

ADDITIVITY OF JORDAN TRIPLE PRODUCT HOMOMORPHISMS ON GENERALIZED MATRIX ALGEBRAS

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ABSTRACT. In this article, it is proved that under some conditions every bijective Jordan triple product homomorphism from generalized matrix algebras onto rings is additive. As a corollary, we obtain that every bijective Jordan triple product homomorphism from $M_n(\mathcal{A})$ (\mathcal{A} is not necessarily a prime algebra) onto an arbitrary ring \mathcal{R}' is additive.

1. Introduction and statements of the results

Let \mathcal{R} be a commutative ring with identity, \mathcal{A} and \mathcal{B} be two associative algebras over \mathcal{R} . Let \mathcal{M} be an $(\mathcal{A}, \mathcal{B})$ -bimodule and \mathcal{N} a $(\mathcal{B}, \mathcal{A})$ -bimodule. Assume that there are two bimodule homomorphisms $\varphi : \mathcal{M} \otimes_{\mathcal{B}} \mathcal{N} \rightarrow \mathcal{A}$ and $\psi : \mathcal{N} \otimes_{\mathcal{A}} \mathcal{M} \rightarrow \mathcal{B}$ satisfying the associativity conditions: $(MN)M' = M(NM')$ and $(NM)N' = N(MN')$ for all $M, M' \in \mathcal{M}$ and $N, N' \in \mathcal{N}$, where we put $MN = \varphi(M \otimes N)$ and $NM = \psi(N \otimes M)$. A *generalized matrix algebra* $\text{Mat}(\mathcal{A}, \mathcal{M}, \mathcal{N}, \mathcal{B})$ is an associative algebra of the form

$$\text{Mat}(\mathcal{A}, \mathcal{M}, \mathcal{N}, \mathcal{B}) = \left\{ \begin{bmatrix} A & M \\ N & B \end{bmatrix} : A \in \mathcal{A}, M \in \mathcal{M}, N \in \mathcal{N}, B \in \mathcal{B} \right\}$$

under the usual matrix-like multiplication, where at least one of the two bimodules \mathcal{M} and \mathcal{N} is distinct from zero. In the above definition of generalized matrix algebras, if \mathcal{M} is faithful as a left \mathcal{A} -module and also as a right \mathcal{B} -module and $\mathcal{N} = \{0\}$, then the associative \mathcal{R} -algebra

$$\text{Tri}(\mathcal{A}, \mathcal{M}, \mathcal{B}) = \left\{ \begin{bmatrix} A & M \\ 0 & B \end{bmatrix} : A \in \mathcal{A}, M \in \mathcal{M}, B \in \mathcal{B} \right\}$$

is usually called a *triangular algebra*. The most important examples of triangular algebras are upper triangular matrix algebras, block upper triangular matrix algebras and nest algebras. Obviously, the triangular algebras and $M_n(\mathcal{A})$, the

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matrix algebra over \mathcal{A} are kinds of generalized matrix algebras. We refer the reader to [22] for some classical examples of generalized matrix algebras.

Let ϕ be a map from \mathcal{A} to \mathcal{B} and A, B, C be arbitrary elements of \mathcal{A} .

- (1) ϕ is said to be *multiplicative* if

$$\phi(AB) = \phi(A)\phi(B).$$

- (2) ϕ is called a *Jordan map* if

$$\phi(AB + BA) = \phi(A)\phi(B) + \phi(B)\phi(A).$$

- (3) ϕ is called a *Jordan triple map* if

$$\phi(ABC + CBA) = \phi(A)\phi(B)\phi(C) + \phi(C)\phi(B)\phi(A).$$

- (4) ϕ is said to be a *Jordan triple product homomorphism* if

$$\phi(ABA) = \phi(A)\phi(B)\phi(A).$$

In studying preservers on algebras or rings, one usually assumes additivity in advance. Recently, however, a growing number of papers began investigating preservers that are not necessarily additive. Characterizing the interrelation between the multiplicative and additive structures of a ring or algebra is an interesting topic. This question was first studied by Martindale [18] who showed the surprising result that every bijective multiplicative map from a prime ring containing a nontrivial idempotent onto an arbitrary ring is necessarily additive. For operator algebras, the same problem was treated in [1, 14, 21]. In the papers [2, 3, 7, 12, 13, 15, 19], the additivity of maps on operator algebras which are multiplicative with respect to other products, such as the Jordan product, the Jordan triple product or Jordan triple product homomorphisms were investigated. Also, the papers [6, 9, 20] studied the similar questions for elementary maps and Jordan elementary maps on rings or operator algebras.

