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Supra b-irresolutness and Supra b-connectedness on Topological Space

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Abstract. In this paper, the concept of supra b-irresolute maps is introduced and several properties of it is investigated. Furthermore, the notion of supra b-connectedness is defined and researched by means of supra b-separated sets.

1. Introduction

Many authors introduced and studied various generalized properties and conditions containing some forms of sets in topological spaces. In 1983, Mashhour et al. [2] developed the supra topological spaces and studied s-continuous maps and $s^\star$-continuous maps. In 2008, Devi et al. [1] introduced and studied a class of sets and maps between topological spaces called supra α-open sets and supra α-continuous maps, respectively. In [3], the present authors introduced the concepts of supra b-open sets, supra b-continuous maps, supra b-open maps and supra b-closed maps and studied their properties. This paper is a continuation of the paper [3]. The purpose of this paper is to introduce the concept of supra b-irresolute maps. Besides developing the basic properties of these maps we study the notion of supra b-connectedness based on supra b-separated sets. We prove that supra b-connectedness is preserved by supra b-irresolute bijections.

Throughout this paper, (X,τ) , (Y,σ) and (Z,υ) (or simply, X , Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated. All sets are assumed to be subsets of topological spaces. The closure

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and the interior of a set A are denoted by $Cl(A)$ and $Int(A)$, respectively. A subcollection $\mu \subset 2^X$ is called a supra topology [2] on a nonempty set X if $X \in \mu$ and μ is closed under arbitrary union. (X, μ) is called a supra topological space. The elements of μ are said to be supra open in (X, μ) and the complement of a supra open set is said to be supra closed. The supra closure of a set A, denoted by $Cl^\mu(A)$, is the intersection of supra closed sets including A. The supra interior of a set A,denoted by $Int^\mu(A)$, is the union of supra open sets included in A. The supra topology μ on X is said to be associated with the topology τ if $\tau \subset \mu$. A set A is said to be supra α-open [1] (resp. supra b-open [3]) if $A \subseteq Int^\mu(Cl^\mu(Int^\mu(A)))$ (resp. $A \subseteq Cl^\mu(Int^\mu(A)) \cup Int^\mu(Cl^\mu(A))$). A set A is said to be supra b-closed if its complement $X - A$ is supra b-open. The supra $b-$closure of a set A, denoted by $Cl_b^\mu(A)$, is the intersection of supra $b-$closed sets including A. The supra $b-$interior of a set A,denoted by $Int_b^\mu(A)$, is the union of supra $b-$open sets included in A. A map $f : X \to Y$ is called a supra b-continuous map [3] if the inverse image of each open set in Y is a supra b-open set in X.

2. Supra b-irresolute Maps

In this section, we introduce a new type of maps called a supra b-irresolute maps and obtain some of their properties and characterizations.

Definition 2.1. A map $f : (X, \tau) \to (Y, \sigma)$ is called a supra b-irresolute map if the inverse image of each supra b-open set in Y is a supra b-open set in X.

Theorem 2.1. *Every supra b-irresolute map is a supra b-continuous map.*

Proof. Straightforward. □

The converse of the above theorem is not true as shown in the following example.

Example 2.1. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$ be a topology on X. The supra topology μ is defined as follows: $\mu = \{X, \phi, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \to (X, \tau)$ be a map defined as follows: $f(a) = b$, $f(b) = c$,and $f(c) = a$. The inverse image of the open set $\{a, b\}$ is $\{a, c\}$ which is a supra b-open set. Hence f is a supra b-continuous map. Furthermore, the inverse image of the supra b-open set $\{a\}$ is $\{c\}$ which is not a supra b-open set. Hence f is not a supra b-irresolute map.

