

# 납기를 고려한 아웃소싱 일정계획문제의 정수계획을 활용한 접근법\*

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Integer Programming Approach for the  
Outsourcing Decision Problem in a Single Machine  
Scheduling Problem with Due Date Constraints

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## ■ Abstract ■

In this paper, we consider the outsourcing decision problem in a single machine scheduling problem. The decision problem is to determine for each job whether to be processed on an in-house manufacturing or external facilities(out-sourcing). Moreover, this paper considers a situation where each job has a due date. The objective of the problem is to minimize the outsourcing cost, subject to the due date constraints. The considered problem is proved to be NP-hard. Some solution properties and valid inequalities are derived, and an effective lower bound is derived based on the LP-relaxation. The results of experimental tests are presented to evaluate the performance of the suggested lower bound.

Keywords : Outsourcing, Scheduling, Integer Programming

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## 1. Introduction

The outsourcing decision in the scheduling problem is concerned with both the selection of jobs to be processed on the in-house or outsourcing, and the sequence of the in-house processing jobs. The problem is to minimize the outsourcing cost subject to the constraints that the completion time of the all the jobs are within the due date.

In our knowledge, relatively few researches have been done in the area of the outsourcing strategy. For example, Cachon et al. [2] and Kaipia [7] have studied why many firms have considered the outsourcing strategy in their management. Kim [8] has investigated the situation in which a manufacturing company outsources its operations to other contract manufacturers. Kolisch [9] has considered the problem of a outsourcing strategy in a make-to-order manufacturing system. The objective of the considered problem is to minimize both holding and setup costs in a supply chain. In his paper, a mixed-integer programming model is derived to solve the problem. Lee [12] have considered an outsourcing in an advanced planning and scheduling (APS) problem. The objective of the problem is to minimize the makespan, a genetic algorithm has been suggested to solve the problem.

Lee et al. [10] and Lee et al. [11] first suggested the outsourcing decision problem in the scheduling area, where jobs in the problem can be processed on either an in-house or external facilities. The objectives of the researches are to minimize the weighted sum of the total completion times and the outsourcing cost with the constraints of outsourcing budget. Tavares et al.

[17], Choi et al. [4] and Qi [16] and Mokhtari et al. [14] considered various scheduling problems with outsourcing alternatives, in various multi-stage production systems. Mishra et al. [13] develop a mixed integer programming model for integrated planning and scheduling with outsourcing alternatives, for which Tabu and simulated annealing method is applied to solve the problem. Coman et al. [6] present an analysis of the outsourcing decision problem. They formulate the outsourcing problem as a linear programming problem and an analytical solution is identified in their work. Chen et al. [3] and Chunga et al. [5] consider the scheduling problem with subcontracting options. They suggest a heuristic algorithm to solve the problem.

The proposed problem can be stated in detail as following. There are a set of jobs that are available at time zero.  $J$  is defined as a set of jobs. Let  $p_j$ ,  $d_j$  and  $o_j$  denote a processing time, a due date and an outsourcing cost of job  $j$ , respectively. Let  $\pi$  denote a schedule and  $O_\pi$  be defined as an outsourced-job set in schedule  $\pi$ . Then, the total outsourcing cost is defined as  $\sum_{j \in O_\pi} o_j$ . Finally, let  $C_j(\pi)$  be the completion time of job  $j$  in a schedule  $\pi$ . Without loss of generality, we assume that  $d_j$ ,  $p_j$  and  $o_j$  have integer values. The customer satisfaction can be measured by how many jobs are completed on time. If the production capacity is not enough, then the customer orders exceeding the capacity can be assigned to outsourcing. The problem is to minimize the outsourcing cost subject to the customer satisfaction as follows :

$$\min \sum_{j \in O_\pi} o_j, \quad \text{Subject To } C_j(\pi) - d_j \leq 0, \quad \forall j \in J.$$

This kind of problem can be considered in the

schedule of a big project. In the industries like ship building or software development, the due date can be very important. The violation of the due date can result in a large amount of penalty cost. To meet the due date of completed jobs, an outsourcing of jobs can be considered. In such industries, the manpower is the most important resource that processes jobs. There may be only one due date for the whole project and the jobs may have precedence relations among them. Let  $D$  be the due date for the project and the jobs are indexed with precedence relations which means that the job  $i$  should be completed before job  $i+1$  starts. If there is  $n$  jobs, due date for the each job can be calculated with  $d_n = D$  and  $d_i = d_{i+1} - p_{i+1}$ . Then, the problem can be reduced to the problem described over.