Ling and Lu [11] studied Jordan maps of nest algebras, a kind of triangular algebras coming from operator theory. They showed that every Jordan bijective map on a standard subalgebra of a nest algebra is additive. This result was extended by Ji [4] to Jordan surjective map pair of triangular algebras. Cheng and Jing [1] proved that every multiplicative bijective map, Jordan bijective map, Jordan triple bijective map and elementary surjective map on triangular algebras is additive. Recently, Li and Xiao [10] extended the results of Ji [4] to generalized matrix algebras. In fact, they proved that every multiplicative bijective map, Jordan bijective map, Jordan triple bijective map on a generalized matrix algebra is additive under some conditions.

For the Jordan triple product homomorphisms, Li and Jing [8] showed that if \mathcal{R} is a 2-torsion free prime ring containing a nontrivial idempotent and \mathcal{R}' is an arbitrary ring, then every bijective Jordan triple product homomorphism is additive. Kuzma [7] described the forms of Jordan triple product homomorphisms on matrix algebras and Molnár [19] obtained the exact forms of Jordan triple product homomorphisms between standard operator algebras.

There is a connection between Jordan triple product homomorphisms and Lie triple product homomorphisms, the \mathcal{R} -linear maps $\phi : \mathcal{A} \rightarrow \mathcal{B}$ such that for every $A, B, C \in \mathcal{A}$, $\phi([[A, B], C]) = [[\phi(A), \phi(B)], \phi(C)]$, where $[A, B] = AB - BA$ is the Lie multiplication. In recent years, there has been some interest in studying Lie triple isomorphisms and Lie triple derivations. We refer the interested readers to [5, 16, 17, 23].

Motivated by the results of [8] and [10], we prove that under some conditions every bijective Jordan triple product homomorphism from generalized matrix algebras onto rings is additive. As a byproduct, we obtain that a bijective Jordan triple product homomorphism from $M_n(\mathcal{A})$ onto an arbitrary ring \mathcal{R}' is additive, where \mathcal{A} is not necessarily a prime algebra. We mention that we use some techniques similar to those of [12].

Throughout this article, let \mathcal{G} be a generalized matrix algebra $\text{Mat}(\mathcal{A}, \mathcal{M}, \mathcal{N}, \mathcal{B})$. For convenience, we set

$$\mathcal{G}_{11} = \left\{ \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} : A \in \mathcal{A} \right\}; \mathcal{G}_{12} = \left\{ \begin{bmatrix} 0 & M \\ 0 & 0 \end{bmatrix} : M \in \mathcal{M} \right\}$$

$$\mathcal{G}_{21} = \left\{ \begin{bmatrix} 0 & 0 \\ N & 0 \end{bmatrix} : N \in \mathcal{N} \right\}; \mathcal{G}_{22} = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & B \end{bmatrix} : B \in \mathcal{B} \right\}.$$

Then every element $A \in \mathcal{G}$ can be written as $A = A_{11} + A_{12} + A_{21} + A_{22}$, where $A_{ij} \in \mathcal{G}_{ij}$ for $1 \leq i, j \leq 2$.

Our main result reads as follows.

Theorem 1.1. *Let \mathcal{R} be a commutative ring with identity, \mathcal{A} and \mathcal{B} be two unital algebras over \mathcal{R} . Let \mathcal{G} be the generalized matrix algebra $\text{Mat}(\mathcal{A}, \mathcal{M}, \mathcal{N}, \mathcal{B})$ which satisfies the following conditions:*

- (1) *for $M \in \mathcal{M}$, $NMN = 0$ for every $N \in \mathcal{N}$ implies $M = 0$,*
- (2) *for $N \in \mathcal{N}$, $MNM = 0$ for every $M \in \mathcal{M}$ implies $N = 0$,*
- (3) *\mathcal{M} is faithful as a left \mathcal{A} -module, that is, for $A \in \mathcal{A}$, $AM = 0$ for every $M \in \mathcal{M}$ implies $A = 0$,*
- (4) *for $B \in \mathcal{B}$, $MBN = 0$ for every $M \in \mathcal{M}$ and $N \in \mathcal{N}$ implies $B = 0$.*

Then every bijective Jordan triple product homomorphism ϕ from \mathcal{G} onto an arbitrary ring \mathcal{R}' is additive.

