KYUNGPOOK Math. J. 53(2013), 333-340
http://dx.doi.org/10.5666/KMJ.2013.53.3.333

Eigenvalues of Type r of the Basic Dirac Operator on Kähler Foliations

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Abstract. In this paper, we prove that on a Kähler spin foliatoin of codimension $q = 2n$, any eigenvalue λ of type r ($r \in \{1, \cdots, [\frac{n+1}{2}]\}$) of the basic Dirac operator D_b satisfies the inequality $\lambda^2 \geq \frac{r}{4r-2} \inf_M \sigma^\nabla$, where σ^∇ is the transversal scalar curvature of $\mathcal{F}$.

1. Introduction

On a Kähler spin foliation $(M, \mathcal{F})$ of codimension $q = 2n$, any eigenvalue λ of the basic Dirac operator D_b satisfies

$$(1.1) \qquad \lambda^2 \geq \begin{cases} \dfrac{n+1}{4n} \inf\limits_M K^\sigma & \text{if } n \text{ is odd [6,7]}, \\ \dfrac{n}{4(n-1)} \inf\limits_M K^\sigma & \text{if } n \text{ is even [4]}, \end{cases}$$

where $K^\sigma = \sigma^\nabla + |\kappa|^2$ with the transversal scalar curvature σ^∇ and the mean curvature form κ of $\mathcal{F}$. In the limiting cases, $\mathcal{F}$ is minimal. For the point foliation, see [9,10]. Since the limiting cases of (1.1) are minimal, the inequalities (1.1) yield the following:

$$(1.2) \qquad \lambda^2 \geq \begin{cases} \dfrac{n+1}{4n} \inf\limits_M \sigma^\nabla & \text{if } n \text{ is odd}, \\ \dfrac{n}{4(n-1)} \inf\limits_M \sigma^\nabla & \text{if } n \text{ is even}. \end{cases}$$

In this paper, we give an estimate of the eigenvalues λ of type r of the basic Dirac operator D_b on a Kähler spin foliation. Recently, G. Habib and K. Richardson [5] proved that the spectrum of the basic Dirac operator does not change with respect to a change of bundle-like metric. And the existence of a bundle-like metric such that $\delta_B \kappa_B = 0$ is assured by A. Mason [11]. Hence we have the following.

Received August 15, 2011; accepted September 17, 2012.
2010 Mathematics Subject Classification: 53C12,53C27, 57R30.
Key words and phrases: Transversal(basic) Dirac operator, Kähler spin foliation.

Theorem 1.1. *Let $(M, g_M, \mathcal{F})$ be a compact Riemannian manifold with a Kähler spin foliation $\mathcal{F}$ of codimension $q = 2n$ and a bundle-like metric g_M. Assume that the transversal scalar curvature is non-negative. Then any eigenvalue λ of type $r(r \in \{1, \cdots, [\frac{n+1}{2}]\})$ of the basic Dirac operator satisfies*

$$\lambda^2 \geq \frac{r}{4r-2} \inf_M \sigma^\nabla. \tag{1.3}$$

From (1.3), if n is odd (i.e., $r = \frac{n+1}{2}$) or n is even (i.e., $r = \frac{n}{2}$), then the inequalities (1.2) are induced.

2. The basic Dirac Operator on a Kähler Spin Foliation

In this section, we summarize some facts on Kähler spin foliations which are studied in [6,7,12]. Let $(M, g_M, \mathcal{F}, S(\mathcal{F}))$ be a Riemannian manifold with a Kähler spin foliation $\mathcal{F}$ of codimension $q = 2n$, a bundle-like metric g_M with respect to $\mathcal{F}$ and a foliated spinor bundle $S(\mathcal{F})$ with a hermitian metric $\langle \cdot, \cdot \rangle$. We recall the exact sequence

$$0 \to L \to TM \xrightarrow{\pi} Q \to 0 \tag{2.1}$$

determined by the tangent bundle L and the normal bundle Q of $\mathcal{F}$. The metric g_M determines an orthogonal decomposition $TM = L \oplus L^\perp$, identifying Q with $L^\perp$ and inducing a metric g_Q on Q. Let ∇ be the transversal Levi-Civita connection on $Q = TM/L$, which is torsion-free and metric with respect to g_Q [8,14]. Let R^∇, ρ^∇ and σ^∇ be the transversal curvature tensor, transversal Ricci tensor and transversal scalar curvature with respect to ∇, respectively. The foliation $\mathcal{F}$ is said to be *minimal* if the mean curvature form κ vanishes. Let $\Omega_B^*(\mathcal{F})$ be the space of all *basic forms* ϕ, which are defined by $i(X)\phi = 0 = i(X)d\phi$ for $X \in \Gamma L$. Then $L^2(\Omega^*(M))$ is decomposed as [1]

$$L^2(\Omega(M)) = L^2(\Omega_B(\mathcal{F})) \oplus L^2(\Omega_B(\mathcal{F}))^\perp. \tag{2.2}$$