## 2. Problem Complexity

In this section, the complexity of the problem is discussed. The standard classification scheme for scheduling problems  $\alpha|\beta|\gamma$  is adopted in this paper where  $\alpha$  describes the shop environment,  $\beta$  describes the shop condition such as details of the processing characteristics, and  $\gamma$  contains the objective function to be minimized. The problem we considered is a single-machine problem, so that  $\alpha=1$ . For  $\beta$ ,  $C_j \leq d_j$  represents that the completion should be less than equal to the due date. For  $\gamma$ , the objective function of the proposed problem is to minimize the outsourcing cost, OC. Therefore, the proposed problem can be described as a  $1|C_j \leq d_j|OC$ .

**Theorem 1 :** *The problem  $1|C_j \leq d_j|OC$  is NP-hard*

*Proof* The Partition Problem is a well-known NP-hard problem. The proof of theorem 1 is easily done by a polynomial time reduction from the Partition Problem to the proposed problem. The Partition Problem is stated as following: Given a set  $N$  and a positive integer  $a_i$  for  $i \in N$ , where  $\sum_{i \in N} a_i = 2B$  and  $0 < a_i < B$ , for  $i \in N$ , the question is that whether there exists a subset  $S \in N$  such that  $\sum_{i \in S} a_i = B$ ,

Let us consider the instance of the given problem  $1|C_j \leq d_j|OC$  with  $p_j = o_j = a_j$ ,  $d_j = B$ , for  $\forall j \in J$  and  $\sum_{j \in J} p_j = 2B$ . It will be proved that there exists an optimal feasible schedule  $\pi$  with the objective function value  $\sum_{j \in o_\pi} o_j = B$  for the problem instance if and only if there exists a solution to the Partition Problem.

- a) For the instance, if there is a subset  $S \in N$  satisfying  $\sum_{i \in S} a_i = B$  to the partition problem then there exists an optimal schedule  $\pi$  with  $\sum_{j \in o_\pi} o_j = B$  for the problem  $1|C_j \leq d_j|OC$ . If the subset  $S$  is the solution to the partition problem, then the subset satisfies both  $\sum_{i \in S} a_i = B$  and  $\sum_{i \in N/S} a_i = B$  which means both  $\sum_{i \in S} p_i = B$  and  $\sum_{i \in N/S} o_i = B$ . If the jobs in subset  $N/S$  is outsourced, then the jobs in  $S$  satisfy the  $C_j \leq d_j$  constraints.
- b) For the instance, if there exists a optimal schedule  $\pi$  with  $\sum_{j \in o_\pi} o_j = B$ , then there is a subset  $S \in N$  satisfying  $\sum_{i \in S} a_i = B$  to the partition problem.

It can be proved by letting the outsourced jobs to be set  $S$  in the partition problem.

## 3. Integer Programming Model for the Problem

In this section, the integer programming

model for the problem is presented. Before we begin to formulate the integer model, the optimality property is developed.

**Proposition 2 :** *For the problem  $1|C_j \leq d_j|OC$ , there is an optimal schedule of all the in-house manufacturing jobs being sequenced in Earliest Due Date(EDD) rule*

*Proof.* It can be easily proved by using interchange arguments.

**Proposition 3 :** *For the problem  $1|C_j \leq d_j|OC$ , there is an optimal schedule of all the in-house manufacturing jobs being sequenced with no idle time in machine processing*

*Proof.* It can be easily proved, so we omit the proof.

From Proposition 2 and 3, the problem is reduced to select jobs to be outsourced. It is assumed that jobs in the set  $J$  in are indexed as the due date order. For example, if  $d_i \leq d_j$ , then  $i < j$ . Let decision variable  $y_j$  represents whether job  $j$  is outsourcing job or not. That is,  $y_j$  equals to 1 if job  $j$  is an outsourced job, otherwise equals to 0. Then, the integer programming model for the model is as follows :

Problem(SO)

$$Z_{\pi(SO)} = \min \sum_{j \in J} o_j y_j$$

s.t.

$$\sum_{i < j} p_i (1 - y_i) + p_j \leq d_j + M y_j, \quad \forall j \in J, \quad (1)$$

$$y_j \in \{0, 1\}, \quad \forall j \in J$$

The objective function of the problem (SO) is to minimize the outsourcing cost. The constraints (1) are to force the job to be outsourced

if the makespan of the precedent jobs plus the processing time of the job is exceed the due date. The number of binary variables is equal to the number of jobs. In the constraints (1), the big- $M$  method is used in the formulation and the minimum value for the value  $M$  is  $\max((\sum_{i \leq j} p_i - d_j), 0)$ .

