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DENSE SETS IN WEAK STRUCTURE AND MINIMAL STRUCTURE

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Abstract. This paper is an attempt to study and introduce the notion of ω-dense set in weak structures and the notion of m-dense set in minimal structures. We have also investigate the relationships between ω-dense sets, m-dense sets, $\sigma(\omega)$ sets, $\pi(\omega)$ sets, $r(\omega)$ sets, $\beta(\omega)$ sets, *m-semiopen* sets and *m-preopen* sets. Further we give some representations of the above generalized sets in minimal structures as well as in weak structures.

1. Introduction

The concept of weak structure has been introduced and studied by Csaszar [5] in 2011 and he defined generalized open sets in the same structure. The other notion of generalization of topological space is minimal structure which was introduced by Maki et al. [8] in 1996. Again the mathematicians like Ozbakir and Yildirim [14] and Min and Kim [10, 11, 12, 13] have further developed this concept in different aspects.

In this paper, we have attempted to introduce a new idea in weak structure as well as in minimal structure which is as like the dense set in topological spaces. We shall discuss the properties of the above sets with the help of generalized open sets in minimal structure and in weak structure. We shall also give the representations of $\sigma(\omega)$ sets, $\pi(\omega)$ sets and *m-semiopen* sets, *m-preopen* sets of the minimal structure as like Levine's *semi-open* [7] set in topological spaces and Mashhour et al's *preopen* [9] set in topological spaces.

2. Preliminaries

We suggest beginning by recalling some notions and notations which have been defined in [5]. Let X be a non-empty set and ω be a collection of subsets of X. Then ω is called a weak structure (briefly WS) on X if $\phi \in \omega$. Then it is clear that each generalized topology (GT) [3, 4] and each minimal structure is a WS. We call the pair (X, ω) a weak structure space (WSS). The elements of ω are called ω-open sets and the complements are called ω-closed sets.

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Let ω be a weak structure on X and $A \subset X$. We define $i_\omega(A)$ as the union of all ω-open subsets of A (e.g. ϕ) and $c_\omega(A)$ as the intersection of all ω-closed sets containing A (e.g. X).

Theorem 2.1 ([5]). *If ω is a WS on X and $A, B \subset X$, then*
(i) $i_\omega(A) \subset A \subset c_\omega(A)$.
(ii) $A \subset B$ *implies* $i_\omega(A) \subset i_\omega(B)$.
(iii) $c_\omega(A) \subset c_\omega(B)$.
(iv) $i_\omega(i_\omega(A)) = i_\omega(A)$ *and* $c_\omega(c_\omega(A)) = c_\omega(A)$.
(v) $i_\omega(X - A) = X - c_\omega(A)$ *and* $c_\omega(X - A) = X - i_\omega(A)$.

Lemma 2.1 ([5]). *If ω is a WS on X, $x \in i_\omega(A)$ if and only if there is a ω-open set $W \subset A$ such that $x \in W$.*

Lemma 2.2 ([5]). *If ω is a WS, we have $x \in c_\omega(A)$ if and only if $W \cap A \neq \phi$ whenever $x \in W \in \omega$.*

Consider again a WS on X. With the help of i_ω and c_ω we can define further structures: for a set $A \subset X$; let (i) $A \in \alpha(\omega)$ if and only if $A \subset i_\omega(c_\omega(i_\omega(A)))$, (ii) $A \in \sigma(\omega)$ if and only if $A \subset c_\omega(i_\omega(A))$, (iii) $A \in \pi(\omega)$ if and only if $A \subset i_\omega(c_\omega(A))$, (iv) $A \in \beta(\omega)$ if and only if $A \subset c_\omega(i_\omega(c_\omega(A)))$.

Theorem 2.2 ([5]). *If ω is a WS, each of the structures $\alpha(\omega), \sigma(\omega), \pi(\omega), \beta(\omega)$ is a generalized topology.*

Theorem 2.3 ([5]). *If ω is a WS, we have $\omega \subset \alpha(\omega) \subset \sigma(\omega) \subset \beta(\omega)$ and $\omega \subset \alpha(\omega) \subset \pi(\omega)$.*

Definition 2.1 ([8]). A family $m \subset \wp(X)$ is said to be a minimal structure if $\phi, X \in m$. In this case (X, m) is called a minimal space.

