# GLOBAL ASYMPTOTIC STABILITY OF POSITIVE STEADY STATES OF AN $n$-DIMENSIONAL RATIO-DEPENDENT PREDATOR-PREY SYSTEM WITH DIFFUSION

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ABSTRACT. The main concern of this paper is to study the dynamics of an $n$-dimensional ratio-dependent predator-prey system with diffusion. We study the dissipativeness, persistence of the system and it is shown that the unique positive constant steady state is globally asymptotically stable under some assumptions.

## 1. Introduction

There is a growing biological and physiological evidence [1, 3, 4, 7] that in some situations, especially when the predator has to search for food and therefore has to share or compete for food, a more suitable general predator-prey theory should be based on the so-called ratio-dependent theory, which can be roughly stated as the per capita predator growth rate being a function of ratio of prey to predator abundance. Denote by $N(t)$ and $P(t)$ the population densities of prey and predator at time $t$, respectively. Then the ratio-dependent type predator-prey model with Michaelis-Menten type functional response is given as follows (after nondimensionalization) [2]:

$$\begin{cases} \dfrac{dN}{dt} = N(1-kN) - \dfrac{\alpha NP}{N+\alpha P}, & t > 0, \\ \dfrac{dP}{dt} = -aP + \dfrac{\alpha NP}{N+\alpha P}, & t > 0, \end{cases}$$

where $k, \alpha$, and $a$ are positive constants.

Consider the following $n$-dimensional ratio-dependent ecological system, in which $n$ different predator species (the $i$-th predator density at time $t$ is denoted

by $P_i(t)$, $i = 1, 2, \ldots, n$, respectively) are competing with a single prey species (the density of prey at time $t$ is denoted by $N(t)$):

$$\begin{cases} \dfrac{dN}{dt} = N(1 - kN) - \dfrac{N(\sum_{i=1}^{n} \alpha_i P_i)}{N + \sum_{i=1}^{n} \alpha_i P_i}, & t > 0, \\ \dfrac{dP_i}{dt} = -a_i P_i + \dfrac{\alpha_i N P_i}{N + \alpha_i P_i}, & t > 0, \ i = 1, 2, \ldots, n, \end{cases}$$

where $k, \alpha_i, a_i$ $(i = 1, 2, \ldots, n)$ are positive constants.

Predators and preys are usually abundant in space with different densities at different positions and they are diffusive. Several papers have focused on the effect of diffusion which plays a crucial role in permanence and stability of population (see [5, 6, 9, 10, 14, 15, 16, 19, 20, 21] and the references therein).

Based on the above reasons, in this paper, we consider an $n$-dimensional ratio-dependent predator-prey system with diffusion as follows:

$$(1.1) \quad \begin{cases} \dfrac{\partial U}{\partial t}(x, t) - D\Delta U(x, t) = F(U(x, t)) & \text{in } \Omega \times (0, \infty), \\ \dfrac{\partial U}{\partial \nu} = (0, \ldots, 0) & \text{on } \partial\Omega \times (0, \infty), \\ U(x, 0) = \varphi(x) = (\varphi_0(x), \ldots, \varphi_n(x)) & \text{in } \bar{\Omega}, \end{cases}$$

where $F = (f_0, f_1, \ldots, f_n)$, $U = (u, v_1, \ldots, v_n)$, $D = \mathrm{diag}[d_0, d_1, \ldots, d_n]$ and

$$(1.2) \qquad f_0 = u(1 - ku) - \frac{u(\sum_{i=1}^{n} \alpha_i v_i)}{u + \sum_{i=1}^{n} \alpha_i v_i} = u^2 \left( \frac{1}{u + \sum_{i=1}^{n} \alpha_i v_i} - k \right),$$

$$(1.3) \qquad\qquad f_i = -a_i v_i + \frac{\alpha_i v_i u}{\alpha_i v_i + u}, \ i = 1, \ldots, n,$$

and $\Omega$ is a bounded domain in $\mathbb{R}^N$, $N \geq 1$, with smooth boundary $\partial\Omega$, $\frac{\partial}{\partial \nu}$ is the outward directional normal derivative to $\partial\Omega$, the parameters $k, d_0, d_i, \alpha_i, a_i$ $(i = 1, \ldots, n)$ are positive constants, and $\varphi_i(x)$ $(i = 0, 1, \ldots, n)$ are continuous non-negative and nontrivial functions. Furthermore, we assume the parameters satisfy

(H1) $\sum_{i=1}^{n} \frac{\alpha_i}{a_i} > n - 1$ and $a_i < \alpha_i$ $(i = 1, \ldots, n)$.
(H2) $\sum_{i=1}^{n} \frac{\alpha_i}{a_i} < 1 + n$ and $a_i < \alpha_i$ $(i = 1, \ldots, n)$.

