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SOME BILATERAL GENERATING FUNCTIONS INVOLVING THE CHAN-CHYAN-SRIVASTAVA POLYNOMIALS AND SOME GENERAL CLASSES OF MULTIVARIABLE POLYNOMIALS

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Abstract. Recently, Liu et al. [Bilateral generating functions for the Chan-Chyan-Srivastava polynomials and the generalized Lauricella function, Integral Transform Spec. Funct. 23 (2012), no. 7, 539–549] investigated, in several interesting papers, some various families of bilateral generating functions involving the Chan-Chyan-Srivastava polynomials. The aim of this present paper is to obtain some bilateral generating functions involving the Chan-Chyan-Sriavastava polynomials and three general classes of multivariable polynomials introduced earlier by Srivastava in [A contour integral involving Fox's H-function, Indian J. Math. 14 (1972), 1–6], [A multilinear generating function for the Konhauser sets of biorthogonal polynomials suggested by the Laguerre polynomials, Pacific J. Math. 117 (1985), 183–191] and by Kaanoğlu and Özarslan in [Two-sided generating functions for certain class of r-variable polynomials, Mathematical and Computer Modelling 54 (2011), 625–631]. Special cases involving the (Srivastava-Daoust) generalized Lauricella functions are also given.

1. Introduction

The Chan-Chyan-Srivastava polynomials $g_n^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r)$ are a multivariable extension of the Laguerre polynomials generated by the following relation [1, p. 140, Eq. (4)]:

$$\prod_{j=1}^{r}(1-x_jz)^{-\alpha_j} = \sum_{n=0}^{\infty} g_n^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r)z^n \quad \left(|z|<\min\{|x_1|^{-1},\ldots,|x_r|^{-1}\}\right). \tag{1.1}$$

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Obviously, setting $r = 2$, $\alpha_1 = \alpha$ and $\alpha_2 = \beta$ in the last equation yields the familiar Lagrange polynomials $g_n^{(\alpha,\beta)}(x_1, x_2)$ which occur in some statistical problems [3, p. 267]. The last generating function (1.1) yields the explicit representation [1, p. 140, Eq. (6)]:

$$g_n^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r) = \sum_{k_1+\cdots+k_r=n} (\alpha_1)_{k_1}\cdots(\alpha_r)_{k_r}\,\frac{x_1^{k_1}}{k_1!}\cdots\frac{x_r^{k_r}}{k_r!} \tag{1.2}$$

or, equivalently, [9, p. 522, Eq. (17)]

$$\begin{aligned} &g_n^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r) \\ &= \sum_{n_{r-1}=0}^{n}\sum_{n_{r-2}=0}^{n_{r-1}}\cdots\sum_{n_1=0}^{n_2}\frac{(\alpha_1)_{n_1}(\alpha_2)_{n_2-n_1}\cdots(\alpha_r)_{n-n_{r-1}}}{n_1!(n_2-n_1)!\cdots(n-n_{r-1})!}x_1^{n_1}x_2^{n_2-n_1}\cdots x_r^{n-n_{r-1}}, \end{aligned} \tag{1.3}$$

where $(\lambda)_n$ denotes the Pochhammer's symbol defined by

$$(\lambda)_n := \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)}; \quad (\lambda)_0 = 1.$$

These polynomials have been extensively investigated first by the work of Chan et al. [1] and subsequently by the works of [2, 4, 5].

Almost four decades ago, Srivastava [12, p. 1, Eq. (1)] introduced and investigated the general class of polynomials $S_n^m(x)$ defined by

$$S_n^m(x) = \sum_{k=0}^{[\frac{n}{m}]}\frac{(-n)_{mk}}{k!}A_{n,k}x^k \qquad (n = 0, 1, 2, \ldots), \tag{1.4}$$

where m is an arbitrary positive integer, the coefficients $A_{n,k}$ $(n, k \geq 0)$ are arbitrary constants, real or complex. By suitably specializing the coefficients $A_{n,k}$, the polynomials $S_n^m(x)$ can be reduced to the classical orthogonal polynomials (Jacobi polynomials, Hermite polynomials, Laguerre polynomials, see for details [12, 18, 19]). Other interesting special cases of the polynomials $S_n^m(x)$ include the generalized hypergeometric polynomials such as the Bessel polynomials $y_n(x, \alpha, \beta)$ investigated by Krall and Frink [8, p. 108, Eq. (34)] and the generalized Hermite polynomials $g_n^m(x, h)$ considered by Gould and Hopper [6, p. 58].

In 1987, Srivastava and Garg [14, p. 686, Eq. (1.4)] introduced the multivariable analogue of the polynomials $S_n^m(x)$. This new class of polynomials $S_n^{m_1,\ldots,m_s}(x_1,\ldots,x_s)$ is defined by

$$\begin{aligned} &S_n^{m_1,\ldots,m_s}(x_1,\ldots,x_s) \\ &= \sum_{k_1,\ldots,k_s=0}^{m_1k_1+\cdots+m_sk_s\leq n}(-n)_{m_1k_1+\cdots+m_sk_s}\Lambda(n;k_1,\ldots,k_s)\frac{x_1^{k_1}\cdots x_s^{k_s}}{k_1!\cdots k_s!}, \end{aligned} \tag{1.5}$$

where $m_1, \ldots, m_s$ are arbitrary positive integers, and the coefficients

$$\Lambda(n; k_1, \ldots, k_s) \qquad (n, k_i \geq 0, i = 1, \ldots, r)$$

are arbitrary constants, real or complex.

Another interesting class of generalized multivariable polynomials, namely the polynomials $S^{n_1,\ldots,n_s}_{m_1,\ldots,m_s}(x_1, \ldots, x_s)$ have been given in 1985 by Srivastava [13, p. 185, Eq. (7)]. These polynomials are defined as follows:

$$(1.6) \qquad = \sum_{k_1=0}^{\left[\frac{n_1}{m_1}\right]} \cdots \sum_{k_s=0}^{\left[\frac{n_s}{m_s}\right]} \frac{(-n_1)_{m_1k_1} \cdots (-n_s)_{m_sk_s}}{k_1! \cdots k_s!} \Omega(n; k_1, \ldots, k_s) x_1^{k_1} \cdots x_s^{k_s},$$

where $m_1, \ldots, m_s$ are arbitrary positive integers, $n_1, \ldots, n_s$ are arbitrary non negative integers and the coefficients

$$\Omega(n; k_1, \ldots, k_s) \qquad (n, k_i \geq 0, i = 1, \ldots, r)$$

are arbitrary constants, real or complex.