A set $A \in m$ is said to be a m-open if $A \in m$. $B \subset X$ is a m-closed set if $X - B \in m$. We set $m\text{-}int(A) = \cup\{U : U \subset A, U \in m\}$ and $m\text{-}cl(A) = \cap\{F : A \subset F, X - F \in m\}$.

Lemma 2.3 ([14]). *Let (X, m) be a minimal space and A a subset of X. Then $x \in m\text{-}cl(A)$ if and only if $U \cap A \neq \phi$ for every $U \in m$ containing x.*

Proposition 2.1 ([14, 1, 2]). *For any two sets A and B,*
(i) $A \subset m\text{-}cl(A)$ *and* $A = m\text{-}cl(A)$ *if A is a m-closed set.*
(ii) $m\text{-}cl(A) \subset m\text{-}cl(B)$ *if* $A \subset B$.
(iii) $(m\text{-}cl(A)) \cup (m\text{-}cl(B)) \subset m\text{-}cl(A \cup B)$.
(iv) $m\text{-}cl(m\text{-}cl(B)) = m\text{-}cl(B)$.
(v) $X - (m\text{-}cl(A)) = m\text{-}int(X - A)$ *and* $X - (m\text{-}int(A)) = m\text{-}cl(X - A)$.

Definition 2.2 ([10, 11]). Let (X, m) be a minimal space and $A \subset X$. Then A is called *m-semiopen* if $A \subset m\text{-}cl(m\text{-}int(A))$.

The collection of all *m-semiopen* sets in (X, m) is denoted as $\sigma(m)$.

We know from [10], any union of *m-semiopen* sets is *m-semiopen*. Then $\sigma(m)$ forms a GT.

Definition 2.3 ([12, 13])**.** Let (X, m) be a minimal space and $A \subset X$. Then A is called *m-preopen* if $A \subset m\text{-}int(m\text{-}cl(A))$.

The collection of all *m-preopen* sets in (X, m) is denoted as $\pi(m)$.

3. ω-dense sets

Definition 3.1. Let ω be a WS on X. Then $A \subset X$ is called ω-*dense* if $c_\omega(D) = X$.

The collection of all ω-dense sets in WSS (X, ω) is denoted as $D^\omega(X, \omega)$. If a WS contains only ϕ, then all nonempty subsets are ω-dense sets.

Following example is related to the existence of ω-dense set.

Example 3.1. Let $X = \{a, b, c\}$ and $\omega = \{\phi, \{a\}, \{a, b\}\}$. Then ω-closed sets are: $\{X, \{c\}, \{b, c\}\}$. Here $\{a\}$, $\{a, c\}$ and X are ω-dense sets.

Theorem 3.1. *Let (X, ω) be a WSS and $D \subset X$. Then D is ω-dense if and only if $D \cap O \neq \phi$ for every nonempty $O \in \omega$.*

Proof. Let D be ω-dense. Then $c_\omega(D) = X$. Let O be any nonempty ω-open set in (X, ω). If possible suppose that $O \cap D = \phi$. Then for $x \in O$, $x \notin c_\omega(D) = X$, a contradiction.

Conversely suppose that $O \cap D \neq \phi$ for every nonempty $O \in \omega$. Let $x \in X$. Then by the hypothesis, there exists ω-open set $O_x \subset O$, $O_x \cap D \neq \phi$, hence $x \in c_\omega(D)$ (from Lemma 2.2). Hence the result. □

Corollary 3.1. *Let (X, ω) be a WSS and $D \subset X$. Then D is ω-dense if and only if $i_\omega(X - D) = \phi$.*

Theorem 3.2. *Let (X, ω) be a WSS and D_1, $D_2 \subset X$. Then for D_1, $D_2 \in D^\omega(X, \omega)$, $D_1 \cup D_2 \in D^\omega(X, \omega)$.*

Proof. Given that $c_\omega(D_1) = X$ and $c_\omega(D_2) = X$. Now $X = c_\omega(D_1) \subseteq c_\omega(D_1 \cup D_2)$. Thus $D_1 \cup D_2 \in D^\omega(X, \omega)$. □

Since $\phi \notin D^\omega(X, \omega)$, so $D^\omega(X, \omega)$ does not form a GT on X.

Following example shows that the intersection of two ω-dense sets may be a ω-dense set.