Theorem 2.2. *Let (X, τ) and (Y, σ) be two topological spaces and μ and ν associated supra topologies with τ and σ, respectively. Let f be a map from X into Y. Then the following are equivalent:*
(1) *f is a supra b-irresolute map;*
(2) *The inverse image of each supra b-closed set in Y is a supra b-closed set in X;*
(3) *$Cl_b^\mu(f^{-1}(B)) \subseteq f^{-1}(Cl_b^\nu(B))$ for every set B in Y;*
(4) *$f(Cl_b^\mu(A)) \subseteq Cl_b^\nu(f(A))$ for every set A in X;*
(5) *$f^{-1}(Int_b^\nu(B)) \subseteq Int_b^\mu(f^{-1}(B))$ for every B in Y.*

Proof. (1)⇒(2): Let B be a supra b-closed set in Y. Then $Y-B$ is a supra b-open set in Y. Hence $f^{-1}(Y-B)=X-f^{-1}(B)$ is a supra b-open set in X. It follows that $f^{-1}(B)$ is a supra b-closed subset of X.
(2)⇒(3): Let B be any subset of Y. Since $Cl_b^{\nu}(B)$ is a supra b-closed set in Y, then $f^{-1}(Cl_b^{\nu}(B))$ is a supra b-closed set in X. Therefore $Cl_b^{\mu}(f^{-1}(B)) \subseteq Cl_b^{\mu}(f^{-1}(Cl_b^{\nu}(B))) = f^{-1}(Cl_b^{\nu}(B))$.
(3)⇒(4): Let A be any subset of X. By (3) we have $f^{-1}(Cl_b^{\nu}(f(A))) \supseteq Cl_b^{\mu}(f^{-1}(f(A))) \supseteq Cl_b^{\mu}(A)$.Therefore $f(Cl_b^{\mu}(A)) \subseteq Cl_b^{\nu}(f(A))$.
(4)⇒(5): Let B be any subset of Y. By (4), $f(Cl_b^{\mu}(X-f^{-1}(B))) \subseteq Cl_b^{\nu}(f(X-f^{-1}(B)))$ and $f(X-Int_b^{\mu}(f^{-1}(B))) \subseteq Cl_b^{\nu}(Y-B) = Y-Int_b^{\nu}(B)$. Therefore we have $X-Int_b^{\mu}(f^{-1}(B)) \subseteq f^{-1}(Y-Int_b^{\nu}(B))$ and hence $f^{-1}(Int_b^{\nu}(B)) \subseteq Int_b^{\mu}(f^{-1}(B))$.

(5)⇒(1): Let B be a supra b-open set in Y and $f^{-1}(Int_b^{\nu}(B)) \subseteq Int_b^{\mu}(f^{-1}(B))$. Then $f^{-1}(B) \subseteq Int_b^{\mu}(f^{-1}(B))$. But, $Int_b^{\mu}(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence $f^{-1}(B) = Int_b^{\mu}(f^{-1}(B))$. Therefore $f^{-1}(B)$ is supra b-open in X. □

Theorem 2.3. *If $f: X \to Y$ and $g: Y \to Z$ are supra b-irresolute maps, then $g \circ f: X \to Z$ is supra b-irresolute.*

Proof. Obvious. □

3. Supra b-separated Sets

In this section, we shall research supra b-separated sets in supra topological spaces.

Definition 3.1. Let (X,μ) be a supra topological space and A, B be non-empty subsets of X. Then A and B are said to be supra b-separated if $A \cap Cl_b^{\mu}(B) = \phi$ and $Cl_b^{\mu}(A) \cap B = \phi$.

The following result is immediate from the above definition.

Theorem 3.1. *Let C and D are non-empty subsets of the supra b-separated sets A and B, respectively. Then C and D are also supra b-separated in X.*

Theorem 3.2. *Let A, B be non-empty subsets of X such that $A \cap B = \phi$ and A, B are either both supra b-open or both supra b-closed. Then A and B are supra b-separated.*

Proof. If both A and B are supra b-closed sets and $A \cap B = \phi$, then A and B are supra b- separated. Let now A and B be supra b-open and $A \cap B = \phi$. Then $A \subseteq X-B$. So $Cl_b^{\mu}(A) \subseteq Cl_b^{\mu}(X-B) = X-Int_b^{\mu}(B) = X-B$. Hence $Cl_b^{\mu}(A) \cap B = \phi$. Similarly, $A \cap Cl_b^{\mu}(B) = \phi$. Thus A and B are supra b-separated. □

Theorem 3.3. *Suppose that A and B are non-empty subsets of X such that either they are both supra b-open or they are both supra b-closed. If $C = A \cap (X-B)$ and $D = B \cap (X-A)$,then C and D are supra b-separated, provided they are non-empty.*

Proof. First suppose A and B are both supra b-open. Now $D = B \cap (X - A)$ implies $D \subseteq X - A$. Then $Cl_b^\mu(D) \subseteq Cl_b^\mu(X - A) = X - Int_b^\mu(A) = X - A$. Hence $A \cap Cl_b^\mu(D) = \phi$. Therefore $C \cap Cl_b^\mu(D) = \phi$. Similarly, $Cl_b^\mu(C) \cap D = \phi$. Thus C and D are supra b-separated.

