

DOUBLE SEMIOPEN SETS ON DOUBLE BITOPOLOGICAL SPACES

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ABSTRACT. We introduce the concepts of double bitopological spaces as a generalization of intuitionistic fuzzy topological spaces in Šostak's sense and Kandil's fuzzy bitopological spaces. Also we introduce the concept of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen sets and double pairwise $(r, s)(u, v)$ -semicontinuous mappings in double bitopological spaces and investigate some of their characteristic properties.

1. Introduction

Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chatopadhyay, Hazra, and Samanta [3], and by Ramadan [11].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Çoker and his colleagues [4, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and M. Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

Kandil [8] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces.

In this paper, we introduce the concepts of double bitopological spaces as a generalization of intuitionistic fuzzy topological spaces in Šostak's

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sense and Kandil's fuzzy bitopological spaces. We also introduce the concept of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen $((r, s)(u, v)$ -semiclosed) sets and double pairwise $(r, s)(u, v)$ -semicontinuous $((r, s)(u, v)$ -semiopen and $(r, s)(u, v)$ -semiclosed, respectively) mappings in double bitopological spaces and investigate some of their characteristic properties.

2. Preliminaries

Let I be the unit interval $[0, 1]$ of the real line. A member μ of I^X is called a fuzzy set of X . For any $\mu \in I^X$, μ^c denotes the complement $1 - \mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

DEFINITION 2.1. [1] *Let A and B be intuitionistic fuzzy sets on X . Then*

- (1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.
- (2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
- (3) $A^c = (\gamma_A, \mu_A)$.
- (4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
- (5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
- (6) $0_{\sim} = (\tilde{0}, \tilde{1})$ and $1_{\sim} = (\tilde{1}, \tilde{0})$.

Let f be a mapping from a set X to a set Y . Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y . Then:

- (1) The image of A under f , denoted by $f(A)$, is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

- (2) The inverse image of B under f , denoted by $f^{-1}(B)$, is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

A *smooth fuzzy topology* on X is a map $T : I^X \rightarrow I$ which satisfies the following properties:

- (1) $T(\tilde{0}) = T(\tilde{1}) = 1$.
- (2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$.
- (3) $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$.

The pair (X, T) is called a *smooth fuzzy topological space*.

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

- (1) $0_{\sim}, 1_{\sim} \in T$.
- (2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
- (3) If $A_i \in T$ for all i , then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space*.

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

DEFINITION 2.2. [12] Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense* $\mathcal{T}^{\mu\gamma} = (\mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ on X is a mapping $\mathcal{T}^{\mu\gamma} : I(X) \rightarrow I \otimes I$ ($\mathcal{T}^{\mu}, \mathcal{T}^{\gamma} : I(X) \rightarrow I$) which satisfies the following properties:

- (1) $\mathcal{T}^{\mu}(0_{\sim}) = \mathcal{T}^{\mu}(1_{\sim}) = 1$ and $\mathcal{T}^{\gamma}(0_{\sim}) = \mathcal{T}^{\gamma}(1_{\sim}) = 0$.
- (2) $\mathcal{T}^{\mu}(A \cap B) \geq \mathcal{T}^{\mu}(A) \wedge \mathcal{T}^{\mu}(B)$ and $\mathcal{T}^{\gamma}(A \cap B) \leq \mathcal{T}^{\gamma}(A) \vee \mathcal{T}^{\gamma}(B)$.
- (3) $\mathcal{T}^{\mu}(\bigcup A_i) \geq \bigwedge \mathcal{T}^{\mu}(A_i)$ and $\mathcal{T}^{\gamma}(\bigcup A_i) \leq \bigvee \mathcal{T}^{\gamma}(A_i)$.

The $(X, \mathcal{T}^{\mu\gamma}) = (X, \mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*. Also, we call $\mathcal{T}^{\mu}(A)$ a *gradation of openness* of A and $\mathcal{T}^{\gamma}(A)$ a *gradation of nonopenness* of A .