Example 3.2. Let $X = \{a, b, c\}$, $\omega = \{\phi, \{a, b\}\}$. Then ω-closed sets are: X, $\{c\}$. Here $\{a, c\}$, $\{b, c\} \in D^\omega(X, \omega)$, but $\{a, c\} \cap \{b, c\} = \{c\} \notin D^\omega(X, \omega)$.

Definition 3.2. A subset A of a WSS (X, ω) is called ω-nowhere dense if $i_\omega(c_\omega(A)) = \phi$.

It is clear from definition, ϕ is always a ω-nowhere dense set. But the following example shows that union of two ω-nowhere dense sets need not be a ω-nowhere dense set. So the collection of all ω-nowhere dense sets of (X, ω) may not be a GT on X but a WS on X.

Example 3.3. Let $X = \{a, b, c, d\}$ $\omega = \{\phi, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}\}$. ω-closed sets are: X, $\{d\}, \{a\}, \{c\}, \{b\}$. Here $\{b\}$ and $\{d\}$ are nowhere dense sets but $\{b, d\}$ is not a nowhere dense set, since $i_\omega(c_\omega(\{b, d\})) \neq \phi$.

Theorem 3.3. *Let (X, ω) be a WSS and $A \subset X$. Then A is ω-nowhere dense if and only if $X - c_\omega(A)$ is ω-dense.*

Proof. Suppose A is ω-nowhere dense in (X, ω). Then $i_\omega(c_\omega(A)) = \phi$ implies $X - i_\omega(c_\omega(A)) = X$. So $c_\omega(X - c_\omega(A)) = X$ (from Theorem 2.1), and hence $(X - c_\omega(A))$ is ω-dense in (X, ω).

Conversely suppose that $X - c_\omega(A)$ is ω-dense in (X, ω). Then $c_\omega(X - c_\omega(A)) = X$ implies $X - i_\omega(c_\omega(A)) = X$. So $i_\omega(c_\omega(A)) = \phi$, and hence A is ω-nowhere dense. □

In [7], Levine gives an interesting representation of semi-open set (Theorem 7) in topological space. But in the present case we are unable to give the similar representation of $A \in \sigma(\omega)$ in WSS, which is evident by the following example.

Example 3.4. Let $X = \{a, b, c\}$, $\omega = \{\phi, \{a\}, \{a, b\}\}$. Then ω-closed sets are: X, $\{b, c\}, \{c\}$. Here, $c_\omega(i_\omega(\{a, d\})) = c_\omega(\{a\}) = X$, so $\{a, d\} \in \sigma(\omega)$. Now $\{a, d\} = \{a\} \cup \{d\}$, $\{a\} \cap \{d\} = \phi$ and $\{a\} \in \omega$ but $\{d\}$ is not a ω-nowhere dense set. Again $\{a, d\}$ is not a ω-open set and $\{a, d\}$ is not a ω-nowhere dense set. So $\{a, d\} \neq O \cup B$, where (i) $O \cap B = \phi$ and (ii) $O \in \omega$ and B is ω-nowhere dense set.

Lemma 3.1. *Let (X, ω) be a WSS. Then*
(a) *for $A \in \sigma(\omega)$, A contains a nonempty ω-interior.*
(b) *for $A \in \alpha(\omega)$, A contains a nonempty ω-interior.*

Proof. The proof is obvious from the definitions of $\sigma(\omega)$, $\alpha(\omega)$, and Lemma 2.1 and Lemma 2.2. □

Theorem 3.4. *Let (X, ω) be a WSS. Then $D^\omega(X, \omega) = D^\omega(X, \sigma(\omega))$.*

Proof. We shall prove only the inclusion $D^\omega(X, \omega) \subset D^\omega(X, \sigma(\omega))$. Let $D \in D^\omega(X, \omega)$ and O be any nonempty $\sigma(\omega)$-open set. Since O contains a nonempty ω-interior (from Lemma 3.1), $O \cap D \neq \phi$. So, $D \in D^\omega(X, \sigma(\omega))$. □

Theorem 3.5. *Let (X, ω) be a WSS. Then $D^\omega(X, \omega) = D^\omega(X, \alpha(\omega))$.*

Proof. The proof is similar with Theorem 3.4. □

Theorem 3.6. *Let (X, ω) be a WSS. Then $D^\omega(X, \beta(\omega)) \subset D^\omega(X, \omega)$.*

Proof. The proof is obvious from the fact that $\omega \subset \beta(\omega)$. □

Following example shows that the reverse inclusion of Theorem 3.6 need not hold in general.