**Proposition 4 :** *If the condition  $(\sum_{i \leq j} p_i - d_j) \leq 0$  is satisfied for a constraint, then the corresponding constraint is redundant.*

*Proof.* If above condition is satisfied, the constraints (1) can be transformed to  $\sum_{i \leq j} p_i - d_j \leq \sum_{i < j} y_i$ . The constraints are always satisfied because the left hand side of the constraint is non-positive.

By proposition 4, if  $\sum_{i \leq j} p_i \geq d_j$ , then the constraints (1) can be replaced with the following constraint.

$$\sum_{i < j} p_i (1 - y_i) + p_j \leq d_j + \left( \sum_{i \leq j} p_i - d_j \right) y_j \quad (2)$$

## 4. Solution Approach

In this section, some valid inequalities are suggested, and the modification of the problem (SO) are derived.

**Proposition 5 :** *For any subset of jobs  $X \subset J$ , the following inequality is valid for Problem (SO).*

$$\sum_{i \in X} p_i (1 - y_i) \leq \max_{i \in X} \{d_i\} \quad (3)$$

*Proof.* For any feasible schedule, note that the inequality  $\sum_{i \in X} p_i (1 - y_i) \leq \max_{i \in X} \{C_i\}$  is valid. The completion time  $C_i$  of job  $i$  less than or equal to  $d_i$ .

Let set  $X_j$  be defined as  $X_j = \{i : i \leq j, i \in J\}$ , then the following inequality is valid from proposition 5.

$$\sum_{i \in X_j} p_i (1 - y_i) \leq d_j \quad (4)$$

For a set  $X_j$ , the valid inequality (4) is same with the constraints (2) except the coefficient of variable  $y_j$ . If we rearrange the constraints so that the variables are moved to the right hand side of constraints, then the coefficient of variable  $y_j$  in constraints (2) and (4) are  $-\sum_{i \leq j} p_i + d_j$  and  $-p_i$ , respectively.

Let  $y'_j$  be the decision variable representing in-house processed job.  $y'_j$  equals to 1 if job  $j$  is an in-house manufacturing job, which means  $y'_j = 1 - y_j$ . Then, the objective function of the problem,  $\min \sum_{j \in J} o_j (1 - y'_j)$ , can be modified to  $\max \sum_{j \in J} o_j y'_j$  and the constraints corresponding to (2) are arranged to  $\sum_{i < j} p_i y'_i + (\sum_{i \leq j} p_i - d_j) y'_j \leq \sum_{i < j} p_i$ . The valid inequalities corresponding to (4) are added in the form of  $\sum_{i \leq j} p_i y'_i \leq d_j$ . Then, the formulation of the modified problem (MSO) can be written as the following.

Problem(MSO)

$$Z_{\pi(MSO)} = \max \sum_{j \in J} o_j y'_j$$

s.t.

$$\sum_{i < j} p_i y'_i + \left( \sum_{i \leq j} p_i - d_j \right) y'_j \leq \sum_{i < j} p_i, \quad \forall j \in J, \quad (5)$$

$$\sum_{i \leq j} p_i y'_i \leq d_j, \quad \forall j \in J, \quad (6)$$

$$y'_j \in \{0, 1\}, \quad \forall j \in J$$

By proposition 5, all the coefficients of the problem (MSO) is positive, then the problem is a multi-constraints 0/1 knapsack problem.

**Proposition 6** : For a job  $j$  satisfying the conditions  $\sum_{i \leq j} p_i > d_j$ , the inequality (7) is valid for

the problem (MSO).

$$\sum_{i \leq j} y'_i \leq j - 1 \quad (7)$$

*Proof* Suppose  $\sum_{i \leq j} y'_i \geq j$  is feasible. This means  $\sum_{i \leq j} p_i > d_j$ , which contradicts the constraints (6).

Let  $E_j$  be a set jobs with  $E_j = \{k : d_k \leq d_j, k \in J\}$ . For a job  $j$  satisfying the conditions  $\sum_{i \in E_j} p_i > d_j$ , let  $A_j \subset E_j$  be a minimal dependent set if conditions  $\sum_{i \in A_j} p_i > d_j$  and  $\sum_{i \in A_j \setminus \{i^*\}} p_i \leq d_j$  are satisfied with  $i^* = \arg \min_{i \in A_j} \{p_i\}$ , referring to Nemhauser et al. [15].

**Proposition 7** : For a set  $A_j$  satisfying the conditions of a minimal dependent set, the inequality (8) is valid for the problem (MSO).

$$\sum_{i \in A_j} y'_i \leq |A_j| - 1 \quad (8)$$

*Proof* It can be easily inferred from Proposition 6.