These classes of polynomials are related to the Srivastava-Daoust generalized Lauricella function [15, p. 37 et seq.] defined as follows:

(1.7)

$$F^{A:B^{(1)};\ldots;B^{(s)}}_{C:D^{(1)};\ldots;D^{(s)}}\left[\begin{array}{llll} [(a):\theta^{(1)},\ldots,\theta^{(s)}]: & [b^{(1)};\phi^{(1)}]; & \ldots; & [b^{(s)};\phi^{(s)}]; \\ & & & & z_1,\ldots,z_s \\ [(c):\psi^{(1)},\ldots,\psi^{(s)}]: & [d^{(1)};\delta^{(1)}]; & \ldots; & [d^{(s)};\delta^{(s)}]; \end{array}\right]$$

$$= \sum_{m_1,\ldots,m_s=0}^{\infty} \Omega(m_1, \ldots, m_s) \frac{z_1^{m_1}}{m_1!} \cdots \frac{z_s^{m_s}}{m_s!},$$

where, for convenience,

(1.8)

$$\Omega(m_1, \ldots, m_s) := \frac{\prod_{j=1}^{A}(a_j)_{m_1\theta_j^{(1)}+\cdots+m_s\theta_j^{(s)}} \prod_{j=1}^{B^{(1)}}(b_j^{(1)})_{m_1\phi_j^{(1)}} \cdots \prod_{j=1}^{B^{(s)}}(b_j^{(s)})_{m_s\phi_j^{(s)}}}{\prod_{j=1}^{C}(c_j)_{m_1\psi_j^{(1)}+\cdots+m_s\psi_j^{(s)}} \prod_{j=1}^{D^{(1)}}(d_j^{(1)})_{m_1\delta_j^{(1)}} \cdots \prod_{j=1}^{D^{(s)}}(d_j^{(s)})_{m_s\delta_j^{(s)}}}.$$

The coefficients

$$\theta_j^{(k)}\ (j = 1, \ldots, A;\ k = 1, \ldots, s), \quad \phi_j^{(k)}\ (j = 1, \ldots, B^{(k)};\ k = 1, \ldots, s),$$

$$\psi_j^{(k)}\ (j = 1, \ldots, C;\ k = 1, \ldots, s) \quad \text{and} \quad \delta_j^{(k)}\ (j = 1, \ldots, D^{(k)};\ k = 1, \ldots, s)$$

are real constants and $(b^{(k)}_{B^{(k)}})$ abbreviates the array of $B^{(k)}$ parameters

$$b_j^{(k)} \quad (j = 1, \ldots, B^{(k)};\ k = 1, \ldots, s)$$

with similar interpretations for other sets of parameters.

By assigning suitably special values to the arbitrary coefficients $\Lambda(n; k_1, \ldots, k_s)$ and $\Omega(n; k_1, \ldots, k_s)$ of equations (1.5) and (1.6) respectively, we arrive to the following special cases.

Setting

$$\Lambda(n;k_1,\ldots,k_s) = \Omega(n;k_1,\ldots,k_s) = \frac{\prod_{j=1}^{A}(a_j)_{m_1\theta_j^{(1)}+\cdots+m_s\theta_j^{(s)}}\prod_{j=1}^{B^{(1)}}(b_j^{(1)})_{m_1\phi_j^{(1)}}\cdots\prod_{j=1}^{B^{(s)}}(b_j^{(s)})_{m_s\phi_j^{(s)}}}{\prod_{j=1}^{C}(c_j)_{m_1\psi_j^{(1)}+\cdots+m_s\psi_j^{(s)}}\prod_{j=1}^{D^{(1)}}(d_j^{(1)})_{m_1\delta_j^{(1)}}\cdots\prod_{j=1}^{D^{(s)}}(d_j^{(s)})_{m_s\delta_j^{(s)}}}$$

in (1.5) and (1.6), we obtain respectively

(1.9)
$$S_n^{m_1,\ldots,m_s}(x_1,\ldots,x_s) = F^{A+1:B^{(1)};\ldots;B^{(s)}}_{C:D^{(1)};\ldots;D^{(s)}}\left[\begin{array}{ll}[(-n):m_1,\ldots,m_s], & [(a):\theta^{(1)},\ldots,\theta^{(s)}]: \\ -, & [(c):\psi^{(1)},\ldots,\psi^{(s)}]:\end{array}\right. \left.\begin{array}{lll}[b^{(1)};\phi^{(1)}]; & \ldots; & [b^{(s)};\delta^{(s)}]; \\ [d^{(1)};\delta^{(1)}]; & \ldots; & [d^{(s)};\delta^{(s)}];\end{array} x_1,\ldots,x_s\right]$$

and

(1.10)
$$S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}(x_1,\ldots,x_s) = F^{A:1+B^{(1)};\ldots;1+B^{(s)}}_{C:D^{(1)};\ldots;D^{(s)}}\left[\begin{array}{l}[(a):\theta^{(1)},\ldots,\theta^{(s)}]: \\ [(c):\psi^{(1)},\ldots,\psi^{(s)}]:\end{array}\right. \left.\begin{array}{llllll}[-n_1;m_1], & [b^{(1)};\phi^{(1)}]; & \ldots; & [-n_s;m_s], & [b^{(s)};\delta^{(s)}]; \\ -, & [d^{(1)};\delta^{(1)}]; & \ldots; & -, & [d^{(s)};\delta^{(s)}];\end{array} x_1,\ldots,x_s\right].$$

Finally, in 2011, Kaanoğlu and Özarslan [7] introduced a certain class of multivariable polynomials $P_n^{m,N_1,\ldots,N_{r-1}}(x_1,\ldots,x_r)$ and obtained two-sided linear generating functions for this class of polynomials. These polynomials are a generalization of the three variable polynomials studied by Srivastava et al. [17]. Explicitly, the polynomials $P_n^{m,N_1,\ldots,N_{r-1}}(x_1,\ldots,x_r)$ are defined by