Example 3.5. Let $X = \{a, b, c, d\}$, $\omega = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then ω-closed sets are: $X, \{b, c, d\}, \{a, d\}, \{d\}$. Here $\{a, c, d\} \in D^{\omega}(X, \omega)$. Now

$$c_{\omega}(i_{\omega}(c_{\omega}(\{b\}))) = \{b, c, d\},$$

so $\{b\} \in \beta(\omega)$. Again $\{a, c, d\} \cap \{b\} = \phi$, so, $\{a, c, d\} \notin D^{\omega}(X, \beta(\omega))$.

For $\pi(\omega)$, we get following results:

Theorem 3.7. *Let (X, ω) be a WSS. Then $D^{\omega}(X, \pi(\omega)) \subset D^{\omega}(X, \omega)$.*

Proof. Since $\omega \subset \pi(\omega)$, therefore $D^{\omega}(X, \pi(\omega)) \subset D^{\omega}(X, \omega)$. □

Reverse inclusion of the above of Theorem need not hold in general.

Example 3.6. Let $X = \{a, b, c, d\}$, $\omega = \{\phi, \{a, b\}, \{c, d\}, X\}$. Here $\{a, c\} \in D^{\omega}(X, \omega)$. Now $\{b, d\} \in \pi(\omega)$, but $\{a, c\} \cap \{b, d\} = \phi$, so $\{a, c\} \notin D^{\omega}(X, \pi(\omega))$.

Definition 3.3. Let (X, ω) be a WSS and $A \subset X$. Then A is called *ωr-open* if $A = i_{\omega}(c_{\omega}(A))$.

The collection of all ωr-open sets in (X, ω) is denoted as $r(\omega)$.

Theorem 3.8. *Let (X, ω) be a WSS. Then $r(\omega) \subseteq \omega$.*

Proof. The proof is obvious. □

The following example shows that the reverse inclusion of Theorem 3.8 need not hold in general.

Example 3.7. Let $X = \{a, b, c\}$, $\omega = \{\phi, \{a\}, \{a, b\}\}$. Then ω-closed sets are: X, $\{b, c\}$, $\{c\}$. We consider the ω-open set $\{a\}$, $\{a\} \neq i_{\omega}(c_{\omega}(\{a\}))$. So $\{a\} \notin r(\omega)$.

Theorem 3.9. *Let (X, ω) be a WSS and $S \in \pi(\omega)$. Then the following stand true*:

(a) *There is a ωr-open set $G \subset X$ such that $S \subset G$ and $c_{\omega}(S) = c_{\omega}(G)$.*
(b) *S is the intersection of a ωr-open set and a ω-dense set.*
(c) *S is the intersection of a ω-open set and a ω-dense set.*

Proof. (a) Let $S \in \pi(\omega)$. Then $S \subset i_{\omega}(c_{\omega}(S))$. Put $G = i_{\omega}(c_{\omega}(S))$. Since $i_{\omega}c_{\omega}$ is idempotent [5], then G is *ωr-open* with $S \subset G$. Hence we get, $c_{\omega}(S) = c_{\omega}(G)$.

(b) Put $D = S \cup (X - G)$. Then we have, $c_{\omega}(D) = c_{\omega}(S \cup (X - G))$ and hence $c_{\omega}(S) \cup c_{\omega}(X - G) \subset c_{\omega}(D)$. Since $G \in r(\omega) \subset \omega$, then $c_{\omega}(S) \cup (X - G) \subset c_{\omega}(D)$. So, $X \subset c_{\omega}(D)$, since $c_{\omega}(S) = c_{\omega}(G)$. Therefore D is ω-dense. Now, we can write that $S = G \cap D$ and get the desired result.

(c) Since $r(\omega) \subset \omega$, then proof is obvious. □

If ω is replaced by the topology τ of the above theorem, then it is coincident with Proposition 1 of [6].

The following example shows that intersection of a ω-open set with a ω-dense set does not represent a member of $\pi(\omega)$.

Example 3.8. Let $X = \{a, b, c, d\}$ and $\omega = \{\phi, X, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}\}$. Then ω-closed sets are: $\phi, X, \{d\}, \{a\}, \{b\}, \{c\}$. Here $\{a, d\}$ is a ω-dense set in (X, ω). Now $\{a, d\} \cap \{a, b, c\} = \{a\}$ and $i_\omega(c_\omega(\{a\})) = i_\omega(\{a\}) = \phi$. Therefore $\{a\}$ is not a subset of $i_\omega(c_\omega(\{a\}))$. Hence $\{a\} \notin \pi(\omega)$.