DEFINITION 2.3. [10] Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space in Šostak's sense $(X, \mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ and $(r, s) \in I \otimes I$. Then A is said to be

- (1) a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set if $\mathcal{T}^{\mu}(A) \geq r$ and $\mathcal{T}^{\gamma}(A) \leq s$,
- (2) a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -closed set if $\mathcal{T}^{\mu}(A^c) \geq r$ and $\mathcal{T}^{\gamma}(A^c) \leq s$.

Let $(X, \mathcal{T}^\mu, \mathcal{T}^\gamma)$ be an intuitionistic fuzzy topological space in Šostak’s sense. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -closure is defined by

$$\begin{aligned} \mathcal{T}^{\mu\gamma}\text{-cl}(A, r, s) &= \bigcap \{B \in I(X) \mid A \subseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r, s)\text{-closed}\} \end{aligned}$$

and the $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -interior is defined by

$$\begin{aligned} \mathcal{T}^{\mu\gamma}\text{-int}(A, r, s) &= \bigcup \{B \in I(X) \mid A \supseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r, s)\text{-open}\}. \end{aligned}$$

LEMMA 2.4. [10] For an intuitionistic fuzzy set A in an intuitionistic fuzzy topological space in Šostak’s sense $(X, \mathcal{T}^\mu, \mathcal{T}^\gamma)$ and $(r, s) \in I \otimes I$, we have:

- (1) $\mathcal{T}^{\mu\gamma}\text{-cl}(A, r, s)^c = \mathcal{T}^{\mu\gamma}\text{-int}(A^c, r, s)$.
- (2) $\mathcal{T}^{\mu\gamma}\text{-int}(A, r, s)^c = \mathcal{T}^{\mu\gamma}\text{-cl}(A^c, r, s)$.

A system $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ consisting of a set X with two intuitionistic fuzzy topologies in Šostak’s sense $\mathcal{T}^{\mu\gamma}$ and $\mathcal{U}^{\mu\gamma}$ on X is called a *double bitopological space*.

Let $(X, \mathcal{T}^{\mu\gamma})$ be an intuitionistic fuzzy topological space in Šostak’s sense. Then it is easy to see that for each $(r, s) \in I \otimes I$, the family $(\mathcal{T}^{\mu\gamma})_{(r,s)}$ defined by

$$(\mathcal{T}^{\mu\gamma})_{(r,s)} = \{A \in I(X) \mid \mathcal{T}^\mu(A) \geq r \text{ and } \mathcal{T}^\gamma(A) \leq s\}$$

is an intuitionistic fuzzy topology on X .

Let (X, T) be an intuitionistic fuzzy topological space and $(r, s) \in I \otimes I$. Then the map $T_{(r,s)}^{\mu\gamma} : I(X) \rightarrow I \otimes I$ defined by

$$T_{(r,s)}^{\mu\gamma}(A) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim \\ (r, s) & \text{if } A \in T - \{0_\sim, 1_\sim\} \\ (0, 1) & \text{otherwise} \end{cases}$$

becomes an intuitionistic fuzzy topology in Šostak’s sense on X .

Hence, we have that if $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ is a double bitopological space and $(r, s), (u, v) \in I \otimes I$, then $(X, (\mathcal{T}^{\mu\gamma})_{(r,s)}, (\mathcal{U}^{\mu\gamma})_{(u,v)})$ is an intuitionistic fuzzy bitopological space. Also, if (X, T, U) is an intuitionistic fuzzy bitopological space and $(r, s), (u, v) \in I \otimes I$, then $(X, (T)_{(r,s)}^{\mu\gamma}, (U)_{(u,v)}^{\mu\gamma})$ is a double bitopological space.

3. $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen sets

DEFINITION 3.1. Let A be an intuitionistic fuzzy set of a double bitopological space $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ and $(r, s), (u, v) \in I \otimes I$. Then A is said to be

- (1) a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set if there is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set B in X such that $B \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(B, u, v)$,
- (2) a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen set if there is a $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -open set B in X such that $B \subseteq A \subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(B, r, s)$,
- (3) a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set if there is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -closed set B in X such that $\mathcal{U}^{\mu\gamma}\text{-int}(B, u, v) \subseteq A \subseteq B$.
- (4) a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed set if there is a $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -closed set B in X such that $\mathcal{T}^{\mu\gamma}\text{-int}(B, r, s) \subseteq A \subseteq B$.