(1.11)
$$P_n^{m,N_1,\ldots,N_{r-1}}(x_1,\ldots,x_r) = \sum_{k_{r-1}=0}^{\left[\frac{n}{N_{r-1}}\right]}\sum_{k_{r-2}=0}^{\left[\frac{k_{r-1}}{N_{r-2}}\right]}\cdots\sum_{k_2=0}^{\left[\frac{k_3}{N_2}\right]}\sum_{k_1=0}^{\left[\frac{k_2}{N_1}\right]} A_{m+n,k_1,\ldots,k_{r-1}}\frac{x_1^{k_1}x_2^{k_2-N_1k_1}\cdots x_r^{n-N_{r-1}k_{r-1}}}{k_1!(k_2-N_1k_1)!\cdots(n-N_{r-1}k_{r-1})!}$$
$$(m,n\in\mathbb{N}_0;\ N_1,N_2,\ldots,N_{r-1}\in\mathbb{N}),$$

where $\{A_{m+n,k_1,\ldots,k_{r-1}}\}$ is a sequence of complex numbers.

Following the works of Liu et al. [9, 10, 11], we propose, in this paper, some bilateral generating functions involving the Chan-Chyan-Srivastava polynomials and the three classes of generalized polynomials defined above in (1.5), (1.6) and (1.11). Some special cases are computed and presented under the form of corollaries.

2. Main results

In this section, we present some bilateral generating functions involving the Chan-Chyan-Srivastava polynomials and the three classes of polynomials respectively defined previously by (1.5), (1.6) and (1.11). Some corollaries are also given as examples of applications of these presumably new generating functions.

The following lemma, given in [9, p. 521, Eq. (13)], will be useful in the sequel.

Lemma 2.1. *The following multiple summation formula*

$$\sum_{n_r=0}^{\infty}\sum_{n_{r-1}=0}^{n_r}\cdots\sum_{n_1=0}^{n_2}A(n_1,n_2,\dots,n_r) = \sum_{n_r=0}^{\infty}\sum_{n_{r-1}=0}^{\infty}\cdots\sum_{n_1}^{\infty}A(n_1,n_1+n_2,\dots,n_1+n_2+\cdots+n_r) \tag{2.1}$$

holds true provided that each of the series involved is absolutely convergent.

For a suitably bounded non-vanishing multiple sequence $\{\Omega(k_1,\dots,k_s)\}_{k_1,\dots,k_s\in\mathbb{N}_0}$ of real or complex parameters, we define a function $\Phi_n(n_2,m_2;\dots;n_s,m_s;y_1,\dots,y_s)$ of s variables where $m_j\in\mathbb{N}$, for $j=2,\dots,s$, and $n_j\in\mathbb{N}_0$, for $j=2,\dots,s$, by

(2.2)

$$\begin{aligned}&\Phi_n(n_2,m_2;\dots;n_s,m_s;y_1,\dots,y_s)\\ &= S^{1,m_2,\dots,m_s}_{n,n_2,\dots,n_s}(y_1,\dots,y_s)\\ &= \sum_{k_1=0}^{n}\sum_{k_2=0}^{\left[\frac{n_2}{m_2}\right]}\cdots\sum_{k_S=0}^{\left[\frac{n_S}{m_S}\right]}\frac{(-n)_{k_1}(-n_2)_{m_2k_2}\cdots(-n_s)_{m_sk_s}}{k_1!\cdots k_s!}\Omega(k_1,\dots,k_s)y_1^{k_1}\cdots y_s^{k_s},\end{aligned}$$

where $S^{1,m_2,\dots,m_s}_{n,n_2,\dots,n_s}(y_1,\dots,y_s)$ denotes the generalized Srivastava polynomials defined by (1.6). As usual, $[x]$ denotes the greatest integer in x and

$$\mathbb{N}_0 := \mathbb{N}\cup\{0\}.$$

Theorem 2.2. *The following bilateral generating function holds true*:

(2.3)

$$\sum_{n=0}^{\infty} g_n^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r) S_{n,n_2,\ldots,n_s}^{1,m_2,\ldots,m_s}(y_1,\ldots,y_s) z^n$$
$$= \prod_{j=1}^{r}(1-x_j z)^{-\alpha_j} \sum_{l_1,\ldots,l_r=0}^{\infty} \sum_{k_2=0}^{\left[\frac{n_2}{m_2}\right]} \cdots \sum_{k_s=0}^{\left[\frac{n_s}{m_s}\right]} \frac{(\alpha_1)_{l_1}\cdots(\alpha_r)_{l_r}}{l_1!\cdots l_r!} \frac{(-n_2)_{m_2k_2}\cdots(-n_s)_{m_sk_s}}{k_2!\cdots k_s!}$$
$$\times \Omega(l_1+\cdots+l_r,\ldots,k_s)\left(\frac{y_1 z x_1}{x_1 z-1}\right)^{l_1}\cdots\left(\frac{y_1 z x_r}{x_r z-1}\right)^{l_r} y_2^{k_2}\cdots y_s^{k_s}.$$

Proof. It is easy to see that

(2.4)

$$\sum_{n=0}^{\infty} g_n^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r) S_{n,n_2,\ldots,n_s}^{1,m_2,\ldots,m_s}(y_1,\ldots,y_s) z^n$$
$$= \sum_{n=0}^{\infty} g_n^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r) \sum_{k_1=0}^{n} \sum_{k_2=0}^{\left[\frac{n_2}{m_2}\right]} \cdots \sum_{k_s=0}^{\left[\frac{n_s}{m_s}\right]} \frac{(-n)_{k_1}(-n_2)_{m_2k_2}\cdots(-n_s)_{m_sk_s}}{k_1!\cdots k_s!}$$
$$\times \Omega(k_1,\ldots,k_s) y_1^{k_1}\cdots y_s^{k_s}$$
$$= \sum_{n=0}^{\infty}\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\left[\frac{n_2}{m_2}\right]} \cdots \sum_{k_s=0}^{\left[\frac{n_s}{m_s}\right]} \binom{n+k_1}{n} g_{n+k_1}^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r) \frac{(-n_2)_{m_2k_2}\cdots(-n_s)_{m_sk_s}}{k_2!\cdots k_s!}$$
$$\times \Omega(k_1,\ldots,k_s)(-y_1 z)^{k_1}\cdots y_s^{k_s}.$$