4. *m*-dense sets

Definition 4.1. Let m be a minimal structure on X and $D \subset X$. Then D is called *m-dense* if $m\text{-}cl(D) = X$.

The collection of all m-dense sets in the minimal space (X, m) is denoted as $D^m(X, m)$.

Following example is related to the existence of m-dense set.

Example 4.1. Let $X = \{a, b, c\}$ and $\omega = \{\phi, \{a\}, X\}$. Then m-closed sets are: $\phi,\ X,\ \{b, c\}$. Here $\{a, b\}$ is m-dense.

It is obvious that $m \subset \omega$ where m and ω are the minimal structure and weak structure respectively on X. Hence $D^\omega(X, \omega) \subset D^m(X, m)$.

Theorem 4.1. *Let (X, m) be a minimal space and $D \subset X$. Then D is m-dense if and only if $D \cap O \neq \phi$ for every nonempty $O \in m$.*

Proof. The proof is similar with Theorem 3.1. □

Corollary 4.1. *Let (X, m) be a minimal space and $D \subset X$. Then D is m-dense if and only if $m\text{-}int(X - D) = \phi$.*

Theorem 4.2. *Let (X, m) be a minimal space and $D_1,\ D_2 \subseteq X$. Then for $D_1,\ D_2 \in D^m(X, m)$, $D_1 \cup D_2 \in D^m(X, m)$.*

Proof. The proof is similar with Theorem 3.2. □

The following example shows that intersection of two m-dense sets may be an m-dense set in general.

Example 4.2. Let $X = \{a, b, c\},\ m = \{\phi, \{a, b\}, X\}$. Then m-closed sets are: $X,\ \{c\}, \phi$. Here $\{a, c\},\ \{b, c\} \in D^m(X, m)$, but $\{a, c\} \cap \{b, c\} = \{c\} \notin D^m(X, m)$.

Definition 4.2. A subset A of a minimal space (X, m) is called *m-nowhere dense* if $m\text{-}int(m\text{-}cl(A)) = \phi$.

It is clear from the definition that ϕ is always a m-nowhere dense set. But the following example shows that union of two m-nowhere dense sets need not be an m-nowhere dense set. So the collection of all m-nowhere dense sets of (X, m) does not form a GT and not a minimal structure on X but a WS on X.

Example 4.3. Let $X = \{a, b, c, d\}, m = \{\phi, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}, X\}$. m-closed sets are: $X,\ \{d\}, \{a\}, \{c\}, \{b\}, \phi$. Here $\{b\}$ and $\{d\}$ are m-nowhere dense sets but $\{b, d\}$ is not a m-nowhere dense set, since $m\text{-}int(m\text{-}cl(\{b, d\})) \neq \phi$.

Theorem 4.3. *Let (X, m) be a minimal space and $A \subset X$. Then A is m-nowhere dense if and only if $(X - m\text{-}cl(A))$ is m-dense.*

Proof. The proof is similar to Theorem 3.3. □

We are also unable to give the similar representation of $A \in \sigma(m)$ in the minimal space (X, m) as like semi-open set of [7], which is evident from the following example.

Example 4.4. Let $X = \{a, b, c\}$, $m = \{\phi, \{a\}, \{a, b\}, X\}$. Then m-closed sets are: X, $\{b, c\}, \{c\}, \phi$. Here, $m\text{-}cl(m\text{-}int(\{a, d\})) = m\text{-}cl(\{a\}) = X$, so $\{a, d\} \in \sigma(m)$. Now $\{a, d\} = \{a\} \cup \{d\}$, $\{a\} \cap \{d\} = \phi$ and $\{a\} \in m$ but $\{d\}$ is not a m-nowhere dense set. Again $\{a, d\}$ is not a m-open set and $\{a, d\}$ is not a m-nowhere dense set. So $\{a, d\} \neq O \cup B$, where (i) $O \cap B = \phi$ and (ii) $O \in m$ and B is m-nowhere dense set.