THEOREM 3.2. Let A be an intuitionistic fuzzy set of a double bitopological space $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ and $(r, s), (u, v) \in I \otimes I$. Then the following statements are equivalent:

- (1) A is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set.
- (2) A^c is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set.
- (3) $\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A, r, s), u, v) \supseteq A$.
- (4) $\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(A^c, r, s), u, v) \subseteq A^c$.

Proof. (1) \Rightarrow (3) Let A be a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set of X . Then there is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set B in X such that $B \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(B, u, v)$. Since $B \subseteq A$, we have

$$B = \mathcal{T}^{\mu\gamma}\text{-int}(B, r, s) \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(A, r, s).$$

Hence $A \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(B, u, v) \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A, r, s), u, v)$.

(3) \Rightarrow (1) Let $\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A, r, s), u, v) \supseteq A$. Suppose that $B = \mathcal{T}^{\mu\gamma}\text{-int}(A, r, s)$. Then B is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set and

$$\begin{aligned} B &= \mathcal{T}^{\mu\gamma}\text{-int}(A, r, s) \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A, r, s), u, v) \\ &= \mathcal{U}^{\mu\gamma}\text{-cl}(B, u, v). \end{aligned}$$

Thus A is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set.

(1) \Leftrightarrow (2) and (3) \Leftrightarrow (4) follow from Lemma 2.4. □

COROLLARY 3.3. Let A be an intuitionistic fuzzy set of a double bitopological space $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ and $(r, s), (u, v) \in I \otimes I$. Then the following statements are equivalent:

- (1) A is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen set.
- (2) A^c is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed set.
- (3) $\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(A, u, v), r, s) \supseteq A$.

$$(4) \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A^c, u, v), r, s) \subseteq A^c.$$

THEOREM 3.4. Let $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ be a double fuzzy topological space and $(r, s), (u, v) \in I \otimes I$.

- (1) Suppose that A is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set. If $\mathcal{T}^{\mu\gamma}\text{-int}(A, r, s) \subseteq B \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(A, u, v)$, then B is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set.
- (2) Suppose that A is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen set. If $\mathcal{U}^{\mu\gamma}\text{-int}(A, u, v) \subseteq B \subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(A, r, s)$, then B is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen set.
- (3) Suppose that A is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set. If $\mathcal{U}^{\mu\gamma}\text{-int}(A, u, v) \subseteq B \subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(A, r, s)$, then B is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set.
- (4) Suppose that A is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed set. If $\mathcal{T}^{\mu\gamma}\text{-int}(A, r, s) \subseteq B \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(A, u, v)$, then B is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed set.

Proof. (1) Let A be a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set and $\mathcal{T}^{\mu\gamma}\text{-int}(A, r, s) \subseteq B \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(A, u, v)$. Then there is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set C such that $C \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(C, u, v)$. It follows that

$$\begin{aligned} C &= \mathcal{T}^{\mu\gamma}\text{-int}(C, r, s) \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(A, r, s) \\ &\subseteq B \\ &\subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(A, u, v) \\ &\subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-cl}(C, u, v), u, v) \\ &= \mathcal{U}^{\mu\gamma}\text{-cl}(C, u, v). \end{aligned}$$

Hence C is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set and $C \subseteq B \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(C, u, v)$. Thus B is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set.

(2) Similar to (1)

(3) Suppose that A be a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set and $\mathcal{U}^{\mu\gamma}\text{-int}(A, u, v) \subseteq B \subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(A, r, s)$. Then there is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -closed set C such that $\mathcal{U}^{\mu\gamma}\text{-int}(C, u, v) \subseteq A \subseteq C$. It follows that

$$\begin{aligned} \mathcal{U}^{\mu\gamma}\text{-int}(C, u, v) &= \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-int}(C, u, v), u, v) \\ &\subseteq \mathcal{U}^{\mu\gamma}\text{-int}(A, u, v) \\ &\subseteq B \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(A, r, s) \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(C, r, s) \\ &= C. \end{aligned}$$

Hence C is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -closed set and $\mathcal{U}^{\mu\gamma}\text{-int}(C, u, v) \subseteq B \subseteq C$. Thus B is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set.