Now, by making use of the following formula [1, p. 143, Eq. (20)]:

$$\sum_{n=0}^{\infty} \binom{n+m}{n} g_{n+m}^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r) z^n = \prod_{j=1}^{r}(1-x_j z)^{-\alpha_j} g_m^{(\alpha_1,\ldots,\alpha_r)}\left(\frac{x_1}{1-x_1 z},\ldots,\frac{x_r}{1-x_r z}\right) \quad (m\in\mathbb{N}_0) \tag{2.5}$$

and equation (1.3), we obtain

(2.6)

$$\sum_{n=0}^{\infty} g_n^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r) S_{n,n_2,\ldots,n_s}^{1,m_2,\ldots,m_s}(y_1,\ldots,y_s) z^n$$
$$= \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\left[\frac{n_2}{m_2}\right]} \cdots \sum_{k_s=0}^{\left[\frac{n_s}{m_s}\right]} \frac{(-n_2)_{m_2k_2}\cdots(-n_s)_{m_sk_s}}{k_2!\cdots k_s!} \Omega(k_1,\ldots,k_s)(-y_1 z)^{k_1}\cdots y_s^{k_s}$$

$$\times \prod_{j=1}^{r}(1-x_jz)^{-\alpha_j} g_{k_1}^{(\alpha_1,\ldots,\alpha_r)}\left(\frac{x_1}{1-x_1z},\ldots,\frac{x_r}{1-x_rz}\right)$$

$$= \prod_{j=1}^{r}(1-x_jz)^{-\alpha_j} \sum_{k_2=0}^{\left[\frac{n_2}{m_2}\right]}\cdots\sum_{k_s=0}^{\left[\frac{n_s}{m_s}\right]} \frac{(-n_2)_{m_2k_2}\cdots(-n_s)_{m_sk_s}}{k_2!\cdots k_s!}$$

$$\times \sum_{k_1=0}^{\infty}\sum_{l_{r-1}=0}^{k_1}\sum_{l_{r-2}=0}^{l_{r-1}}\cdots\sum_{l_1=0}^{l_2}\Omega(k_1,\ldots,k_s)\frac{(\alpha_1)_{l_1}(\alpha_2)_{l_2-l_1}\cdots(\alpha_r)_{k_1-l_{r-1}}}{l_1!(l_2-l_1)!\cdots(k_1-l_{r-1})!}$$

$$\times (-y_1z)^{k_1}\cdots y_s^{k_s}\left(\frac{x_1}{1-x_1z}\right)^{l_1}\left(\frac{x_2}{1-x_2z}\right)^{l_2-l_1}\cdots\left(\frac{x_r}{1-x_rz}\right)^{k_1-l_{r-1}}.$$

With the help of Lemma 2.1, we thus arrive to the desired result. □

Corollary 2.3. *In view of equations* (1.10) *and* (2.3), *we have the following relation*

(2.7)

$$\sum_{n=0}^{\infty} g_n^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r) F^{A:1+B^{(1)};\ldots;1+B^{(s)}}_{C:D^{(1)};\ldots;D^{(s)}}\left[\begin{array}{l}[(a):\theta^{(1)},\ldots,\theta^{(s)}]:\\ [(c):\psi^{(1)},\ldots,\psi^{(s)}]:\end{array}\right.$$

$$\left.\begin{array}{llllll}[-n;1], & [b^{(1)};\phi^{(1)}]; & \ldots; & [-n_s;m_s], & [b^{(s)};\delta^{(s)}]; & \\ & & & & & y_1,\ldots,y_s\\ -, & [d^{(1)};\delta^{(1)}]; & \ldots; & -, & [d^{(s)};\delta^{(s)}]; & \end{array}\right] z^n$$

$$= \prod_{j=1}^{r}(1-x_jz)^{-\alpha_j} F^{A+B:1;\ldots;1;1+B^{(2)};\ldots;1+B^{(s)}}_{C+D:0;\ldots;0;D^{(2)};\ldots;D^{(s)}}$$

$$\left[\begin{array}{lllll}[(e):\varphi^{(1)},\ldots,\varphi^{(r+s-1)}]: & [\alpha_1;1]; & \ldots; & [\alpha_r;1]; & [-n_2;m_2],\\ [(f):\Theta^{(1)},\ldots,\Theta^{(r+s-1)}]: & -; & \ldots; & -; & -,\end{array}\right.$$

$$\left.\begin{array}{lllll}[b^{(2)};\phi^{(2)}]; & \ldots; & [-n_s;m_s], & [b^{(s)};\delta^{(s)}]; & \\ & & & & \frac{y_1zx_1}{x_1z-1},\ldots,\frac{y_1zx_r}{x_rz-1},y_2,\ldots,y_s\\ [d^{(2)};\delta^{(2)}]; & \ldots; & -, & [d^{(s)};\delta^{(s)}]; & \end{array}\right],$$

where the coefficients $e_j, f_j, \varphi_j^{(k)}$ *and* $\Theta_j^{(k)}$ *are given by*

$$e_j = \begin{cases} a_j & (1 \le j \le A)\\ b_{j-A} & (A < j \le A+B),\end{cases}$$

$$f_j = \begin{cases} c_j & (1 \le j \le E)\\ d_{j-A} & (E < j \le E+D),\end{cases}$$

$$\varphi_j^{(k)} = \begin{cases} \theta_j^{(1)} & (1 \leq j \leq A; 1 \leq k \leq r) \\ \theta_j^{(k-r+1)} & (1 \leq j \leq A; r < k \leq r+s-1) \\ \phi_{j-A} & (A < j \leq A+B; 1 \leq k \leq r) \\ 0 & (A < j \leq A+B; r < k \leq r+s-1) \end{cases}$$

and

$$\Theta_j^{(k)} = \begin{cases} \Psi_j^{(1)} & (1 \leq j \leq E; 1 \leq k \leq r) \\ \Psi_j^{(k-r+1)} & (1 \leq j \leq E; r < k \leq r+s-1) \\ \delta_{j-A} & (E < j \leq E+D; 1 \leq k \leq r) \\ 0 & (E < j \leq E+D; r < k \leq r+s-1), \end{cases}$$

respectively.

