact, this method can also be applied to lower-order modulations. Furthermore, we derive the closed-form expression of the minimum distance for BPSK and quadrature phase-shift keying (QPSK) and design a method to obtain the minimum distance for other modulations that is based on look-up tables. Simulation results show that the proposed scheme can improve the SER performance for two-way relay networks.

## II. System Model

Consider a two-way relay system, where A and B exchange their information via  $K$  relays  $R_1, R_2, \dots, R_K$ . It is assumed that each node is equipped with a single antenna and satisfies the half-duplex constraint. Therefore, two phases, that is, the multiple access (MA) phase and the broadcasting (BC) phase, are required to complete a round of information exchange. In the MA phase, A and B transmit their information to the  $K$

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relays simultaneously. It is assumed that all channels experience block fading so that the channel coefficients remain unchanged during the two phases. Let  $S_A$  and  $S_B$  denote the source data from A and B; let  $\mathcal{M}(S_A)$  and  $\mathcal{M}(S_B)$  denote the modulated signals with  $E(|\mathcal{M}(S_A)|^2) = E(|\mathcal{M}(S_B)|^2) = E_S$ . Then, the received signal at the  $k$ -th relay  $R_k$  can be written as

$$y_k = f_k \mathcal{M}(S_A) + g_k \mathcal{M}(S_B) + n_k, \quad (1)$$

where  $n_k$  is additive white Gaussian noise (AWGN) with a variance of  $N_0$  and  $f_k$  and  $g_k$  denote the channel coefficients between A and  $R_k$  and between B and  $R_k$ , respectively. These fading coefficients are assumed to be independent circular symmetric complex Gaussian random variables with zero mean and unit variance. In the BC phase, only one relay is selected to help the information exchange between A and B. Without loss of generality, if the selected relay is  $R_i$ ,  $R_i$  will detect the received signal according to maximum likelihood (ML) detection as

$$(\widehat{S}_A, \widehat{S}_B) = \arg \min_{(S_1, S_2) \in Z_M \times Z_M} |y_i - (f_i \mathcal{M}(S_1) + g_i \mathcal{M}(S_2))|^2, \quad (2)$$

where  $\widehat{S}_A$  and  $\widehat{S}_B$  are the estimated signals of  $S_A$  and  $S_B$ , respectively. After that,  $R_i$  encodes  $(\widehat{S}_A, \widehat{S}_B)$  by a bitwise XOR operation and broadcasts a coded signal to A and B. Let  $n_A$  and  $n_B$  be the AWGN at A and B, respectively. Then, the received signal at A and B can be given by

$$y_A = f_i \mathcal{M}(\widehat{S}_A \oplus \widehat{S}_B) + n_A, \quad y_B = g_i \mathcal{M}(\widehat{S}_A \oplus \widehat{S}_B) + n_B, \quad (3)$$

where  $E(|\mathcal{M}(\widehat{S}_A \oplus \widehat{S}_B)|^2) = E_S$ . Since A knows its own signal  $S_A$ , it can decode the desired signal  $S_B$  by ML detection as

$$\widehat{S}_B = \arg \min_{S \in Z_M} |y_A - f_i \mathcal{M}(S_A \oplus S)|^2, \quad (4)$$

where  $\widehat{S}_B$  denotes the detected signal of  $S_B$ . In a similar way, B decodes the desired signal  $\widehat{S}_A$ .

The SER performance for the two-way relay system is decided by the decoding performance at the A, B, and relay. In terms of (2) and (4), one can derive that the decoding performance has close relationships with  $f_i$  and  $g_i$ . Furthermore, different  $R_i$  are associated with different  $f_i$  and  $g_i$ . Therefore, the SER performance can be improved by selecting the best relay.

### III. Relay Selection Scheme

In this section, we discuss the relay selection scheme to improve the end-to-end SER performance for two-way relay networks with PNC. No closed-form expressions for SER have been derived for two-way relay PNC with higher-order modulations. Nevertheless, at the high SNR region, the SER expression can be approximated to be  $N_{\min} Q(d_{\min} \sqrt{1/2N_0})$  [8], where  $d_{\min}$  is the shortest distance between different points

in the constellation and  $N_{\min}$  is the number of  $d_{\min}$ . Due to the characteristic of the Q-function, it can be derived that SER is mainly dominated by  $d_{\min}$ . Therefore, we propose a relay selection scheme in which the minimum distance between the different points of the constellation at relays is maximized.