It is clear $M_n(\mathcal{A})$ satisfies the conditions of Theorem 1 for (not necessarily prime) unital algebras \mathcal{A} . Hence we have the following corollary.

Corollary 1.2. *Let \mathcal{A} be a unital (not necessarily prime) algebra over a commutative ring \mathcal{R} . Then every bijective Jordan triple product homomorphism ϕ from $M_n(\mathcal{A})$ onto an arbitrary ring \mathcal{R}' is additive for $n \geq 2$.*

2. Proofs

We prove the theorem in a series of lemmas.

Lemma 2.1. $\phi(0) = 0$.

Proof. Since ϕ is surjective there exists an element $A \in \mathcal{G}$ such that $\phi(A) = 0$. Then $\phi(0) = \phi(0A0) = \phi(0)\phi(A)\phi(0) = 0$. \square

Lemma 2.2. $\phi(A_{11} + A_{12} + A_{21} + A_{22}) = \phi(A_{11}) + \phi(A_{12}) + \phi(A_{21}) + \phi(A_{22})$ for every $A_{ij} \in \mathcal{G}_{ij}$.

Proof. Let $S \in \mathcal{G}$ be an element such that $\phi(S) = \phi(A_{11}) + \phi(A_{12}) + \phi(A_{21}) + \phi(A_{22})$. Then we have for every $X_{ij}, 1 \leq i, j \leq 2$

$$\begin{aligned} \phi(X_{ij}SX_{ij}) &= \phi(X_{ij})\phi(S)\phi(X_{ij}) \\ &= \phi(X_{ij}) \sum_{l,t} \phi(A_{lt})\phi(X_{ij}) \\ &= \sum_{l,t} \phi(X_{ij}A_{lt}X_{ij}) \\ &= \phi(X_{ij}A_{ji}X_{ij}). \end{aligned}$$

Hence it follows that $X_{ij}SX_{ij} = X_{ij}A_{ji}X_{ij}$, that is, $X_{ij}(S - A_{ji})X_{ij} = 0$. From $X_{ii}(S - A_{ii})X_{ii} = 0$, since \mathcal{A} and \mathcal{B} are unital algebras we have $S_{11} = A_{11}$ and $S_{22} = A_{22}$. From $X_{12}(S - A_{21})X_{12} = 0$, $S_{21} = A_{21}$ by condition (2). Similarly, we have $S_{12} = A_{12}$. \square

Lemma 2.3. ϕ is additive on \mathcal{G}_{12} .

Proof. Let $A_{11} = I_{\mathcal{A}}$, the identity element of \mathcal{A} and $C_{22} = I_{\mathcal{B}}$ be that of \mathcal{B} . Then we have

$$\begin{aligned} \phi(A_{11}) + \phi(A_{12} + B_{12}) &= \phi(A_{11} + A_{12} + B_{12}C_{22}) \text{ by Lemma 2.2} \\ &= \phi((A_{11} + C_{22} + A_{12})(A_{11} + B_{12})(A_{11} + C_{22} + A_{12})) \\ &= \phi(A_{11} + C_{22} + A_{12})\phi(A_{11})\phi(A_{11} + C_{22} + A_{12}) \\ &\quad + \phi(A_{11} + C_{22} + A_{12})\phi(B_{12})\phi(A_{11} + C_{22} + A_{12}) \\ &= \phi(A_{11} + A_{12}) + \phi(A_{11}B_{12}C_{22}) \\ &= \phi(A_{11}) + \phi(A_{12}) + \phi(A_{11}B_{12}) \\ &= \phi(A_{11}) + \phi(A_{12}) + \phi(B_{12}), \end{aligned}$$

from which it follows that $\phi(A_{12} + B_{12}) = \phi(A_{12}) + \phi(B_{12})$. \square

Lemma 2.4. ϕ is additive on \mathcal{G}_{21} .