Let $P : L^2(\Omega^*(M)) \to L^2(\Omega_B^*(\mathcal{F}))$ be the orthogonal projection onto basic forms [13], which preserves smoothness in the case of Riemannian foliations. For any r-form ω, we put the basic part of ω as $\omega_B := P\omega$. Then it is well-known [1] that $\kappa_B := P\kappa$ is closed. Let $\Delta_B = d_B\delta_B + \delta_B d_B$ be the *basic Laplacian*, where δ_B is the formal adjoint operator of $d_B = d|_{\Omega_B^*(\mathcal{F})}$.

Let $J : Q \to Q$ be an almost complex structure on Q. Then the basic Kähler form $\Omega \in \Omega_B^2(\mathcal{F})$ is defined by

$$\Omega(X, Y) = g_Q(X, JY), \quad \forall X, Y \in \Gamma Q. \tag{2.3}$$

By the Clifford multiplication in the fibers of $S(\mathcal{F})$ for any vector field X in Q and any transversal spinor field Ψ, the Clifford product $X \cdot \Psi$, which is also a transversal spinor field, is defined. This product has the following properties: for all $X, Y \in \Gamma Q$ and $\Phi, \Psi \in \Gamma S(\mathcal{F})$,

$$(X \cdot Y + Y \cdot X)\Psi = -2g_Q(X, Y)\Psi, \tag{2.4}$$

$$\langle X \cdot \Psi, \Phi\rangle + \langle \Psi, X \cdot \Phi\rangle = 0, \tag{2.5}$$

$$\nabla^S_Y(X \cdot \Psi) = (\nabla_Y X) \cdot \Psi + X \cdot (\nabla^S_Y \Psi), \tag{2.6}$$

where ∇^S is a metric covariant derivation on $S(\mathcal{F})$, i.e., for all $X \in \Gamma Q$, and all $\Psi, \Phi \in \Gamma S(\mathcal{F})$, it holds

$$X\langle \Psi, \Phi\rangle = \langle \nabla^S_X \Psi, \Phi\rangle + \langle \Psi, \nabla^S_X \Phi\rangle. \tag{2.7}$$

If we have no confusion, we use the same notation ∇. Moreover, if we define the Clifford product $\xi \cdot \Psi$ of a 1-form $\xi \in Q^*$ and a transversal spinor field Ψ as $\xi \cdot \Psi \equiv \xi^\sharp \cdot \Psi$, where $\xi^\sharp$ is a g_Q-dual vector of ξ, then any basic r-form can be considered as an endomorphism of $S(\mathcal{F})$. So Ω is locally expressed as

$$\Omega = -\frac{1}{2}\sum_{a=1}^{2n} E_a \cdot JE_a = \frac{1}{2}\sum_{a=1}^{2n} JE_a \cdot E_a, \tag{2.8}$$

where $\{E_a\}_{a=1,\cdots,2n}$ is a local orthonormal basic frame in Q. From (2.4), (2.5) and (2.8), Ω is skew-symmetric and, for any $X \in \Gamma Q$,

$$X \cdot \Omega - \Omega \cdot X = 2JX. \tag{2.9}$$

Note that, by the action of the Kähler form, the foliated spinor bundle $S(\mathcal{F})$ splits into the orthogonal direct sum

$$S(\mathcal{F}) = S_0 \oplus S_1 \oplus \cdots \oplus S_n, \tag{2.10}$$

where the fiber $(S_r)_x$ of the subbundle S_r is just defined as the eigenspace corresponding to the eigenvalue $i\mu_r,\ \mu_r = n - 2r (r = 0, \cdots, n)$ of Ω_x [7,9]. Let $p_r : S(\mathcal{F}) \to S_r$ be the projection. Then, for any $X \in \Gamma Q$,

$$Xp_s = p_{s-1}Xp_s + p_{s+1}Xp_s, \tag{2.11}$$

$$J(X)p_s = -ip_{s-1}Xp_s + ip_{s+1}Xp_s \quad (s \in \mathbb{N}), \tag{2.12}$$

where $p_s = 0$ for $s \notin \{0, 1, \cdots, n\}$. Moreover, the foliated spinor bundle of a Kähler spin foliation carries an anti-linear map j satisfying the relations [7]:

$$\nabla j = 0,\ [X, j] = 0,\ [\Omega, j] = 0,\ j^2 = (-1)^{\frac{n(n+1)}{2}} \tag{2.13}$$

$$\langle j\Psi, j\Phi\rangle = \langle \Phi, \Psi\rangle, \quad jp_r = p_{n-r}j. \tag{2.14}$$