Let  $d_{\min}^{(R_k)}$  denote the minimum distance of the constellation at relay  $R_k$ . Mathematically, the proposed relay selection scheme can be given by

$$i = \arg \max_{k=1 \dots K} (d_{\min}^{(R_k)}). \quad (5)$$

Since the position of the transmitted signal pair  $(S_A, S_B)$  in the constellation of  $R_k$  is  $f_k \mathcal{M}(S_A) + g_k \mathcal{M}(S_B)$ , the distance between transmitted signal  $(S_A, S_B)$  and its candidate signal  $(\widehat{S}_A, \widehat{S}_B)$  can be given by

$$d_{(S_A, S_B) - (\widehat{S}_A, \widehat{S}_B)} = |f_k (\mathcal{M}(S_A) - \mathcal{M}(\widehat{S}_A)) + g_k (\mathcal{M}(S_B) - \mathcal{M}(\widehat{S}_B))|. \quad (6)$$

A transmission error occurs when  $\text{XOR}(S_A, S_B) \neq \text{XOR}(\widehat{S}_A, \widehat{S}_B)$ , so the minimum distance between different points of the constellation at relay  $R_k$  can be given by

$$d_{\min}^{(R_k)} = \min_{\text{XOR}(S_A, S_B) \neq \text{XOR}(\widehat{S}_A, \widehat{S}_B)} d_{(S_A, S_B) - (\widehat{S}_A, \widehat{S}_B)}. \quad (7)$$

For different modulations,  $d_{\min}^{(R_k)}$  has different expressions.

#### 1. BPSK

For BPSK modulation, the transmitted signals  $S_A, S_B \in \{0, 1\}$  and the modulated signals  $\mathcal{M}(S_A), \mathcal{M}(S_B) \in \{-1, 1\}$ . Consequently, the possible values of  $f_k \mathcal{M}(S_A) + g_k \mathcal{M}(S_B)$  are  $-f_k - g_k, -f_k + g_k, f_k - g_k,$  and  $f_k + g_k$ . We plot the four values into the constellation of  $R_k$ , which is shown in Fig. 1. It can be seen that the constellation is distorted and varies according to carrier phase offset and the amplitude variation of  $|f_k|$  and  $|g_k|$ . In terms of (7), our goal is to find the minimum distance between the pairs of points in the constellation with different XOR results. Since  $\text{XOR}(0, 0) = \text{XOR}(1, 1)$  and  $\text{XOR}(0, 1) = \text{XOR}(1, 0)$ , the distances between point (0, 0) and point (1, 1) and between point (0,1) and point (1,0) do not need to be considered. The distance between point (1, 0) and point (0, 0) is  $|f_k - g_k - (-f_k - g_k)| = 2|f_k|$ . Similarly, the distances of the other three segments are  $2|f_k|, 2|g_k|,$  and  $2|g_k|$ , respectively. Thus, for BPSK modulation, the minimum distance at  $R_k$  can be given by

$$d_{\min}^{(R_k)} = \min(2|f_k|, 2|g_k|). \quad (8)$$

By using (8) as a substitute in (7), the relay selection scheme for BPSK can be further simplified as

$$i = \arg \max_{k=1 \dots K} (\min(|f_k|, |g_k|)). \quad (9)$$

Note that (9) is the same as (7) in [7]. This means that although

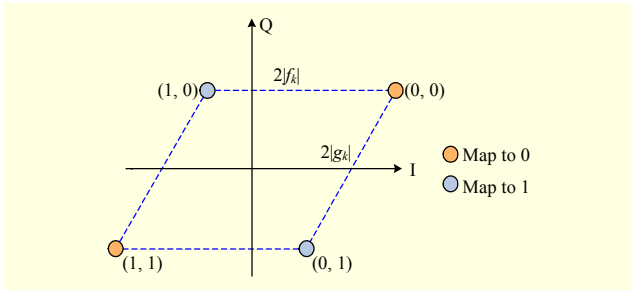


Fig. 1. Constellation at relay for BPSK.

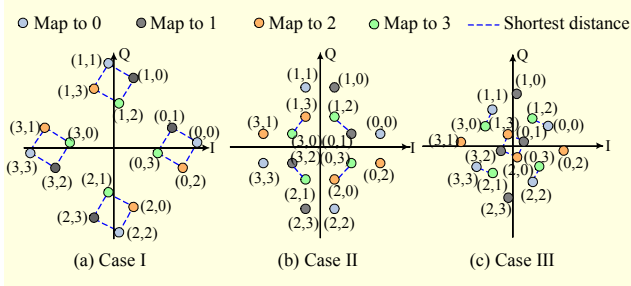


Fig. 2. Constellation at relay for QPSK.

the scheme is proposed for higher-order modulation, it can also be applied to BPSK modulation.