It is easy to see that problem (1.1) has a unique positive constant steady state $\tilde{U} = (\tilde{u}, \tilde{v}_1, \ldots, \tilde{v}_n)$ if and only if (H1) holds and

$$(1.4) \qquad \tilde{u} = \frac{1}{k\left(1 - n + \sum_{i=1}^{n} \frac{\alpha_i}{a_i}\right)}, \quad \tilde{v}_i = \frac{(\alpha_i - a_i)}{a_i \alpha_i}\tilde{u}, \ i = 1, \ldots, n.$$

In recent years, the study of the dynamics of predator-prey system attracts a lot of authors' interests (see [8, 11, 12, 13, 17] and reference therein). The main goal of this paper is to study dissipativeness, persistence of the system (1.1) and globally asymptotical stability of the positive constant steady-state $\check{U}$. Our main results are:

(1) The system (1.1) is dissipative and persistent if (H2) holds.

(2) The constant steady-state $\tilde{U}$ is globally asymptotically stable if (H1) and (H2) hold.

The organization of this paper is as follows. In Section 2, we study dissipativeness and persistence and in Section 3, we study the globally asymptotical stability of $\tilde{U}$.

## 2. Dissipativeness and persistence of system (1.1)

The main concern of this section is dissipativeness and persistence of system (1.1) and the main result is as follows.

**Theorem 2.1.** *Assume* (H2) *holds and let* $(u, v_1, \ldots, v_n)$ *be a solution of problem* (1.1). *Then*

$$\frac{1}{k}\left(1 + n - \sum_{i=1}^{n}\frac{\alpha_i}{a_i}\right) \le \liminf_{t\to+\infty}\min_{\bar{\Omega}} u(\cdot, t) \le u(x, t)$$

(2.1)

$$\le \limsup_{t\to+\infty}\max_{\bar{\Omega}} u(\cdot, t) \le \frac{1}{k},$$

$$\frac{(\alpha_i - a_i)}{ka_i\alpha_i}\left(1 + n - \sum_{i=1}^{n}\frac{\alpha_i}{a_i}\right) \le \liminf_{t\to+\infty}\min_{\bar{\Omega}} v_i(\cdot, t) \le v_i(x, t)$$

(2.2)

$$\le \limsup_{t\to+\infty}\max_{\bar{\Omega}} v_i(\cdot, t) \le \frac{(\alpha_i - a_i)}{ka_i\alpha_i},$$

*where* $i = 1, \ldots, n$.

In order to prove the above results, we first introduce the following lemma [17, 18].

**Lemma 2.2.** *Assume* $f(s) \in C^1([0, +\infty))$, $d > 0, \beta \ge 0, T \in [0, +\infty)$ *are constants, and* $w \in C^{2,1}(\Omega \times (T, +\infty)) \cap C^{1,0}(\bar{\Omega} \times [T, +\infty))$ *is positive. Then we have*

(1) *If* $w$ *satisfies*

$$\begin{cases} \dfrac{\partial w}{\partial t} - d\Delta w \le (\ge)w^{1+\beta}f(w)(\alpha - w) & \text{in } \Omega \times (T, +\infty), \\ \dfrac{\partial w}{\partial \nu} = 0 & \text{on } \partial\Omega \times (T, +\infty), \end{cases}$$

*where* $\alpha > 0$ *is a constant, we have*

$$\limsup_{t\to+\infty}\max_{\bar{\Omega}} w(\cdot, t) \le \alpha \quad (\liminf_{t\to+\infty}\min_{\bar{\Omega}} w(\cdot, t) \ge \alpha).$$

(2) *If $w$ satisfies*

$$\begin{cases} \dfrac{\partial w}{\partial t} - d\Delta w \leq w^{1+\beta} f(w)(\alpha - w) & \text{in } \Omega \times (T, +\infty), \\[2mm] \dfrac{\partial w}{\partial \nu} = 0 & \text{on } \partial\Omega \times (T, +\infty), \end{cases}$$

*where $\alpha \leq 0$ is a constant, we have*

$$\limsup_{t \to +\infty} \max_{\bar{\Omega}} w(\cdot, t) \leq 0.$$

Now, we can prove Theorem 2.1.