(4) Similar to (3) □

It is obvious that every $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open $((r, s)$ -closed) set is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen $((r, s)(u, v)$ -semiclosed) set and every $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -open $((u, v)$ -closed) set is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen $((u, v)(r, s)$ -semiclosed) set but the converses need not be true which is shown by the following example.

EXAMPLE 3.5. Let $X = \{x, y\}$ and let A_1, A_2, A_3 and A_4 be intuitionistic fuzzy sets of X defined as

$$\begin{aligned} A_1(x) &= (0.0, 0.7), & A_1(y) &= (0.4, 0.3); \\ A_2(x) &= (0.5, 0.2), & A_2(y) &= (0.6, 0.1); \\ A_3(x) &= (0.1, 0.4), & A_3(y) &= (0.7, 0.1); \end{aligned}$$

and

$$A_4(x) = (0.6, 0.1), \quad A_4(y) = (0.8, 0.0).$$

Define $\mathcal{T}^{\mu\gamma} : I(X) \rightarrow I \otimes I$ and $\mathcal{U}^{\mu\gamma} : I(X) \rightarrow I \otimes I$ by

$$\mathcal{T}^{\mu\gamma}(A) = (\mathcal{T}^\mu(A), \mathcal{T}^\gamma(A)) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{5}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}^{\mu\gamma}(A) = (\mathcal{U}^\mu(A), \mathcal{U}^\gamma(A)) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{3}, \frac{1}{4}) & \text{if } A = A_2, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Then clearly $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ is a double bitopological space on X . Since

$$\begin{aligned} \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A_3, \frac{1}{2}, \frac{1}{5}), \frac{1}{3}, \frac{1}{4}) &= \mathcal{U}^{\mu\gamma}\text{-cl}(A_1, \frac{1}{3}, \frac{1}{4}) \\ &= 1_\sim \\ &\supseteq A_3, \end{aligned}$$

the intuitionistic fuzzy set A_3 is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$ -semiopen set which is not a $\mathcal{T}^{\mu\gamma}$ -fuzzy $(\frac{1}{2}, \frac{1}{5})$ -open set. Also, A_3^c is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$ -semiclosed set which is not a $\mathcal{T}^{\mu\gamma}$ -fuzzy $(\frac{1}{2}, \frac{1}{5})$ -closed set. Since

$$\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(A_4, \frac{1}{3}, \frac{1}{4}), \frac{1}{2}, \frac{1}{5}) = \mathcal{T}^{\mu\gamma}\text{-cl}(A_2, \frac{1}{2}, \frac{1}{5}) = 1_\sim \supseteq A_4,$$

the intuitionistic fuzzy set A_4 is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$ -semiopen set which is not a $\mathcal{U}^{\mu\gamma}$ -fuzzy $(\frac{1}{3}, \frac{1}{4})$ -open set. Also, A_4^c is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$ -semiclosed set which is not a $\mathcal{U}^{\mu\gamma}$ -fuzzy $(\frac{1}{3}, \frac{1}{4})$ -closed set.

THEOREM 3.6. Let $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ be a double bitopological space and $(r, s), (u, v) \in I \otimes I$.

- (1) If $\{A_k\}$ is a family of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen sets of X , then $\bigcup A_k$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set.
- (2) If $\{A_k\}$ is a family of $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen sets of X , then $\bigcup A_k$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen set.
- (3) If $\{A_k\}$ is a family of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed sets of X , then $\bigcap A_k$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set.
- (4) If $\{A_k\}$ is a family of $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed sets of X , then $\bigcap A_k$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed set.