## 2. QPSK

For QPSK modulation, the original signals  $S_A, S_B \in \{00, 01, 11, 10\}$ ; therefore, the modulated signals  $\mathcal{M}(S_A), \mathcal{M}(S_B) \in \{1, i, -1, -i\}$ . Hence, there are 16 possible values for  $f_k \mathcal{M}(S_A) + g_k \mathcal{M}(S_B)$ , and the constellation at  $R_k$  contains 16 points. Let  $\theta$  denote the carrier-phase offset between  $f_k$  and  $g_k$ . Without loss of generality, we assume  $|f_k|$  is no less than  $|g_k|$  and  $\theta$  is between 0 and  $\pi/2$ . With the change of  $|f_k|$ ,  $|g_k|$ , and  $\theta$ , the possible constellations are as shown in Fig. 2, and we can derive that the shortest distances in Figs. 2(a), 2(b), and 2(c) are  $d_1 = \sqrt{2} |g_k|$ ,

$$d_2 = \sqrt{2 |f_k|^2 + 4 |g_k|^2 - 4 |f_k g_k| (\sin \theta + \cos \theta)}, \quad \text{and}$$

$$d_3 = \sqrt{2 |f_k|^2 + 2 |g_k|^2 - 4 |f_k g_k| \sin \theta}, \quad \text{respectively.}$$

Let  $x = |g_k| \cos \theta / |f_k|$ ,  $y = |g_k| \sin \theta / |f_k|$ . Through calculation, we find that  $d_1$  is less than  $d_2$  and  $d_3$  when  $y \leq 0.5$  and  $(x-1)^2 + (y-1)^2 \geq 1$  and that  $d_3$  is less than  $d_1$  and  $d_2$  when  $(x-1)^2 + y^2 > 1$  and  $y > 0.5$ . Therefore,  $d_{\min}^{(R_k)}$  can be given by

$$d_{\min}^{(R_k)} = \begin{cases} d_1 & y \leq 0.5 \ \& \ (x-1)^2 + (y-1)^2 \geq 1 \\ d_2 & (x-1)^2 + (y-1)^2 < 1 \ \& \ (x-1)^2 + y^2 \leq 1. \\ d_3 & (x-1)^2 + y^2 > 1 \ \& \ y > 0.5 \end{cases} \quad (10)$$

Figure 3 plots the distribution of  $d_{\min}^{(R_k)}$  in the  $xy$  plane. If  $(x, y)$  falls in the area of I,  $d_{\min}^{(R_k)} = d_1$ , if  $(x, y)$  falls in the

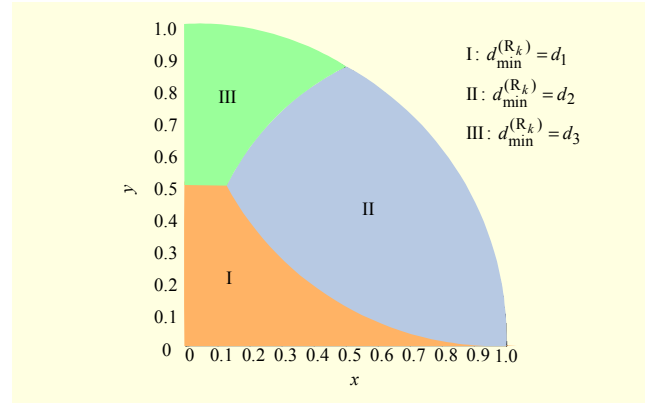


Fig. 3. Distribution of  $d_{\min}^{(R_k)}$  in  $xy$  plane for QPSK modulation, where  $x = |g_k| \cos \theta / |f_k|$  and  $y = |g_k| \sin \theta / |f_k|$ .

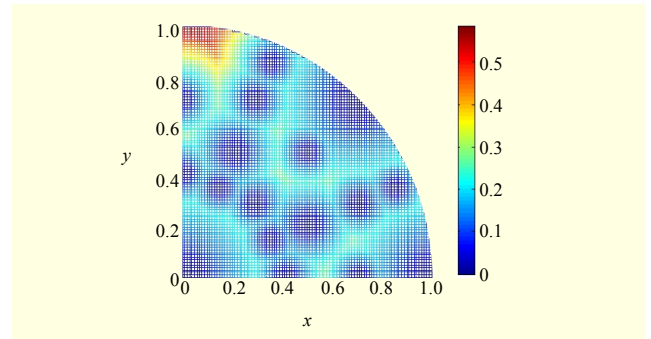


Fig. 4. Look-up table for 8PSK modulation.

area of II,  $d_{\min}^{(R_k)} = d_2$ , and if  $(x, y)$  falls in the area of III,  $d_{\min}^{(R_k)} = d_3$ .

## 3. Other Higher-Order Modulations

For other higher-order modulations, such as 8PSK, 16QAM, 64PSK, and 64QAM, the constellation becomes more complex and the expression of  $d_{\min}^{(R_k)}$  is difficult to obtain. If an exhaustive search is applied, to obtain  $d_{\min}^{(R_k)}$ , the computational complexity is  $O(M^2)$ , where  $M$  is the modulation order. For example, for 8PSK, there are 64 points in the constellation of  $R_k$ . Therefore, to find the  $d_{\min}^{(R_k)}$ ,  $C_{64}^2 = 2,016$  possible distances need to be calculated. For 16 QAM,  $C_{256}^2 = 32,640$  possible distances must be calculated. Thus, it costs too much for an exhaustive search to be applied in a real system.