Next, suppose that A and B are both supra b-closed sets. Then $C = A \cap (X - B)$ implies $C \subseteq A$. Hence $Cl_b^\mu(C) \subseteq Cl_b^\mu(A) = A$. Therefore $Cl_b^\mu(C) \cap D = \phi$. Similarly, $C \cap Cl_b^\mu(D) = \phi$. Thus C and D are supra b-separated. □

Theorem 3.4. *Non-empty subsets A and B of X are supra b-separated if and only if there exist supra b-open sets U and V such that $A \subseteq U$, $B \subseteq V$, $A \cap V = \phi$, and $B \cap U = \phi$.*

Proof. Suppose that A and B are supra b-separated. Now, $A \cap Cl_b^\mu(B) = \phi$ and $Cl_b^\mu(A) \cap B = \phi$. Then $A \subseteq X - Cl_b^\mu(B) = U$ (say); and $B \subseteq X - Cl_b^\mu(A) = V$ (say). Since both $Cl_b^\mu(A)$ and $Cl_b^\mu(B)$ are supra b-closed, then both U and V are supra b-open. Therefore $A \subseteq Cl_b^\mu(A) = X - V$ and $B \subseteq Cl_b^\mu(B) = X - U$. Hence $A \cap V = \phi$ and $B \cap U = \phi$.

Conversely, let U and V be supra b-open sets such that $A \subseteq U$, $B \subseteq V$, $A \cap V = \phi$ and $B \cap U = \phi$. Then $X - U$ and $X - V$ are supra b-closed. Also, $A \cap V = \phi$ implies $A \subseteq X - V$. Therefore $Cl_b^\mu(A) \subseteq Cl_b^\mu(X - V) = X - V$. Hence $Cl_b^\mu(A) \cap V = \phi$ and $Cl_b^\mu(A) \cap B = \phi$. Similarly, $U \cap Cl_b^\mu(B) = \phi$ and $A \cap Cl_b^\mu(B) = \phi$. Thus A and B are supra b-separated. □

4. Supra b-connectedness

In this section, we research supra b-connectedness by means of supra b-separated sets.

Definition 4.1. A subset A of X is said to be supra b-connected if it can't be represented as a union of two non-empty supra b-separated sets. If X is supra b-connected, then X is called a supra b-connected space.

Theorem 4.1. *A non-empty subset C of X is supra b-connected if and only if for every pair of supra b-separated sets A and B in X with $C \subseteq A \cup B$, one of the following possibilities holds:*
(a) $C \subseteq A$ and $C \cap B = \phi$,
(b) $C \subseteq B$ and $C \cap A = \phi$.

Proof. Let C be supra b-connected. Since $C \subseteq A \cup B$, then both $C \cap A = \phi$ and $C \cap B = \phi$ can not hold simultaneously. If $C \cap A \neq \phi$ and $C \cap B \neq \phi$, then by Theorem 3.1 they are also supra b-separated and $C = (C \cap A) \cup (C \cap B)$ which goes against the supra b-connectedness of C. Now, if $C \cap A = \phi$, then $C \subseteq B$, while $C \subseteq A$ holds if $C \cap B = \phi$.
Conversely, suppose that the given condition holds. Assume that C is not supra b-connected. Then there exist non-empty supra b-separated sets A and B in X

such that $C = A \cup B$. By hypothesis, either $C \cap A = \phi$ or $C \cap B = \phi$. So, either $A = \phi$ or $B = \phi$, none of which is true. Thus C is supra b-connected. □

Theorem 4.2. *The following properties are equivalent:*
(1) *A space X is not supra b-connected.*
(2) *There exist non-empty supra b-closed sets A and B such that $A \cup B = X$ and $A \cap B = \phi$.*
(3) *There exist non-empty supra b-open sets A and B such that $A \cup B = X$ and $A \cap B = \phi$.*