Proof. (1) Let $\{A_k\}$ be a collection of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen sets. Then for each k , there is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set B_k such that $B_k \subseteq A_k \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(B_k, u, v)$. Since B_k is $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open, $\mathcal{T}^\mu(B_k) \geq r$ and $\mathcal{T}^\gamma(B_k) \leq s$ for each k . So $\mathcal{T}^\mu(\bigcup B_k) \geq \bigwedge \mathcal{T}^\mu(B_k) \geq r$ and $\mathcal{T}^\gamma(\bigcup B_k) \leq \bigvee \mathcal{T}^\gamma(B_k) \leq s$. Hence $\bigcup B_k$ is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set. Also, we have

$$\begin{aligned} \bigcup B_k &\subseteq \bigcup A_k \subseteq \bigcup \mathcal{U}^{\mu\gamma}\text{-cl}(B_k, u, v) \\ &\subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\bigcup B_k, u, v). \end{aligned}$$

Hence $\bigcup B_k$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set.

(2) Let $\{A_k\}$ be a collection of $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen sets. Then for each k , there is a $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -open set B_k such that $B_k \subseteq A_k \subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(B_k, r, s)$. Since B_k is $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -open, $\mathcal{U}^\mu(B_k) \geq u$ and $\mathcal{U}^\gamma(B_k) \leq v$ for each k . So $\mathcal{U}^\mu(\bigcup B_k) \geq \bigwedge \mathcal{U}^\mu(B_k) \geq u$ and $\mathcal{U}^\gamma(\bigcup B_k) \leq \bigvee \mathcal{U}^\gamma(B_k) \leq v$. Hence $\bigcup B_k$ is a $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -open set. Also, we have

$$\begin{aligned} \bigcup B_k &\subseteq \bigcup A_k \subseteq \bigcup \mathcal{T}^{\mu\gamma}\text{-cl}(B_k, r, s) \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\bigcup B_k, r, s). \end{aligned}$$

Hence $\bigcup B_k$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen set.

(3) It follows from (1) using Theorem 3.2.

(4) It follows from (2) using Corollary 3.3 □

Let $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \rightarrow (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ be a mapping from a double bitopological space X to a double bitopological space Y and $(r, s), (u, v) \in I \otimes I$. Then f is called a *double pairwise $(r, s)(u, v)$ -continuous* ($(r, s)(u, v)$ -open and $(r, s)(u, v)$ -closed, respectively) mapping if the induced mapping $f : (X, \mathcal{T}^{\mu\gamma}) \rightarrow (Y, \mathcal{V}^{\mu\gamma})$ is fuzzy (r, s) -continuous ((r, s) -open and (r, s) -closed, respectively) and the induced mapping $f : (X, \mathcal{U}^{\mu\gamma}) \rightarrow (Y, \mathcal{W}^{\mu\gamma})$ is fuzzy (u, v) -continuous ((u, v) -open and (u, v) -closed, respectively).

DEFINITION 3.7. Let $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \rightarrow (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ be a mapping from a double bitopological space X to a double bitopological space Y and $(r, s), (u, v) \in I \otimes I$. Then f is called

- (1) *double pairwise $(r, s)(u, v)$ -semicontinuous* if $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set of X for each $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -open set A of Y and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen set of X for each $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -open set B of Y ,
- (2) *double pairwise $(r, s)(u, v)$ -semiopen* if $f(C)$ is a $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set of Y for each $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set C of X and $f(D)$ is a $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen set of Y for each $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -open set D of X ,
- (3) *double pairwise $(r, s)(u, v)$ -semiclosed* if $f(C)$ is a $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set of Y for each $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -closed set C of X and $f(D)$ is a $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed set of Y for each $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -closed set D of X .

THEOREM 3.8. Let $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \rightarrow (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ be a mapping and $(r, s), (u, v) \in I \otimes I$. Then the following statements are equivalent:

- (1) f is a double pairwise $(r, s)(u, v)$ -semicontinuous mapping.
- (2) $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set of X for each $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -closed set A of Y and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed set of X for each $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -closed set B of Y .
- (3) $\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A), r, s), u, v) \subseteq f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A, r, s))$ and $\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(A), u, v), r, s) \subseteq f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A, u, v))$ for each intuitionistic fuzzy set A of Y .
- (4) $f(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(C, r, s), u, v) \subseteq \mathcal{V}^{\mu\gamma}\text{-cl}(f(C), r, s)$ and $f(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(C, u, v), r, s) \subseteq \mathcal{W}^{\mu\gamma}\text{-cl}(f(C), u, v)$ for each intuitionistic fuzzy set C of X .