Considering now a suitably bounded non-vanishing multiple sequence

$$\{\Omega(n, k_1; n_2, k_2; \ldots; n_s, k_s)\}_{k_1,\ldots,k_s \in \mathbb{N}_0}$$

of real or complex parameters where $n, n_2, \ldots, n_s$ are fixed nonnegative integers, we define a function $\Xi_n(m_1; n_2, m_2; \ldots; n_s, m_s; y_1, \ldots, y_s)$ of s variables where $m_j \in \mathbb{N}$, for $j = 1, \ldots, s$, by

(2.8)

$$\begin{aligned} &\Xi_n(m_1; n_2, m_2; \ldots; n_s, m_s; y_1, \ldots, y_s) \\ &= \frac{S_{n,n_2,\ldots,n_s}^{m_1,\ldots,m_s}(y_1, \ldots, y_s)}{\prod_{j=1}^r (1-\alpha_j)_n} \\ &= \sum_{k_1=0}^{\left[\frac{n}{m_1}\right]} \sum_{k_2=0}^{\left[\frac{n_2}{m_2}\right]} \cdots \sum_{k_s=0}^{\left[\frac{n_s}{m_s}\right]} \frac{(-n)_{m_1k_1} \cdots (-n_s)_{m_sk_s}}{k_1! \cdots k_s!} \frac{\Omega(n, k_1; n_2, k_2; \ldots; n_s, k_s)}{\prod_{j=1}^r (1-\alpha_j)_n} y_1^{k_1} \cdots y_s^{k_s}, \end{aligned}$$

where $S_{n,n_2,\ldots,n_s}^{m_1,\ldots,m_s}(y_1, \ldots, y_s)$ denotes the generalized Srivastava polynomials defined by (1.6).

Theorem 2.4. *The following bilateral generating function holds true*:

$$\begin{aligned} (2.9) \quad &\sum_{n=0}^{\infty} g_n^{(\alpha_1-n,\ldots,\alpha_r-n)}(x_1, \ldots, x_r) \Xi_n(m_1; n_2, m_2; \ldots; n_s, m_s; y_1, \ldots, y_s) z^n \\ &= \sum_{l_1,\ldots,l_r=0}^{\infty} \frac{(-x_1z)^{l_1} \cdots (-x_rz)^{l_r}}{l_1! \cdots l_r! \, ((1-\alpha))_\zeta} S_{l_1+\cdots+l_r,n_2,\ldots,n_s}^{m_1,\ldots,m_s}(y_1, \ldots, y_s) \end{aligned}$$

or equivalently,

$$(2.10) \qquad \sum_{n=0}^{\infty} g_n^{(\alpha_1-n,\ldots,\alpha_r-n)}(x_1, \ldots, x_r) \frac{S_{n,n_2,\ldots,n_s}^{m_1,\ldots,m_s}(y_1, \ldots, y_s)}{\prod_{j=1}^r (1-\alpha_j)_n} z^n$$

$$= \sum_{l_1,\ldots,l_r=0}^{\infty} \frac{(-x_1 z)^{l_1} \cdots (-x_r z)^{l_r}}{l_1! \cdots l_r! \ ((1-\alpha))_\zeta} S^{m_1,\ldots,m_s}_{l_1+\cdots+l_r,n_2,\ldots,n_s}(y_1,\ldots,y_s),$$

where

(2.11) $$((1-\alpha))_\zeta = \prod_{j=1}^{r}(1-\alpha_j)_{\zeta_j} \quad \textit{and} \quad \zeta_j = \sum_{k=1\ (k\neq j)}^{r} l_k.$$

Proof. By using (1.3), we have

(2.12)
$$\sum_{n=0}^{\infty} g_n^{(\alpha_1-n,\ldots,\alpha_r-n)}(x_1,\ldots,x_r)\Xi_n(m_1;n_2,m_2;\ldots;n_s,m_s;y_1,\ldots,y_s)z^n$$
$$= \sum_{n=0}^{\infty}\sum_{l_{r-1}=0}^{n}\sum_{l_{r-2}=0}^{l_{r-1}}\cdots\sum_{l_1=0}^{l_2} \frac{(\alpha_1-n)_{l_1}(\alpha_2-n)_{l_2-l_1}\cdots(\alpha_r-n)_{n-l_{r-1}}}{l_1!(l_2-l_1)!\cdots(n-l_{r-1})!}$$
$$\times \frac{x_1^{l_1}x_2^{l_2-l_1}\cdots x_r^{n-l_{r-1}}}{\prod_{j=1}^{r}(1-\alpha_j)_n} S^{m_1,\ldots,m_s}_{n,n_2,\ldots,n_s}(y_1,\ldots,y_s)z^n$$
$$= \sum_{n=0}^{\infty}\sum_{l_{r-1}=0}^{n}\sum_{l_{r-2}=0}^{l_{r-1}}\cdots\sum_{l_1=0}^{l_2} \frac{(-x_1z)^{l_1}(-x_2z)^{l_2-l_1}\cdots(-x_rz)^{n-l_{r-1}}}{l_1!(l_2-l_1)!\cdots(n-l_{r-1})!}$$
$$\times \frac{S^{m_1,\ldots,m_s}_{n,n_2,\ldots,n_s}(y_1,\ldots,y_s)}{(1-\alpha_1)_{n-l_1}(1-\alpha_2)_{n-l_2+l_1}\cdots(1-\alpha_r)_{l_{r-1}}}.$$

Applying Lemma 2.1 to the last relation, we find

(2.13)
$$\sum_{n=0}^{\infty} g_n^{(\alpha_1-n,\ldots,\alpha_r-n)}(x_1,\ldots,x_r)\Xi_n(m_1;n_2,m_2;\ldots;n_s,m_s;y_1,\ldots,y_s)z^n$$
$$= \sum_{l_1,\ldots,l_r=0}^{\infty} \frac{(-x_1 z)^{l_1} \cdots (-x_r z)^{l_r}}{l_1! \cdots l_r! \ ((1-\alpha))_\zeta} S^{m_1,\ldots,m_s}_{l_1+\cdots+l_r,n_2,\ldots,n_s}(y_1,\ldots,y_s)$$

which completes the proof. □

Corollary 2.5. *In view of equations* (1.10) *and* (2.9)*, we have the following relation*