Lemma 4.1. *Let (X, m) be a minimal space. Then for $A \in \sigma(m)$, A contains a nonempty m-interior.*

Proof. Proof is obvious from the definition of $\sigma(m)$ and Lemma 2.3. □

Theorem 4.4. *Let (X, m) be a minimal space. Then*

$$D^m(X, m) = D^m(X, \sigma(m)).$$

Proof. The proof is obvious from Lemma 4.1. □

Definition 4.3. Let (X, m) be a minimal space and $A \subset X$. Then A is called *mr-open* if $A = m\text{-}int(m\text{-}cl(A))$. The collection of all mr-open sets in (X, m) is denoted as $r(m)$.

Theorem 4.5. *Let (X, m) be a minimal space. Then $r(m) \subseteq m$.*

Proof. Proof is obvious. □

The following example shows that the reverse inclusion need not hold in general.

Example 4.5. Let $X = \{a, b, c\}$, $m = \{\phi, \{a\}, \{a, b\}, X\}$. Then m-closed sets are: X, $\{b, c\}$, $\{c\}, \phi$. We consider the m-open set $\{a\}$, $\{a\} \neq m\text{-}int(m\text{-}cl(\{a\}))$. So $\{a\} \notin r(m)$.

Theorem 4.6. *Let (X, m) be a minimal space and $S \in \pi(m)$. Then the following stand true:*

(a) *There is an mr-open set $G \subset X$ such that $S \subset G$ and $m\text{-}cl(S) = m\text{-}cl(G)$.*

(b) *S is the intersection of an mr-open set and a m-dense set.*

(c) *S is the intersection of an m-open set and a m-dense set.*

Proof. The proof is similar with Theorem 3.9. □

The following example shows that the converse of the above theorem need not hold in general.

Example 4.6. Let $X = \{a, b, c, d\}$ and $m = \{\phi, X, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}\}$. Then m-closed sets are: $\phi, X, \{d\}, \{a\}, \{b\}, \{c\}$. Here $\{a, d\}$ is a m-dense set in (X, m). Now $\{a, d\} \cap \{a, b, c\} = \{a\}$ and $m\text{-}int(m\text{-}cl(\{a\})) = m\text{-}int(\{a\}) = \phi$. Therefore $\{a\}$ is not a subset of $m\text{-}int(m\text{-}cl(\{a\}))$. Hence $\{a\} \notin \pi(m)$.

References

[1] M. Alimohammady and M. Roohi, *Fixed point in minimal spaces*, Nonlinear Anal. Model. Control **10** (2005), no. 4, 305–314.

[2] ———, *Linear minimal spaces*, Chaos Solitons Fractals **33** (2007), no. 4, 1348–1354.

[3] A. Csaszar, *Generalized open sets*, Acta Math. Hungar. **97** (1997), no. 1-2, 65–87.

[4] ———, *Generalized topology, generalized continuity*, Acta Math. Hungar. **96** (2002), no. 4, 351–357.

[5] ———, *Weak structures*, Acta Math. Hungar. **131** (2011), no. 1-2, 193–195.

[6] M. Ganster, *Preopen sets and resolvable spaces*, Kyungpook Math. J. **27** (1987), no. 2, 135–142.

[7] N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly **70** (1963), 36–41.

[8] H. Maki, J. Umehara, and T. Noiri, *Every topological space is pre T1/2*, Men. Fac. Sci. Kochi Univ. Ser. A Math. **17** (1996), 33–42.

[9] A. S. Mashhour, M. E. AbD El-Mosef, and S. N. El-Deeb, *On precontinuous and weak precontinuous mappings*, Proc. Math. Phys. Soc. Egypt **53** (1982), 47–53.

[10] W. K. Min, *m-semiopen sets and M-semicontinuous functions on spaces with minimal structures*, Honam Math. J. **31** (2009), no. 2, 239–245.

[11] ———, *On minimal semicontinuous functions*, Commun. Korean Math. Soc. **27** (2012), no. 2, 341–345.

[12] W. K. Min and Y. K. Kim, *On minimal precontinuous functions*, J. Chun. Math. Soc. **22** (2009), no. 4, 667–673.

[13] ———, *m-preopen sets and M-precontinuity on spaces with minimal structures*, Adv. Fuzzy Sets Syst. **4** (2009), no. 3, 237–245.

[14] O. B. Ozbakir and E. D. Yildirim, *On some closed sets in ideal minimal spaces*, Acta. Math. Hungar. **125** (1009), no. 3, 227–235.

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