Proof. (1)⇒(2): Suppose that X is not supra b-connected. Then there exist non-empty subsets A and B such that $Cl_b^{\mu}(A) \cap B = A \cap Cl_b^{\mu}(B) = \phi$ and $A \cup B = X$. It follows that $Cl_b^{\mu}(A) = Cl_b^{\mu}(A) \cap (A \cup B) = (Cl_b^{\mu}(A) \cap A) \cup (Cl_b^{\mu}(A) \cap B) = A \cup \phi = A$. Hence A is a supra b-closed set. Similarly, B is supra b-closed. Thus (2) is held.
(2)⇒(3) and (3)⇒(1): Obvious. □

Corollary 4.1. *The following are properties equivalent:*
(1) *A space X is supra b-connected.*
(2) *If A and B are supra b-open sets, $A \cup B = X$ and $A \cap B = \phi$, then $A = \phi$ or $B = \phi$.*
(3) *If A and B are supra b-closed sets, $A \cup B = X$ and $A \cap B = \phi$, then $A = \phi$ or $B = \phi$.*

Theorem 4.3. *For a subset G of X, the following conditions are equivalent:*
(1) *G is supra b-connected.*
(2) *There do not exist two supra b-closed sets A and B such that $A \cap G \neq \phi$, $B \cap G \neq \phi, G \subseteq A \cup B$ and $A \cap B \cap G = \phi$.*
(3) *There do not exist two supra b-closed sets A and B such that $G \nsubseteq A$, $G \nsubseteq B, G \subseteq A \cup B$ and $A \cap B \cap G = \phi$.*

Proof. (1)⇒(2): Suppose that G is supra b-connected and there exist two supra b-closed sets A and B such that $A \cap G \neq \phi$, $B \cap G \neq \phi, G \subseteq A \cup B$ and $A \cap B \cap G = \phi$. Then $(A \cap G) \cup (B \cap G) = (A \cup B) \cap G = G$. Also, $Cl_b^{\mu}(A \cap G) \cap (B \cap G) \subseteq Cl_b^{\mu}(A) \cap (B \cap G) = A \cap B \cap G = \phi$. Similarly, $(A \cap G) \cap Cl_b^{\mu}(B \cap G) = \phi$. This shows that G is not supra b-connected, which is a contradiction.
(2)⇒(3): Suppose that there exist two supra b-closed sets A and B such that $G \nsubseteq A$, $G \nsubseteq B, G \subseteq A \cup B$ and $A \cap B \cap G = \phi$. Then $A \cap G \neq \phi$ and $B \cap G \neq \phi$. This is a contradiction.
(3)⇒(1): Suppose that (3) is satisfied and G is not supra b-connected. Then there exist two non-empty supra b-separated sets C and D such that $G = C \cup D$. Thus $Cl_b^{\mu}(C) \cap D = C \cap Cl_b^{\mu}(D) = \phi$. Assume that $A = Cl_b^{\mu}(C)$ and $B = Cl_b^{\mu}(D)$. Hence $G \subseteq A \cup B$ and $Cl_b^{\mu}(C) \cap Cl_b^{\mu}(D) \cap (C \cup D) = (Cl_b^{\mu}(C) \cap Cl_b^{\mu}(D) \cap C)$ $\cup (Cl_b^{\mu}(C) \cap Cl_b^{\mu}(D) \cap D) \subset (Cl_b^{\mu}(D) \cap C) \cup (Cl_b^{\mu}(C) \cap D) = \phi \cup \phi = \phi$. Now we prove that $G \nsubseteq A$ and $G \nsubseteq B$. In fact, if $G \subseteq A$, then $Cl_b^{\mu}(D) \cap G = B \cap G$ $= B \cap (G \cap A) = \phi$, a contradiction. Thus $G \nsubseteq A$. Analogously we have $G \nsubseteq B$. This contradicts (3). Therefore G is supra b-connected. □

Corollary 4.2. *A space X is supra b-connected if and only if there do not exist two non-empty supra b-closed sets A and B such that $A \cup B = X$ and $A \cap B = \phi$.*