(2.14)
$$\sum_{n=0}^{\infty} \frac{g_n^{(\alpha_1-n,\ldots,\alpha_r-n)}(x_1,\ldots,x_r)}{\prod_{j=1}^{r}(1-\alpha_j)_n} F^{A:1+B^{(1)};\ldots;1+B^{(s)}}_{C:D^{(1)};\ldots;D^{(s)}} \left[\begin{array}{c} [(a):\theta^{(1)},\ldots,\theta^{(s)}]: \\ [(c):\psi^{(1)},\ldots,\psi^{(s)}]: \end{array}\right.$$
$$\left.\begin{array}{cccccc} [-n;m_1], & [b^{(1)};\phi^{(1)}]; & \ldots; & [-n_s;m_s], & [b^{(s)};\delta^{(s)}]; & \\ -, & [d^{(1)};\delta^{(1)}]; & \ldots; & -, & [d^{(s)};\delta^{(s)}]; & \end{array} y_1,\ldots,y_s \right] z^n$$

$$= \sum_{l_1,\dots,l_r=0}^{\infty} \frac{(-x_1 z)^{l_1}\cdots(-x_r z)^{l_r}}{l_1!\cdots l_r!\ ((1-\alpha))_\zeta} F^{A:1+B^{(1)};\dots;1+B^{(s)}}_{C:D^{(1)};\dots;D^{(s)}} \left[\begin{array}{c} [(a):\theta^{(1)},\dots,\theta^{(s)}]: \\ \\ [(c):\psi^{(1)},\dots,\psi^{(s)}]: \end{array} \right.$$

$$\left. \begin{array}{cccccc} [-(l_1+\cdots+l_r);m_1], & [b^{(1)};\phi^{(1)}]; & \dots; & [-n_s;m_s], & [b^{(s)};\delta^{(s)}]; & \\ & & & & & y_1,\dots,y_s \\ -, & [d^{(1)};\delta^{(1)}]; & \dots; & -, & [d^{(s)};\delta^{(s)}]; & \end{array} \right],$$

where

$$((1-\alpha))_\zeta = \prod_{j=1}^{r} (1-\alpha_j)_{\zeta_j} \quad \textit{and} \quad \zeta_j = \sum_{k=1\ (k\neq j)}^{r} l_k.$$

For a suitably bounded non-vanishing multiple sequence $\{\Lambda(n;k_1,\dots,k_s)\}_{n,k_1,\dots,k_s\in\mathbb{N}_0}$ of real or complex parameters, we define a function $\Psi_n(m_1,\dots,m_s;y_1,\dots,y_s)$ of s variables where $m_j\in\mathbb{N}$, for $j=1,\dots,s$, by

$$\begin{aligned} (2.15)\qquad & \Psi_n(m_1,\dots,m_s;y_1,\dots,y_s) \\ &= \frac{S_n^{m_1,\dots,m_s}(y_1,\dots,y_s)}{\prod_{j=1}^{r}(1-\alpha_j)_n} \\ &= \sum_{k_1,\dots,k_s=0}^{m_1k_1+\cdots+m_sk_s\leq n} \frac{(-n)_{m_1k_1+\cdots+m_sk_s}}{\prod_{j=1}^{r}(1-\alpha_j)_n} \Lambda(n;k_1,\dots,k_s) \frac{y_1^{k_1}\cdots y_s^{k_s}}{k_1!\cdots k_s!}. \end{aligned}$$

Theorem 2.6. *The following bilateral generating function holds true*:

$$\begin{aligned} (2.16)\qquad & \sum_{n=0}^{\infty} g_n^{(\alpha_1-n,\dots,\alpha_r-n)}(x_1,\dots,x_r)\Psi_n(m_1,\dots,m_s;y_1,\dots,y_s)z^n \\ &= \sum_{l_1,\dots,l_r=0}^{\infty} \frac{(-x_1 z)^{l_1}\cdots(-x_r z)^{l_r}}{l_1!\cdots l_r!\ ((1-\alpha))_\zeta} S^{m_1,\dots,m_s}_{l_1+\cdots+l_r}(y_1,\dots,y_s), \end{aligned}$$

where

$$(2.17)\qquad ((1-\alpha))_\zeta = \prod_{j=1}^{r} (1-\alpha_j)_{\zeta_j} \quad \textit{and} \quad \zeta_j = \sum_{k=1\ (k\neq j)}^{r} l_k.$$

Proof. From (1.3), we have

$$\begin{aligned} (2.18)\qquad & \sum_{n=0}^{\infty} g_n^{(\alpha_1-n,\dots,\alpha_r-n)}(x_1,\dots,x_r)\Psi_n(m_1,\dots,m_s;y_1,\dots,y_s)z^n \\ &= \sum_{n=0}^{\infty}\sum_{l_{r-1}=0}^{n}\sum_{l_{r-2}=0}^{l_{r-1}}\cdots\sum_{l_1=0}^{l_2} \frac{(\alpha_1-n)_{l_1}(\alpha_2-n)_{l_2-l_1}\cdots(\alpha_r-n)_{n-l_{r-1}}}{l_1!(l_2-l_1)!\cdots(n-l_{r-1})!} \\ &\quad\times \frac{x_1^{l_1}x_2^{l_2-l_1}\cdots x_r^{n-l_{r-1}}}{\prod_{j=1}^{r}(1-\alpha_j)_n} S_n^{m_1,\dots,m_s}(y_1,\dots,y_s)z^n \end{aligned}$$

$$= \sum_{n=0}^{\infty} \sum_{l_{r-1}=0}^{n} \sum_{l_{r-2}=0}^{l_{r-1}} \cdots \sum_{l_1=0}^{l_2} \frac{(-x_1 z)^{l_1}(-x_2 z)^{l_2-l_1} \cdots (-x_r z)^{n-l_{r-1}}}{l_1!(l_2-l_1)! \cdots (n-l_{r-1})!}$$
$$\times \frac{S_n^{m_1,\ldots,m_s}(y_1,\ldots,y_s)}{(1-\alpha_1)_{n-l_1}(1-\alpha_2)_{n-l_2+l_1} \cdots (1-\alpha_r)_{l_{r-1}}}.$$

With the help of Lemma 2.1, the result follows easily. □

The next lemma [16, p. 102, Eq. (17)] is required to establish Corollary 2.8.