Proof. Let S be an element of \mathcal{G} such that $\phi(S) = \phi(A_{21}) + \phi(B_{21})$. Then for every X_{ij} , we have

$$(2.1) \quad \begin{aligned} \phi(X_{ij}SX_{ij}) &= \phi(X_{ij})(\phi(A_{21}) + \phi(B_{21}))\phi(X_{ij}) \\ &= \phi(X_{ij}A_{21}X_{ij}) + \phi(X_{ij}B_{21}X_{ij}). \end{aligned}$$

Then by Lemma 2.3, we have

$$\begin{aligned} \phi(X_{12}SX_{12}) &= \phi(X_{12}A_{21}X_{12}) + \phi(X_{12}B_{21}X_{12}) \\ &= \phi(X_{12}A_{21}X_{12} + X_{12}B_{21}X_{12}). \end{aligned}$$

Hence we have

$$X_{12}SX_{12} = X_{12}A_{21}X_{12} + X_{12}B_{21}X_{12}.$$

Then by condition (2), we get $S_{21} = A_{21} + B_{21}$. By (2.1) we have

$$\phi(X_{11}SX_{11}) = \phi(X_{22}SX_{22}) = \phi(X_{21}SX_{21}) = 0.$$

Then it follows that $S_{11} = S_{22} = 0$ and $S_{12} = 0$ by condition (1), and hence

$$\phi(A_{21} + B_{21}) = \phi(S_{21}) = \phi(S) = \phi(A_{21}) + \phi(B_{21}). \quad \square$$

Lemma 2.5. $\phi(A_{11} + B_{11}) = \phi(A_{11}) + \phi(B_{11})$ for every $A_{11}, B_{11} \in \mathcal{G}_{11}$.

Proof. We first claim that for every $C_{11} \in \mathcal{G}_{11}$ and $D_{12} \in \mathcal{G}_{12}$, we have that

$$(2.2) \quad \phi(C_{11}D_{12}) = \phi(P_{11})\phi(C_{11})\phi(D_{12}) + \phi(D_{12})\phi(C_{11})\phi(P_{11}),$$

where P_{11} is the identity element of \mathcal{A} . By Lemma 2.2, we have

$$\begin{aligned} \phi(C_{11}) + \phi(C_{11}D_{12}) &= \phi(C_{11} + C_{11}D_{12}) \\ &= \phi((P_{11} + D_{12})C_{11}(P_{11} + D_{12})) \\ &= \phi(P_{11} + D_{12})\phi(C_{11})\phi(P_{11} + D_{12}) \\ &= (\phi(P_{11}) + \phi(D_{12}))\phi(C_{11})(\phi(P_{11}) + \phi(D_{12})) \\ &= \phi(P_{11})\phi(C_{11})\phi(P_{11}) + \phi(D_{12})\phi(C_{11})\phi(D_{12}) \\ &\quad + \phi(P_{11})\phi(C_{11})\phi(D_{12}) + \phi(D_{12})\phi(C_{11})\phi(P_{11}) \\ &= \phi(C_{11}) + \phi(P_{11})\phi(C_{11})\phi(D_{12}) + \phi(D_{12})\phi(C_{11})\phi(P_{11}), \end{aligned}$$

from which it follows that (2.2) holds.

Now choose $S \in \mathcal{G}$ such that $\phi(S) = \phi(A_{11}) + \phi(B_{11})$. Since for every $X_{ij} \in \mathcal{G}_{ij}$

$$\phi(X_{ij}SX_{ij}) = \phi(X_{ij})\phi(S)\phi(X_{ij}) = \phi(X_{ij}A_{11}X_{ij}) + \phi(X_{ij}B_{11}X_{ij}),$$

we have

$$\phi(X_{12}SX_{12}) = \phi(X_{21}SX_{21}) = \phi(X_{22}SX_{22}) = 0.$$

Hence we get $S_{12} = S_{21} = S_{22} = 0$ by conditions (1) and (2). We also have

$$\begin{aligned} \phi(S_{11}X_{12}) &= \phi(P_{11})\phi(S_{11})\phi(X_{12}) + \phi(X_{12})\phi(S_{11})\phi(P_{11}) \\ &= \phi(P_{11})(\phi(A_{11}) + \phi(B_{11}))\phi(X_{12}) + \phi(X_{12})(\phi(A_{11}) + \phi(B_{11}))\phi(P_{11}) \\ &= \phi(A_{11}X_{12}) + \phi(B_{11}X_{12}) \\ &= \phi(A_{11}X_{12} + B_{11}X_{12}), \end{aligned}$$

where the first and third equations hold by (2.2) and the last equation comes by the additivity of ϕ on \mathcal{G}_{12} . Then we get $S_{11} = A_{11} + B_{11}$ by condition (3), and hence

$$\phi(A_{11} + B_{11}) = \phi(A_{11}) + \phi(B_{11}). \quad \square$$

Lemma 2.6. ϕ is additive on \mathcal{G}_{22} .