To reduce the computational complexity, we propose a method based on look-up tables for other higher-order modulations, in which we can derive and pre-store a look-up table composed of the minimum distances for the system offline. For example, Fig. 4 shows the look-up table for 8PSK when the norm of the ratio of the two channel coefficients  $r$  is between 0 and 1 and the angle difference  $\theta$  is between 0 and  $\pi/2$ , where  $x = r \cos \theta$ ,  $y = r \sin \theta$ . Therefore, for a given pair of  $f_k$

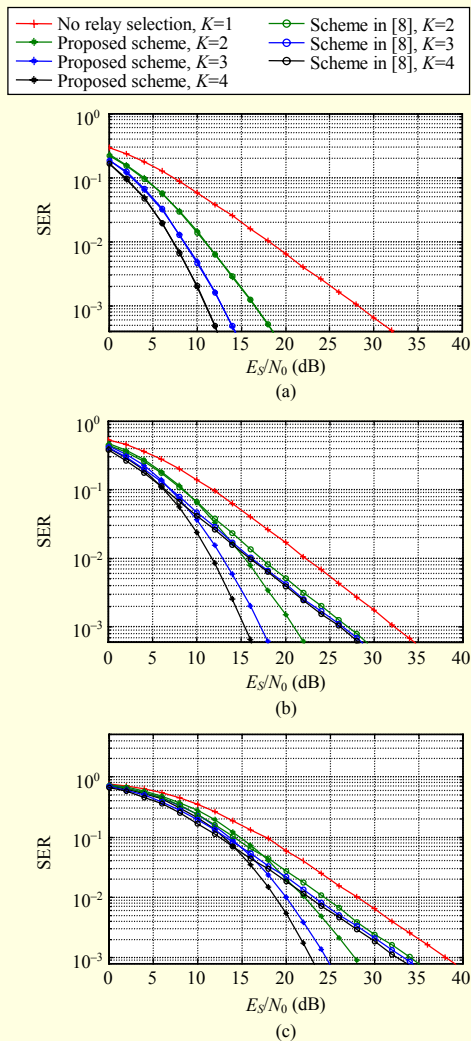


Fig. 5. SER of relay selection for (a) BPSK, (b) QPSK, and (c) 8PSK.

and  $g_k$  ( $|f_k| \geq |g_k|$ ),  $d_{\min}^{(R_k)}$  can be derived by  $|f_k| \times Z(x, y)$ , where  $Z(x, y)$  can be looked up in the pre-stored table. So, the computational complexity of the proposed method is  $O(1)$ . For a given pair of  $(x, y)$ , if we assume the channel coefficient from A to  $R_k$  is 1, then the channel coefficient from B to  $R_k$  is  $x+iy$ . Therefore, by applying the exhaustive search as mentioned previously, the shortest distance  $Z(x, y)$  can be derived. Thus, by calculating the shortest distance for all the possible pairs  $(x, y)$ , we can derive the whole look-up table  $Z(x, y)$ .

#### IV. Simulation Results

In this section, the SER of the proposed scheme is presented using the Monte Carlo simulation. The traditional no-relay selection scheme and the scheme in [6] are also compared.

As shown in Fig. 5(a), when BPSK is applied, our scheme and the scheme in [6] have similar SER performance because

our scheme is actually equivalent to the MinMax scheme. Moreover, with the increase of  $K$ , the diversity gain is achieved for the two schemes. Figure 5(b) shows the SER performance for QPSK. It can be seen that the SER performance of our scheme at a high SNR region is much better than the performance of the scheme proposed in [6]. With the increase of  $K$ , a diversity gain is achieved in our scheme. In contrast, no diversity gain can be obtained by the scheme in [6]. When  $K=4$ , to ensure that the SER is lower than  $10^{-3}$ ,  $E_s/N_0$  must only be 15 dB for our scheme. However, for the scheme in [6],  $E_s/N_0$  must be no less than 28 dB. Figure 5(c) shows the SER performance for 8PSK. Similar to the simulation results of QPSK, the SER performance of our scheme with 8PSK is also much better than that of the scheme in [6].

#### V. Conclusion

We proposed a relay selection scheme to improve the SER performance for two-way relay with PNC at a high SNR region. The scheme can be realized by maximizing the minimum distance between different points in its constellation at relays. We also analyzed the minimum distance for BPSK and QPSK and pointed out that the relay selection can be achieved by a look-up table method for higher-order modulations.

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