*Proof of Theorem 2.1.* Throughout the proof, $(1.1)_u$ means the equation satisfying by $u$ in $(1.1)$ and $(1.1)_{v_i}$ means the equation satisfying by $v_i$ in $(1.1)$. By $(1.1)_u$, we obtain $u_t - d_0 \Delta u \leq u(1 - ku)$. So, by virtue of Lemma 2.2, we obtain

$$(2.3) \qquad \limsup_{t \to +\infty} \max_{\bar{\Omega}} u(\cdot, t) \leq \frac{1}{k} := \bar{u}^{(1)}.$$

So, for any $\epsilon > 0$, there exists $T_1^\epsilon \gg 1$ such that $u(x, t) \leq \bar{u}^{(1)} + \epsilon$ for any $x \in \bar{\Omega}$ and $t \geq T_1^\epsilon$. By $(1.1)_{v_i}$ and for any $(x, t) \in \Omega \times (T_1^\epsilon, +\infty)$, we obtain

$$v_{it} - d_i \Delta v_i \leq -a_i v_i + \frac{\alpha_i v_i (\bar{u}^{(1)} + \epsilon)}{\alpha_i v_i + (\bar{u}^{(1)} + \epsilon)} = v_i \left[ \frac{(\alpha_i - a_i)(\bar{u}^{(1)} + \epsilon) - a_i \alpha_i v_i}{\alpha_i v_i + (\bar{u}^{(1)} + \epsilon)} \right].$$

By Lemma 2.2, we have $\limsup_{t \to +\infty} \max_{\bar{\Omega}} v_i(\cdot, t) \leq \frac{(\alpha_i - a_i)(\bar{u}^{(1)} + \epsilon)}{a_i \alpha_i}$. By the arbitrariness of $\epsilon > 0$, we obtain

$$(2.4) \qquad \limsup_{t \to +\infty} \max_{\bar{\Omega}} v_i(\cdot, t) \leq \frac{(\alpha_i - a_i)\bar{u}^{(1)}}{a_i \alpha_i} := \bar{v}_i^{(1)}.$$

Direct computation implies

$$(2.5) \qquad n + 1 > \sum_{i=1}^{n} \frac{\alpha_i}{a_i} \Rightarrow \frac{1}{k} > \sum_{i=1}^{n} \alpha_i \bar{v}_i^{(1)}.$$

So, (H2) and (2.5) implies there exists $\epsilon_0 > 0$ such that for $\epsilon \in (0, \epsilon_0]$, $1/k > \sum_{i=1}^{n} \alpha_i (\bar{v}_i^{(1)} + \epsilon)$ and there exists $T_2^\epsilon \gg 1$ such that for any $(x, t) \in \bar{\Omega} \times [T_2^\epsilon, +\infty)$, $v_i(x, t) \leq \bar{v}_i^{(1)} + \epsilon$. Then by $(1.1)_u$ and for any $(x, t) \in \Omega \times (T_2^\epsilon, +\infty)$, we obtain

$$u_t - d_0 \Delta u \geq u^2 \left( \frac{1}{u + \sum_{i=1}^{n} \alpha_i (\bar{v}_i^{(1)} + \epsilon)} - k \right)$$

$$= u^2 \left[ \frac{1 - k \sum_{i=1}^{n} \alpha_i (\bar{v}_i^{(1)} + \epsilon) - ku}{u + \sum_{i=1}^{n} \alpha_i (\bar{v}_i^{(1)} + \epsilon)} \right].$$