**Proposition 8** : If  $p_i \leq d_i$  for all  $i \in J$ , the convex hull of the integer feasible solution of Problem (MSO) is full dimensional.

*Proof* The following  $|J|$  independent integer solutions are in the convex hull of the integer feasible solution of Problem (MSO); For each  $j \in J$ ,  $y'_j = 1$ ,  $y'_i = 0$  with  $i \in J \setminus \{j\}$ . There are  $|J|$  of these.

From Proposition 8, the valid inequality satisfying  $|J|$  affinely independent solutions in equality is the facet for the problem (MSO).

**Proposition 9** : For a set  $E_j$  satisfying all of the following conditions, the inequality (8) is a facet for the problem (MSO).

1.  $|E_j|=|A_j|$ , where  $A_j$  is a minimal dependent set,
2.  $\sum_{i \in E_j} p_i \leq d_j$ , for  $\forall l \in E_j \setminus \{j\}$ ,
3.  $\max_{i \in E_j} \{p_i\} \geq \max_{i \in \bigcup_{j \in J} E_j} \{p_i\}$ ,

*Proof.* The following  $|J|$  affinely independent solution satisfy (8) at equality.

1. For each  $l \in A_j$ , the solution  $y'_l = 0$  and  $y'_i = 1$  for  $\forall i \in A_j \setminus \{l\}$  and  $y'_i = 0$  for  $\forall i \in J \setminus A_j$  is feasible and the inequality (8) is satisfied at equality. There are  $|A_j|$  of these.
2. For each  $l \in J \setminus A_j$ , let  $k = \arg_i \{\max_{i \in E_j} \{p_i\}\}$ , then the solution  $y'_l = 1$  and  $y'_i = 1$  for  $\forall i \in A_j \setminus \{k\}$  and  $y'_k = 0$  and  $y'_i = 0$  for  $\forall i \in J \setminus (A_j \cup \{l\})$  is feasible and the inequality (8) is satisfied at equality. There are  $|J \setminus A_j|$  of these.

**Proposition 10 :** For a minimum dependent set  $A_j$  satisfying the condition  $\min_{i \in E_j \setminus A_j} \{p_i\} \geq \max_{i \in A_j} \{p_i\}$ , the inequality (9) is valid for the problem (MSO).

$$\sum_{i \in E_j} y'_i \leq |A_j| - 1 \quad (9)$$

*Proof.* Suppose  $\sum_{i \in E_j} y'_i \geq |A_j|$  is feasible. By the condition  $\min_{i \in E_j \setminus A_j} \{p_i\} \geq \max_{i \in A_j} \{p_i\}$ , the inequality  $\sum_{i \in E_j} p_i y'_i \geq \sum_{i \in A_j} p_i \geq d_j$  is valid, which contradicts the constraints (6).

For the problem, this paper provides a LP-relaxation bound by solving the LP-relaxed problem of (MSO), via LP solve package. The derived lower bound could be used in each node of the branch-and-bound tree. Thus, the lower bound can improve the performance of ranch-and-bound method.

## 5. Numerical Experiments

The numerical experiments are conducted to evaluate the performance of the presented LP-relaxed lower bound. The processing times  $p_j$ 's, the outsourcing costs  $o_j$ 's are generated from uniform distributions U [1, 10] and U [1, 30], respectively. Let  $P_s$  be  $\sum_{j \in J} p_j$ , the sum of processing times. SDD and TF represent the span of due date and the tardiness factor. Then, the due date  $d_j$ 's have uniform distribution between  $P_s$  (1-SDD/2-TF) and  $P_s$  (1+SDD/2-TF) which means that the due date  $d_j$ 's are depends on the value of  $P_s$  and two parameters SDD and TF. Especially, the difficulties of the problems may depend on the values of SDD and TF [10]. for each job size between 60 and 140, SDD and TF are selected from the set {0.2, 0.4, 0.6, 0.8, 1.0}. 25 combinations of SDD and TF are tested for each job size. 30 problems are randomly generated for the test of the proposed procedure.

For the numerical experiments, the binary integer problem was solved by lp solve package Berkelaar et al. [1] with which some sets of computational experiments were carried out on a PC.