Theorem 4.4. *For a subset G of X, the following conditions are equivalent:*
(1) *G is supra b-connected.*
(2) *For any supra b-separated sets A and B with $G \subseteq A \cup B$, we have $G \cap A = \phi$ or $G \cap B = \phi$.*
(3) *For any supra b-separated sets A snd B with $G \subseteq A \cup B$, we have $G \subseteq A$ or $G \subseteq B$.*

Proof. (1)⇒(2): Suppose that A and B are supra b-separated and $G \subseteq A \cup B$. Then by Theorem 3.1 we have $G \cap A$ and $G \cap B$ are also supra b-separated. Since G is supra b-connected and $G = G \cap (A \cup B) = (G \cap A) \cup (G \cap B)$, then $G \cap A = \phi$ or $G \cap B = \phi$.
(2)⇒(3): If $G \cap A = \phi$, then $G = G \cap (A \cup B) = (G \cap A) \cup (G \cap B) = G \cap B$. So, $G \subseteq B$. Similarly, $G \cap B = \phi$ implies $G \subseteq A$.

(3)⇒(1): Suppose that A and B are supra b-separated and $G = A \cup B$. Then by (3) either $G \subseteq A$ or $G \subseteq B$. If $G \subseteq A$, then $B = B \cap G \subseteq B \cap A \subseteq B \cap Cl_b^\mu(A) = \phi$. Similarly, if $G \subseteq B$, then $A = \phi$. So G can not be represented as a union of two non-empty supra b-separated sets. Therefore G is supra b-connected. □

Theorem 4.5. *Let G be a supra b-connected subset of X. If $G \subseteq H \subseteq Cl_b^\mu(G)$, then H is also supra b-connected.*

Proof. Suppose that H is not supra b-connected. By Theorem 4.3 there exist two supra b-closed sets A and B such that $H \nsubseteq A$, $H \nsubseteq B$, $H \subseteq A \cup B$ and $A \cap B \cap H = \phi$. Since $G \subseteq H$, then $G \subseteq A \cup B$ and $A \cap B \cap G = \phi$. Now we prove that $G \nsubseteq A$ and $G \nsubseteq B$. In fact, if $G \subseteq A$, then $Cl_b^\mu(G) \subseteq Cl_b^\mu(A) = A$. Therefore by hypothesis $H \subseteq A$ which is a contradiction. Hence $G \nsubseteq A$. Similarly, $G \nsubseteq B$. This contradicts that G is supra b-connected. □

Theorem 4.6. *Let G and H be supra b-connected. If G and H are not supra b-separated, then $G \cup H$ is supra b-connected.*

Proof. Suppose that $G \cup H$ is not supra b-connected. By Theorem 4.3 there exist two supra b-closed sets A and B such that $G \cup H \nsubseteq A, G \cup H \nsubseteq B, G \cup H \subseteq A \cup B$ and $(G \cup H) \cap (A \cap B) = \phi$. So, either $G \nsubseteq A$ or $H \nsubseteq A$. Assume $G \nsubseteq A$. Then $G \subseteq B$ because G is supra b-connected. Hence $H \nsubseteq B$ and $H \subseteq A$. Thus $A \cap G \subseteq A \cap B \cap (G \cup H) = \phi$. Therefore $Cl_b^\mu(H) \cap G \subseteq Cl_b^\mu(A) \cap G = A \cap G = \phi$. Similarly, $H \cap Cl_b^\mu(G) = \phi$. This shows that G and H are supra b-separated, a contradiction. □

Theorem 4.7. *Let $\{G_i\}_{i \in I}$ be a family of supra b-connected subsets of X. If there is $j \in I$ such that G_i and G_j are not supra b-separated for each $i \neq j$, then $\bigcup_{i \in I} G_i$ is supra b-connected.*

Proof. Suppose that $\bigcup_{i \in I} G_i$ is not supra b-connected. Then there exist non-empty supra b-separated subsets A and B of X such that $\bigcup_{i\in I} G_i = A \cup B$. For each $i \in I$, G_i is supra b-connected and $G_i \subseteq A \cup B$. Then by Theorem 4.1 either $G_i \subseteq A$ and $G_i \cap B = \phi$, or else $G_i \subseteq B$ and $G_i \cap A = \phi$. If possible, let for some $r, s \in I$ with $r \neq s, G_r \subseteq A$ and $G_s \subseteq B$. Then G_r, G_s being non-empty of supra b-separated sets which is not the case. Thus either $G_i \subseteq A$ with $G_i \cap B = \phi$ for each $i \in I$ or else $G_i \subseteq B$ with $G_i \cap A = \phi$ for each $i \in I$. In the first case $B = \phi$ (since $B \subseteq \bigcup_{i\in I} G_i$) and in the second case $A = \phi$. Non of which is true. Thus $\bigcup_{i\in I} G_i$ is supra b-connected. □