By Lemma 2.2, we obtain $\liminf_{t\to+\infty} \min_{\bar{\Omega}} u(\cdot,t) \geq \frac{1}{k} - \sum_{i=1}^{n} \alpha_i(\bar{v}_i^{(1)}+\epsilon)$. By the arbitrariness of $\epsilon \in (0,\epsilon_0]$, we obtain

$$(2.6) \qquad \liminf_{t\to+\infty} \min_{\bar{\Omega}} u(\cdot,t) \geq \frac{1}{k} - \sum_{i=1}^{n} \alpha_i \bar{v}_i^{(1)} := \underline{u}^{(1)} > 0.$$

So for any $\epsilon \in (0,\underline{u}^{(1)})$, there exists $T_3^\epsilon >> 1$ such that for any $(x,t) \in \bar{\Omega} \times [T_3^\epsilon,+\infty)$, $u(x,t) \geq \underline{u}^{(1)}-\epsilon$. Then by $(1.1)_{v_i}$ and for any $(x,t) \in \Omega \times (T_3^\epsilon,+\infty)$, we obtain

$$v_{it} - d_i\Delta v_i \geq -a_i v_i + \frac{\alpha_i v_i(\underline{u}^{(1)} - \epsilon)}{\alpha_i v_i + (\underline{u}^{(1)} - \epsilon)} = v_i \left[ \frac{(\alpha_i - a_i)(\underline{u}^{(1)} - \epsilon) - a_i \alpha_i v_i}{\alpha_i v_i + (\underline{u}^{(1)} - \epsilon)} \right].$$

By Lemma 2.2, we have $\liminf_{t\to+\infty} v_i(\cdot,t) \geq \frac{(\alpha_i-a_i)(\underline{u}^{(1)}-\epsilon)}{a_i\alpha_i}$. By the arbitrariness of $\epsilon \in (0,\underline{u}^{(1)})$, we have

$$(2.7) \qquad \liminf_{t\to+\infty} \min_{\bar{\Omega}} v_i(\cdot,t) \geq \frac{(\alpha_i - a_i)\underline{u}^{(1)}}{a_i\alpha_i} := \underline{v}_i^{(1)} > 0.$$

By (2.3) and (2.6), we have (2.1) and, by (2.4) and (2.7), we have (2.2). The proof is completed. $\qquad\square$

## 3. Global stability of the positive constant steady state $\tilde{U}$

The main concern of this section is to prove the global stability of $\tilde{U}$ defined in (1.4). The main result is as follows.

**Theorem 3.1.** *If* (H1) *and* (H2) *hold, then the positive constant steady state* $\tilde{U}$ *of problem* (1.1) *defined in* (1.4) *is globally asymptotically stable.*

*Proof.* In order to prove the theorem, we will use basically the method of upper and lower solutions combined with the monotone iterative method. More concretely, if we can construct $2n+2$ sequences, namely $\{\bar{u}^{(j)}\}_{j=1}^{+\infty}$, $\{\underline{u}^{(j)}\}_{j=1}^{+\infty}$, $\{\bar{v}_i^{(j)}\}_{j=1}^{+\infty}$, $\{\underline{v}_i^{(j)}\}_{j=1}^{+\infty}$ for $i = 1,\dots,n$, which satisfy the following properties:

    i) $\underline{u}^{(j)} \leq \liminf_{t\to+\infty} \min_{\bar{\Omega}} u(\cdot,t) \leq u(x,t) \leq \limsup_{t\to+\infty} \max_{\bar{\Omega}} u(\cdot,t) \leq \bar{u}^{(j)}$, $j = 1,2,\dots$,

    ii) $\underline{v}_i^{(j)} \leq \liminf_{t\to+\infty} \min_{\bar{\Omega}} v_i(\cdot,t) \leq v_i(x,t) \leq \limsup_{t\to+\infty} \max_{\bar{\Omega}} v_i(\cdot,t) \leq \bar{v}_i^{(j)}$, $i = 1,\dots,n$, $j = 1,2,\dots$,

    iii) $\{\bar{u}^{(j)}\}_{j=1}^{+\infty}$ and $\{\bar{v}_i^{(j)}\}_{j=1}^{+\infty}$ are nonincreasing in $j$ for $i = 1,\dots,n$,

    iv) $\{\underline{u}^{(j)}\}_{j=1}^{+\infty}$ and $\{\underline{v}_i^{(j)}\}_{j=1}^{+\infty}$ are nondecreasing in $j$ for $i = 1,\dots,n$,

    v) $\lim_{j\to+\infty} \bar{u}^{(j)} = \tilde{u} = \lim_{j\to+\infty} \underline{u}^{(j)}$ and $\lim_{j\to+\infty} \bar{v}_i^{(j)} = \tilde{v}_i = \lim_{j\to+\infty} \underline{v}_i^{(j)}$ for $i = 1,\dots,n$,

then we obtain Theorem 3.1. So, in the following, our task is to construct the sequences which satisfy i)-v).