Proof. (1) \Rightarrow (2) Let A be any $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -closed set and B any $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -closed set of Y . Then A^c is a $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -open set and B^c is a $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -open set of Y . Since f is double pairwise

$(r, s)(u, v)$ -semicontinuous, $f^{-1}(A^c)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set and $f^{-1}(B^c)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen set of X . But $f^{-1}(A^c) = f^{-1}(A)^c$ and $f^{-1}(B^c) = f^{-1}(B)^c$. By Theorem 3.2 and Corollary 3.3, $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed set of X .

(2) \Rightarrow (1) Let A be any $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -open set and B any $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -open set of Y . Then A^c is a $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -closed set and B^c is a $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -closed set of Y . By (2), $f^{-1}(A^c)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set and $f^{-1}(B^c)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed set of X . But $f^{-1}(A^c) = f^{-1}(A)^c$ and $f^{-1}(B^c) = f^{-1}(B)^c$. By Theorem 3.2 and Corollary 3.3, $f^{-1}(A)$ is $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen and $f^{-1}(B)$ is $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen of X . Thus f is a double pairwise $(r, s)(u, v)$ -semicontinuous mapping.

(2) \Rightarrow (3) Let A be any intuitionistic fuzzy set of Y . Then $\mathcal{V}^{\mu\gamma}\text{-cl}(A, r, s)$ is a $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -closed set and $\mathcal{W}^{\mu\gamma}\text{-cl}(A, u, v)$ is a $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -closed set of Y . By (2), $f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A, r, s))$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set and $f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A, u, v))$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed set of X . By Theorem 3.2 and Corollary 3.3,

$$\begin{aligned} f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A, r, s)) & \supseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A, r, s)), r, s), u, v) \\ & \supseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A), r, s), u, v) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A, u, v)) & \supseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A, u, v)), u, v), r, s) \\ & \supseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(A), u, v), r, s). \end{aligned}$$

(3) \Rightarrow (4) Let C be an intuitionistic fuzzy set of X . Then $f(C)$ is an intuitionistic fuzzy set of Y . By (3),

$$\begin{aligned} f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C), r, s)) & \supseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}f(C), r, s), u, v) \\ & \supseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(C, r, s), u, v) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C), u, v)) & \supseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}f(C), u, v), r, s) \\ & \supseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(C, u, v), r, s). \end{aligned}$$

Thus we have

$$\begin{aligned} \mathcal{V}^{\mu\gamma}\text{-cl}(f(C), r, s) &\supseteq f f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C), r, s)) \\ &\supseteq f(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(C, r, s), u, v)) \end{aligned}$$

and

$$\begin{aligned} \mathcal{W}^{\mu\gamma}\text{-cl}(f(C), u, v) &\supseteq f f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C), u, v)) \\ &\supseteq f(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(C, u, v), r, s)). \end{aligned}$$

(4) \Rightarrow (2) Let A be any $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -closed set and B any $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -closed set of Y . Then $f^{-1}(A)$ and $f^{-1}(B)$ are intuitionistic fuzzy sets of X . By (4),

$$\begin{aligned} f(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A), r, s), u, v)) &\subseteq \mathcal{V}^{\mu\gamma}\text{-cl}(f f^{-1}(A), r, s) \\ &\subseteq \mathcal{V}^{\mu\gamma}\text{-cl}(A, r, s) \\ &= A \end{aligned}$$

and

$$\begin{aligned} f(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(B), u, v), r, s)) &\subseteq \mathcal{W}^{\mu\gamma}\text{-cl}(f f^{-1}(B), u, v) \\ &\subseteq \mathcal{W}^{\mu\gamma}\text{-cl}(B, u, v) \\ &= B. \end{aligned}$$

So we have

$$\begin{aligned} \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A), r, s), u, v) &\subseteq f^{-1}f(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A), r, s), u, v)) \\ &\subseteq f^{-1}(A) \end{aligned}$$

and

$$\begin{aligned} \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(B), u, v), r, s) &\subseteq f^{-1}f(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(B), u, v), r, s)) \\ &\subseteq f^{-1}(B). \end{aligned}$$

Thus $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed set of X . \square

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