Corollary 4.3. *Let $\{G_i\}_{i\in I}$ be a family of supra b-connected sets. If $\bigcap_{i\in I} G_i \neq \phi$, then $\bigcup_{i\in I} G_i$ is supra b-connected.*

Theorem 4.8. *A non-empty subset G of X is supra b-connected if and only if for any two elements x and y in G there exists a supra b-connected set H such that $x, y \in H \subseteq G$.*

Proof. The necessity is obvious. Now we prove the sufficiency. Suppose by contrary that G is not supra b-connected. Then there exist two non-empty supra b-separated P, Q in X such that $G = P \cup Q$. Choose $x \in P$ and $y \in Q$. So, $x, y \in G$ and hence by hypothesis there exists a supra b-connected set H such that $x, y \in H \subseteq G$. Thus $H \cap P$ and $H \cap Q$ are non-empty supra b-separated sets with $H = (P \cap H) \cup (Q \cap H)$. This is contrary to the supra b-connectedness of H. □

Theorem 4.9. *If $f : X \longrightarrow Y$ is a supra b-irresolute surjective map and C, D are supra b-separated sets in Y, then $f^{-1}(C)$ and $f^{-1}(D)$ are supra b-separated in X.*

Proof. Since C and D are supra b-separated, we have $Cl_b^v(C) \cap D = C \cap Cl_b^v(D) = \phi$. By Theorem 2.2, $Cl_b^\mu(f^{-1}(C)) \cap f^{-1}(D) \subset f^{-1}(Cl_b^v(C)) \cap f^{-1}(D) = f^{-1}(Cl_b^v(C)) \cap D) = \phi$. Similarly, we have $f^{-1}(C) \cap Cl_b^\mu(f^{-1}(D)) = \phi$. Therefore, $f^{-1}(C)$ and $f^{-1}(D)$ are supra b-separated. □

Theorem 4.10. *If $f : X \longrightarrow Y$ is supra b-irresolute bijective and A is supra b-connected in X, then f(A) is supra b-connected in Y.*

Proof. Suppose that $f(A)$ is not supra b-connected in Y. Then $f(A) = C \cup D$, where C and D are two non-empty supra b-separated in Y. By Theorem 4.9 $f^{-1}(C)$ and $f^{-1}(D)$ are supra b-separated in X. Since f is bijective, then $A = f^{-1}(f(A)) = f^{-1}(C) \cup f^{-1}(D)$. Hence A is not supra b-connected in X. This is a contradiction. Thus $f(A)$ is supra b-connected in Y. □

Theorem 4.11. *If $f : X \longrightarrow Y$ is a supra b-irresolute map and G is supra b-connected in X, then f(G) is supra b-connected in Y.*

Proof. Suppose that $f(G)$ is not supra b-connected in Y. Then, by Theorem 4.3, there exist two supra b-closed sets A and B in Y such that $f(G) \nsubseteq A$, $f(G) \nsubseteq B$, $f(G) \subseteq A \cup B$ and $f(G) \cap A \cap B = \phi$. Hence we have that $G \nsubseteq f^{-1}(A)$, $G \nsubseteq f^{-1}(B)$, $G \subseteq f^{-1}(A) \cup f^{-1}(B)$ and $G \cap f^{-1}(A) \cap f^{-1}(B) \subseteq f^{-1}(f(G)) \cap f^{-1}(A) \cap f^{-1}(B) = f^{-1}(f(G) \cap A \cap B) = \phi$. This implies that G is not supra b-connected, a contradiction. Therefore $f(G)$ is supra b-connected in Y. □

Corollary 4.4. *If $f : X \longrightarrow Y$ is a supra b-irresolute surjective map and X is supra b-connected, then so is Y.*

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