Let  $LP(Z_{\pi(SO)})$  be the optimal objective function value of the LP relaxed problem of the problem (SO). In the <Table 1>, symbol  $|J|$  stands for the number of jobs. Symbol  $a, b, c$  stand for the average gap between of  $LP(Z_{\pi(SO)})$  and  $Z_{\pi(MSO)}$ , the average gap between of  $LP(Z_{\pi(MSO)})$  and  $Z_{\pi(MSO)}$  and the average gap between of  $LP(Z_{\pi(MSO)})$  with constraints (8) and (9) and  $Z_{\pi(MSO)}$ , respectively. Symbol  $d, e, f$  stand for the number the problem  $LP(Z_{\pi(SO)})$ ,  $LP(Z_{\pi(MSO)})$  and  $LP(Z_{\pi(MSO)})$  with constraints (8) and (9) among the generated

problems (750) that are equal to the  $Z_{\pi(MSO)}$ , respectively. Symbol  $g$  stand for the computational time of the problem (MSO) in seconds. To find the integer solution of  $Z_{\pi(MSO)}$ , branch-and-bound method is used. Because we assume all the parameters are integer value, the integer value greater than or equal to the optimal objective function value of the LP relaxed Problem (MSO)

can be used as the bound for the optimal objective function value of Problem (MSO). In the Table, it is observed that the average gap between LP bound and the optimal objective function value has a relatively small value, which means that the LP relaxed solution of the problem gives a good bound for the problem (MSO). From column  $g$  in the <Table 1>, it is observed that the average

<Table 1> Computational Results for Jobs between 60 and 140

$ J $	a	b	c	d	e	f	g
60	29.33%	5.05%	4.13%	1	20	60	0.00
80	28.49%	3.62%	3.15%	3	16	55	0.03
100	27.54%	2.71%	2.43%	2	14	56	0.10
120	26.61%	2.51%	2.20%	4	34	116	2.94
140	26.43%	1.97%	1.96%	5	19	63	5.07

<Table 2> Computational Results for Various SDD and TF

SDD	TF	a	b	c	d	e	f	g
0.2	0.2	7.84%	1.79%	1.79%	1	17	17	0
	0.4	27.92%	0.58%	0.58%	0	14	14	0
	0.6	42.38%	0.42%	0.42%	0	5	5	0.16
	0.8	51.56%	0.38%	0.38%	0	6	6	0.32
	1	79.94%	1.92%	1.91%	0	0	0	0.01
0.4	0.2	5.28%	5.28%	0.83%	0	0	168	0
	0.4	14.98%	1.11%	1.11%	0	16	16	0
	0.6	26.17%	0.70%	0.70%	0	13	13	0.14
	0.8	29.37%	1.20%	1.16%	0	3	3	1.17
	1	61.32%	2.86%	2.86%	0	0	0	0.4
0.6	0.2	0.00%	0.00%	0.00%	0	0	0	0
	0.4	6.78%	2.09%	2.09%	1	12	12	0
	0.6	14.58%	1.34%	1.34%	0	2	2	0.08
	0.8	26.03%	4.07%	4.02%	0	0	0	1.72
	1	48.65%	4.07%	4.07%	0	0	0	18.73
0.8	0.2	0.00%	0.00%	0.00%	0	0	0	0
	0.4	7.38%	6.27%	3.49%	9	9	75	0
	0.6	13.03%	4.63%	4.32%	0	2	2	0.29
	0.8	35.51%	6.71%	6.67%	0	0	0	4.31
	1	46.25%	4.56%	4.56%	0	0	0	4.84
1	0.2	0.00%	0.00%	0.00%	0	0	0	0
	0.4	3.29%	2.64%	1.06%	4	4	17	0
	0.6	40.44%	11.80%	11.56%	0	0	0	0.33
	0.8	48.80%	7.12%	7.10%	0	0	0	0.18
	1	53.75%	5.38%	5.38%	0	0	0	15.21

computation time of the branch and bound method is less than one minute. The suggested procedure can solve the practical size of the problem. From the Table, column  $f$  represents the number of problems that find the optimal bound with LP relaxed problem with suggested valid constraints (8) and (9). It can be noticed that there are much improvement compared to column  $e$  in which the constrains (8) and (9) are not considered,

<Table 2> is the experiments result with varying SDD and TF. Lower value of TF means the due date is more tight compared to the sum of the processing of the all jobs. The SDD value represents the how wide the due date is distributed, the lower SDD value means the due date of the jobs are concentrated in a narrower time interval.

The numerical experiments shows that the performance of the suggested procedure depends on the SDD and TF value in terms of computational time.

## 6. Conclusions

This paper considers a outsourcing decision in schedule problem. The objective is to minimize the outsourcing cost with due date constraints. The NP-hardness of the problem is proved, and some solution properties and valid inequalities are derived. An effective lower bound is derived based on the LP-relaxation of the formulation (MSO). Numerical tests are conducted to evaluate performance of the suggested lower bound. Further study of any better tight lower bounds or a more efficient optimal algorithm may be interesting to solve much larger problem instances.

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