Let $\phi(s_1,\dots,s_n) = 1/k - \sum_{i=1}^{n} \alpha_i s_i$ and $\psi_i(s) = \frac{\alpha_i - a_i}{a_i\alpha_i} s$, where $s, s_1,\dots,s_n \in (0,+\infty)$ and $i = 1,\dots,n$. It is easy to see that $\frac{\partial\phi}{\partial s_i} < 0$ and $\psi_i'(s) > 0$.

Then we construct the sequences $\{\bar{u}^{(j)}\}_{j=1}^{+\infty}$, $\{\underline{u}^{(j)}\}_{j=1}^{+\infty}$, $\{\bar{v}_i^{(j)}\}_{j=1}^{+\infty}$, $\{\underline{v}_i^{(j)}\}_{j=1}^{+\infty}$ as follows:

$$\bar{u}^{(1)} = \frac{1}{k}, \quad \bar{v}_i^{(1)} = \frac{(\alpha_i - a_i)\bar{u}^{(1)}}{a_i\alpha_i}, \quad i = 1,\ldots,n,$$

$$\underline{u}^{(1)} = \frac{1}{k} - \sum_{i=1}^{n} \alpha_i \bar{v}_i^{(1)}, \quad \underline{v}_i^{(1)} = \frac{(\alpha_i - a_i)\underline{u}^{(1)}}{a_i\alpha_i}, \quad i = 1,\ldots,n,$$

$$\bar{u}^{(j+1)} = \phi(\underline{v}_1^{(j)},\ldots,\underline{v}_n^{(j)}), \ j = 1,2,\ldots, \quad \bar{v}_i^{(j)} = \psi_i(\bar{u}^{(j)}), \ i = 1,\ldots,n, \ j = 2,\ldots$$

$$\underline{u}^{(j)} = \phi(\bar{v}_1^{(j)},\ldots,\bar{v}_n^{(j)}), \ j = 1,2,\ldots, \quad \underline{v}_i^{(j)} = \psi_i(\underline{u}^{(j)}), \ i = 1,\ldots,n, \ j = 2,\ldots.$$

First, we prove the sequences $\{\bar{u}^{(j)}\}_{j=1}^{+\infty}$, $\{\underline{u}^{(j)}\}_{j=1}^{+\infty}$, $\{\bar{v}_i^{(j)}\}_{j=1}^{+\infty}$, $\{\underline{v}_i^{(j)}\}_{j=1}^{+\infty}$ satisfy i) and ii) by induction. By Theorem 2.1, we obtain i) and ii) hold for $j = 1$, then we assume i) and ii) hold for $j = m > 1$. Since $\liminf_{t\to+\infty} \min_{\bar{\Omega}} v_i(\cdot,t) \geq \underline{v}_i^{(m)} > 0$, then for any $\epsilon \in (0,\underline{v}_i^{(m)})$, there exists $\hat{T}_1^\epsilon \gg 1$ such that $v_i(x,t) \geq \underline{v}_i^{(m)} - \epsilon$ for any $(x,t) \in \bar{\Omega} \times [\hat{T}_1^\epsilon,+\infty)$. Then by $(1.1)_u$ and for any $(x,t) \in \Omega \times (\hat{T}_1^\epsilon,+\infty)$, we have

$$u_t - d_0\Delta u \leq u^2\left(\frac{1}{u + \sum_{i=1}^n \alpha_i(\underline{v}_i^{(m)} - \epsilon)} - k\right)$$
$$= u^2\left[\frac{1 - k\sum_{i=1}^n \alpha_i(\underline{v}_i^{(m)} - \epsilon) - ku}{u + \sum_{i=1}^n \alpha_i(\underline{v}_i^{(m)} - \epsilon)}\right].$$

By Lemma 2.2 and arbitrariness of $\epsilon$, we obtain $\limsup_{t\to+\infty} \max_{\bar{\Omega}} u(\cdot,t) \leq 1/k - \sum_{i=1}^n \alpha_i\underline{v}_i^{(m)} = \bar{u}^{(m+1)}$. Then repeat the proof of Theorem 2.1 by replace $\bar{u}^{(1)}$ with $\bar{u}^{(m+1)}$, we can prove i) and ii) hold for $j = m + 1$.