Proof. Let A_{22} and B_{22} be elements of \mathcal{G}_{22} and $S \in \mathcal{G}$ be an element such that $\phi(S) = \phi(A_{22}) + \phi(B_{22})$. Since for every $X_{ij} \in \mathcal{G}_{ij}$

$$(2.3) \quad \phi(X_{ij}SX_{ij}) = \phi(X_{ij})\phi(S)\phi(X_{ij}) = \phi(X_{ij}A_{22}X_{ij}) + \phi(X_{ij}B_{22}X_{ij}),$$

we have

$$\phi(X_{11}SX_{11}) = \phi(X_{11})\phi(A_{22})\phi(X_{11}) + \phi(X_{11})\phi(B_{22})\phi(X_{11}) = 0.$$

Hence $X_{11}SX_{11} = 0$ and $X_{11}S_{11}X_{11} = 0$. Then it follows that $S_{11} = 0$. Since $\phi(X_{12}SX_{12}) = 0$ by (2.3), we have $X_{12}SX_{12} = 0$ and hence $S_{21} = 0$. Similarly, $S_{12} = 0$. Hence

$$\phi(S_{22}) = \phi(A_{22}) + \phi(B_{22}).$$

Considering, for every $X_{12}, Y_{21} \in \mathcal{G}_{12}$

$$\begin{aligned} \phi(X_{12}S_{22}Y_{21}) &= \phi((X_{12} + Y_{21})S_{22}(X_{12} + Y_{21})) \\ &= \phi(X_{12} + Y_{21})\phi(S_{22})\phi(X_{12} + Y_{21}) \\ &= \phi(X_{12} + Y_{21})(\phi(A_{22}) + \phi(B_{22}))\phi(X_{12} + Y_{21}) \\ &= \phi((X_{12} + Y_{21})A_{22}(X_{12} + Y_{21})) + \phi((X_{12} + Y_{21})B_{22}(X_{12} + Y_{21})) \\ &= \phi(X_{12}A_{22}Y_{21}) + \phi(X_{12}B_{22}Y_{21}) \\ &= \phi(X_{12}A_{22}Y_{21} + X_{12}B_{22}Y_{21}), \end{aligned}$$

we have

$$X_{12}S_{22}Y_{21} = X_{12}A_{22}Y_{21} + X_{12}B_{22}Y_{21}.$$

Therefore by condition (4), it follows that $S_{22} = A_{22} + B_{22}$, and hence

$$\phi(A_{22} + B_{22}) = \phi(A_{22}) + \phi(B_{22}). \quad \square$$

We now prove our main result.

Proof of Theorem 1.1. Let $A = A_{11} + A_{12} + A_{21} + A_{22}$ and $B = B_{11} + B_{12} + B_{21} + B_{22}$ be elements of \mathcal{G} . Then by Lemmas 2.2, 2.3, 2.4, 2.5, and 2.6, we have

$$\begin{aligned} \phi(A + B) &= \phi(A_{11} + B_{11} + A_{12} + B_{12} + A_{21} + B_{21} + A_{22} + B_{22}) \\ &= \phi(A_{11} + B_{11}) + \phi(A_{12} + B_{12}) + \phi(A_{21} + B_{21}) + \phi(A_{22} + B_{22}) \\ &= \phi(A_{11}) + \phi(B_{11}) + \phi(A_{12}) + \phi(B_{12}) \\ &\quad + \phi(A_{21}) + \phi(B_{21}) + \phi(A_{22}) + \phi(B_{22}) \\ &= \phi(A_{11}) + \phi(A_{12}) + \phi(A_{21}) + \phi(A_{22}) \\ &\quad + \phi(B_{11}) + \phi(B_{12}) + \phi(B_{21}) + \phi(B_{22}) \\ &= \phi(A) + \phi(B). \end{aligned}$$

This completes the proof. □

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