Lemma 2.7. *The following multiple summation formula*

$$\sum_{n=0}^{\infty} \sum_{k_1,\ldots,k_s=0}^{m_1k_1+\cdots+m_sk_s \le n} A(k_1,\ldots,k_s;n) \tag{2.19}$$
$$= \sum_{n=0}^{\infty} \sum_{k_1,\ldots,k_s=0}^{\infty} A(k_1,\ldots,k_s;n+m_1k_1+\cdots+m_sk_s)$$

holds true for positive integers $m_1,\ldots,m_s$.

Corollary 2.8. *The following bilateral generating function holds true*:

(2.20)
$$\sum_{l_1,\ldots,l_r=0}^{\infty} \frac{(-x_1 z)^{l_1} \cdots (-x_r z)^{l_r}}{l_1! \cdots l_r! \; ((1-\alpha))_{\zeta}} S_{l_1+\cdots+l_r}^{m_1,\ldots,m_s}(y_1,\ldots,y_s)$$
$$= \sum_{n,k_1,\ldots,k_s=0}^{\infty} \frac{g_{n+m_1k_1+\cdots+m_sk_s}^{(\alpha_1-n-m_1k_1-\cdots-m_sk_s,\ldots,\alpha_r-n-m_1k_1-\cdots-m_sk_s)}(x_1,\ldots,x_r)}{\prod_{j=1}^{r}(1-\alpha_j)_{n+m_1k_1+\cdots+m_sk_s}}$$
$$\times \Lambda(n+m_1k_1+\cdots+m_sk_s,k_1,\ldots,k_s)$$
$$\times (1+n)_{m_1k_1+\cdots+m_sk_s} \frac{\left((-z)^{m_1}y_1\right)^{k_1} \cdots \left((-z)^{m_s}y_s\right)^{k_s} z^n}{k_1! \cdots k_s!},$$

where

$$((1-\alpha))_{\zeta} = \prod_{j=1}^{r}(1-\alpha_j)_{\zeta_j} \quad \textit{and} \quad \zeta_j = \sum_{k=1\ (k\neq j)}^{r} l_k.$$

Proof. Applying Lemma 2.7 to the left hand side of equation (2.16) yields the result. □

Corollary 2.9. *In view of equations* (1.9) *and* (2.16), *we have the following relation*

(2.21)
$$\sum_{n=0}^{\infty} \frac{g_n^{(\alpha_1-n,\ldots,\alpha_r-n)}(x_1,\ldots,x_r)}{\prod_{j=1}^{r}(1-\alpha_j)_n} F_{C:D^{(1)};\ldots;D^{(s)}}^{A+1:B^{(1)};\ldots;B^{(s)}} \left[\begin{array}{c} [(-n):m_1,\ldots,m_s], \\ -, \end{array} \right.$$

$$\left[\begin{array}{llll} [(a):\theta^{(1)},\ldots,\theta^{(s)}]: & [b^{(1)};\phi^{(1)}]; & \ldots; & [b^{(s)};\delta^{(s)}]; \\ [(c):\psi^{(1)},\ldots,\psi^{(s)}]: & [d^{(1)};\delta^{(1)}]; & \ldots; & [d^{(s)};\delta^{(s)}]; \end{array} y_1,\ldots,y_s\right] z^n$$

$$= \sum_{l_1,\ldots,l_r=0}^{\infty} \frac{(-x_1 z)^{l_1}\cdots(-x_r z)^{l_r}}{l_1!\cdots l_r!\ ((1-\alpha))_\zeta} F^{A+1:B^{(1)};\ldots;B^{(s)}}_{C:D^{(1)};\ldots;D^{(s)}} \left[\begin{array}{llll} [(-l_1-\cdots-l_r):m_1,\ldots,m_s], \\ -, \\ [(a):\theta^{(1)},\ldots,\theta^{(s)}]: & [b^{(1)};\phi^{(1)}]; & \ldots; & [b^{(s)};\delta^{(s)}]; \\ [(c):\psi^{(1)},\ldots,\psi^{(s)}]: & [d^{(1)};\delta^{(1)}]; & \ldots; & [d^{(s)};\delta^{(s)}]; \end{array} y_1,\ldots,y_s\right],$$

where

$$((1-\alpha))_\zeta = \prod_{j=1}^{r}(1-\alpha_j)_{\zeta_j} \quad \textit{and} \quad \zeta_j = \sum_{k=1\ (k\neq j)}^{r} l_k.$$

Let us shift our focus on two special cases of Theorem 2.6. First of all, setting $s=1$ and using the fact that the Gould-Hopper polynomials [6, p. 58] $g_n^m(y,h)$ defined by

$$g_n^m(y,h) = \sum_{k=0}^{\left[\frac{n}{m}\right]} \frac{n!}{k!(n-mk)!} h^k y^{n-mk} \tag{2.22}$$

are related to the polynomials $S_n^m(y)$ (see [18, p. 161, Eq. (1.15)]) by

$$S_n^m(y) = (-1)^n \left(\frac{y}{h}\right)^{n/m} g_n^m\left(-\left(\frac{h}{y}\right)^{1/m}, h\right). \tag{2.23}$$

Thus, we obtain the following relationship between the Chan-Chyan-Srivastava polynomials and the Gould-Hopper polynomials:

(2.24)

$$\sum_{n=0}^{\infty} \frac{g_n^{(\alpha_1-n,\ldots,\alpha_r-n)}(x_1,\ldots,x_r)}{\prod_{j=1}^{r}(1-\alpha_j)_n} (-1)^n \left(\frac{y}{h}\right)^{n/m} g_n^m\left(-\left(\frac{h}{y}\right)^{1/m}, h\right) z^n$$

$$= \sum_{l_1,\ldots,l_r=0}^{\infty} \frac{(x_1 z)^{l_1}\cdots(x_r z)^{l_r}}{l_1!\cdots l_r!\ ((1-\alpha))_\zeta} \left(\frac{y}{h}\right)^{\frac{l_1+\cdots+l_r}{m}} g^m_{l_1+\cdots+l_r}\left(-\left(\frac{h}{y}\right)^{1/m}, h\right),$$

where

$$((1-\alpha))_\zeta = \prod_{j=1}^{r}(1-\alpha_j)_{\zeta_j} \quad \text{and} \quad \zeta_j = \sum_{k=1\ (k\neq j)}^{r} l_k.$$