Next, we will prove iii) and iv) hold by induction. Recall the monotone properties of $\phi$ and $\psi_i$, we obtain $\bar{v}_i^{(2)} = \psi_i(\bar{u}^{(2)}) < \psi_i(\bar{u}^{(1)}) = \bar{v}_i^{(1)}$ since $\bar{u}^{(2)} < \bar{u}^{(1)}$, and $\underline{u}^{(2)} = \phi(\bar{v}_1^{(2)},\ldots,\bar{v}_n^{(2)}) > \phi(\bar{v}_1^{(1)},\ldots,\bar{v}_n^{(1)}) = \underline{u}^{(1)}$, and $\underline{v}_i^{(2)} = \psi_i(\underline{u}^{(2)}) > \psi_i(\underline{u}^{(1)}) = \underline{v}_i^{(1)}$. Assume $\bar{u}^{(m+1)} < \bar{u}^{(m)}$, $\bar{v}_i^{(m+1)} < \bar{v}_i^{(m)}$, $\underline{u}^{(m+1)} > \underline{u}^{(m)}$ and $\underline{v}_i^{(m+1)} > \underline{v}_i^{(m)}$, then repeat the above proof, we have $\bar{u}^{(m+2)} < \bar{u}^{(m+1)}$, $\bar{v}_i^{(m+2)} < \bar{v}_i^{(m+1)}$, $\underline{u}^{(m+2)} > \underline{u}^{(m+1)}$ and $\underline{v}_i^{(m+2)} > \underline{v}_i^{(m+1)}$.

Finally, we prove v) holds. Assume $\lim_{j\to+\infty} \underline{u}^{(j)} = \underline{u}$, $\lim_{j\to+\infty} \underline{v}_i^{(j)} = \underline{v}_i$, $\lim_{j\to+\infty} \bar{u}^{(j)} = \bar{u}$ and $\lim_{j\to+\infty} \bar{v}_i^{(j)} = \bar{v}_i$. It is obvious that $0 < \underline{u} \leq \bar{u}$, $0 < \underline{v}_i \leq \bar{v}_i$ and $\underline{u}, \underline{v}_i, \bar{u}, \bar{v}_i$ satisfy

(3.1)
$$\underline{u} = \phi(\bar{v}_1,\ldots,\bar{v}_n), \ \bar{u} = \phi(\underline{v}_1,\ldots,\underline{v}_n), \ \underline{v}_i = \psi_i(\underline{u}), \ \bar{v}_i = \psi_i(\bar{u}), \ i = 1,\ldots,n.$$

Obviously, (3.1) implies

$$(3.2) \quad \underline{u} = \frac{1}{k} - \sum_{i=1}^n \alpha_i\bar{v}_i, \ \bar{u} = \frac{1}{k} - \sum_{i=1}^n \alpha_i\underline{v}_i, \ \underline{v}_i = \frac{\alpha_i - a_i}{a_i\alpha_i}\underline{u}, \ \bar{v}_i = \frac{\alpha_i - a_i}{a_i\alpha_i}\bar{u}.$$

By virtue of (3.2), we obtain

$$\underline{u} = \frac{1}{k} - \sum_{i=1}^{n} \frac{\alpha_i - a_i}{a_i} \bar{u}, \ \bar{u} = \frac{1}{k} - \sum_{i=1}^{n} \frac{\alpha_i - a_i}{a_i} \underline{u},$$

which implies $(1 + n - \sum_{i=1}^{n} \alpha_i/a_i)(\bar{u} - \underline{u}) = 0$, and we have $\bar{u} = \underline{u}$ by (H2). Then we get $\bar{v} = \underline{v}$ by (3.2). The proof is completed. □

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