Next, putting $s=1$, $y_1=u$ and considering the relation established by Srivastava and Singh [18, p. 160, Eq. (1.13)] between the generalized Bessel polynomials $y_n(y,\gamma,\beta)$ introduced by Krall and Frink [8, p. 108, Eq. (34)] and defined by

$$y_n(y,\gamma,\beta)=\sum_{k=0}^{n}\binom{n}{k}\binom{n+\gamma+k-2}{k}k!\left(\frac{y}{\beta}\right)^k \tag{2.25}$$

and the Srivastava polynomials $S_n^m(y)$, namely,

$$S_n^1(y)=y_n(-\beta y,\gamma,\beta). \tag{2.26}$$

This relation is obtained by replacing $A_{n,k}$ by $(\gamma+n-1)_k$ and m by 1 in (1.4). Therefore, we have the next relation between the Chan-Chyan-Srivastava polynomials and the generalized Bessel polynomials:

$$\begin{aligned}\sum_{n=0}^{\infty}&\frac{g_n^{(\alpha_1-n,\ldots,\alpha_r-n)}(x_1,\ldots,x_r)}{\prod_{j=1}^{r}(1-\alpha_j)_n}y_n(-\beta u,\gamma,\beta)z^n\\&=\sum_{l_1,\ldots,l_r=0}^{\infty}\frac{(-x_1z)^{l_1}\cdots(-x_rz)^{l_r}}{l_1!\cdots l_r!\ ((1-\alpha))_\zeta}y_{l_1+\cdots+l_r}(-\beta u,\gamma,\beta).\end{aligned} \tag{2.27}$$

We end this paper by giving a bilateral generating function involving the class of polynomials $P_n^{m,N_1,\ldots,N_{r-1}}(x_1,\ldots,x_r)$ defined by (1.11). For a suitably bounded non-vanishing multiple sequence $\{A_{m+n,k_1,\ldots,k_{r-1}}\}_{n,m,k_1,\ldots,k_{s-1}\in\mathbb{N}_0}$ of real or complex parameters, we define a function $\Delta_n(m,N_1,\ldots,N_{s-1};y_1,\ldots,y_s)$ of s-variables where $N_j\in\mathbb{N}$, for $j=1,\ldots,s-1$, by

(2.28)

$$\begin{aligned}&\Delta_n(m,N_1,\ldots,N_{s-1};y_1,\ldots,y_s)\\&=\frac{P_n^{m,N_1,\ldots,N_{s-1}}(y_1,\ldots,y_s)}{\prod_{j=1}^{r}(1-\alpha_j)_n}\\&=\sum_{k_{s-1}=0}^{\left[\frac{n}{N_{s-1}}\right]}\sum_{k_{s-2}=0}^{\left[\frac{k_{s-1}}{N_{s-2}}\right]}\cdots\sum_{k_2=0}^{\left[\frac{k_3}{N_2}\right]}\sum_{k_1=0}^{\left[\frac{k_2}{N_1}\right]}\frac{A_{m+n,k_1,\ldots,k_{s-1}}}{\prod_{j=1}^{r}(1-\alpha_j)_n}\frac{y_1^{k_1}y_2^{k_2-N_1k_1}\cdots y_s^{n-N_{s-1}k_{s-1}}}{k_1!(k_2-N_1k_1)!\cdots(n-N_{s-1}k_{s-1})!}.\end{aligned}$$

Theorem 2.10. *The following bilateral generating function holds true*:

$$\begin{aligned}\sum_{n=0}^{\infty}&g_n^{(\alpha_1-n,\ldots,\alpha_r-n)}(x_1,\ldots,x_r)\Delta_n(m,N_1,\ldots,N_{s-1};y_1,\ldots,y_s)z^n\\&=\sum_{l_1,\ldots,l_r=0}^{\infty}\frac{(-x_1z)^{l_1}\cdots(-x_rz)^{l_r}}{l_1!\cdots l_r!\ ((1-\alpha))_\zeta}P_{l_1+\cdots+l_r}^{m,N_1,\ldots,N_{s-1}}(y_1,\ldots,y_s),\end{aligned} \tag{2.29}$$

where

$$((1-\alpha))_\zeta=\prod_{j=1}^{r}(1-\alpha_j)_{\zeta_j}\quad\textit{and}\quad\zeta_j=\sum_{k=1\ (k\neq j)}^{r}l_k.$$

Proof. The proof is omitted since it is almost the same as the one of Theorem 2.6. □

Setting $N_1 = N_2 = \cdots = N_{s-1} = 1$ and

$$A_{m+n,k_1,\ldots,k_{s-1}} = (\alpha_1)_{k_1}(\alpha_2)_{k_2-k_1}\cdots(\alpha_{s-1})_{k_{s-1}-k_{s-2}}(\alpha_s)_m(\alpha_s+m)_{n-k_{s-1}},$$

we find from [7, p. 627, Eq. (1.7)] that

$$P_n^{m,1,\ldots,1}(y_1,\ldots,y_s) = (\alpha_s)_m\, g_n^{(\alpha_1,\ldots,\alpha_s+m)}(y_1,\ldots,y_s). \tag{2.30}$$

Putting $s = r$, $m = 0$ and substituting $x_j = y_j$ for $j = 1,\ldots,r$, we obtain the next relation.

Corollary 2.11. *The following bilateral generating function involving the product of two Chan-Chyan-Srivastava polynomials* $g_n^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r)$ *holds true*:

$$\begin{aligned}\sum_{n=0}^{\infty}&\frac{g_n^{(\alpha_1-n,\ldots,\alpha_r-n)}(x_1,\ldots,x_r)\, g_n^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r)}{\prod_{j=1}^{r}(1-\alpha_j)_n}z^n \\ &= \sum_{l_1,\ldots,l_r=0}^{\infty}\frac{(-x_1z)^{l_1}\cdots(-x_rz)^{l_r}}{l_1!\cdots l_r!\ ((1-\alpha))_\zeta}\, g_{l_1+\cdots+l_r}^{(\alpha_1,\ldots,\alpha_r)}(x_1,\ldots,x_r),\end{aligned} \tag{2.31}$$

where

$$((1-\alpha))_\zeta = \prod_{j=1}^{r}(1-\alpha_j)_{\zeta_j} \quad \textit{and} \quad \zeta_j = \sum_{k=1\ (k\neq j)}^{r} l_k.$$

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