The transversal Dirac operator D_{tr} acting on sections of the foliated spinor bundle $S(\mathcal{F})$ is locally given by [2,3]

$$D_{tr}\Psi = \sum_a E_a \cdot \nabla_{E_a}\Psi - \frac{1}{2}\kappa_B^\sharp \cdot \Psi. \tag{2.15}$$

At any point $x \in M$, we choose normal coordinates at this point so that $(\nabla E_a)(x) = 0$, for all a. From now on, all the computations in this paper will be made in such charts. Associated with J, there is a transversally elliptic self-adjoint operator $\tilde{D}_{tr}$ locally defined by [6,7]

$$\tilde{D}_{tr}\Psi = \sum_a JE_a \cdot \nabla_{E_a}\Psi - \frac{1}{2}J\kappa_B^\sharp \cdot \Psi. \tag{2.16}$$

Then we have the following theorem.

Theorem 2.1([7]). *Let $(M, g_M, \mathcal{F})$ be a Riemannian manifold with a Kähler spin foliation $\mathcal{F}$ and a bundle-like metric g_M such that $\delta_B\kappa_B = 0$. Suppose the basic part of κ is transversally holomorphic, i.e., $\nabla_{JX}\kappa_B^\sharp = J\nabla_X\kappa_B^\sharp$ for all $X \in \Gamma Q$. Then, for any $\Psi \in S(\mathcal{F})$,*

$$D_{tr}^2\Psi = \tilde{D}_{tr}^2\Psi = \nabla_{tr}^*\nabla_{tr}\Psi + \frac{1}{4}K^\sigma\Psi, \tag{2.17}$$

$$D_{tr}\tilde{D}_{tr} + \tilde{D}_{tr}D_{tr} = 0, \tag{2.18}$$

where $K^\sigma = |\kappa_B|^2 + \sigma^\nabla$ and $\nabla_{tr}^*\nabla_{tr}\Psi = -\sum_a \nabla_{E_a}\nabla_{E_a}\Psi + \nabla_{\kappa_B^\sharp}\Psi$.

Also, we can easily check the following relations [7]:

$$[D_{tr}, \Omega] = 2\tilde{D}_{tr},\ [\tilde{D}_{tr}, \Omega] = -2D_{tr},\ [D_{tr}, j] = 0,\ [\tilde{D}_{tr}, j] = 0. \tag{2.19}$$

Now, we define the subspace $\Gamma_B S(\mathcal{F})$ of *basic* or *holonomy invariant* sections of $S(\mathcal{F})$ by

$$\Gamma_B S(\mathcal{F}) = \{\Psi \in \Gamma S(\mathcal{F}) |\ \nabla_X\Psi = 0 \quad \text{for any } X \in \Gamma L\}.$$

Let $D_b = D_{tr}|_{\Gamma_B S(\mathcal{F})} : \Gamma_B S(\mathcal{F}) \to \Gamma_B S(\mathcal{F})$. This operator D_b is called the *basic Dirac operator* on (smooth) basic sections. It is well-known [2] that D_b has a discrete spectrum. Hence we have the following.

Theorem 2.2([6]). *If $\lambda \neq 0$ is an eigenvalue of the basic Dirac operator D_b on a Kähler spin foliation $\mathcal{F}$ of codimension $q = 2n$, then the corresponding eigenspace $E^\lambda(D_b)$ splits into the orthogonal direct sum*

$$E^\lambda(D_b) = \oplus_{r=1}^n E_r^\lambda(D_b), \tag{2.20}$$

where each eigenspinor $\Psi \in E_r^\lambda(D_b)$ has a decomposition $\Psi = \Psi_{r-1} + \Psi_r$ with $\Psi_i \in \Gamma_B S_i = S_i \cap \Gamma_B S(\mathcal{F})(i = r-1, r)$ such that the equations

$$D_b\Psi_{r-1} = \lambda\Psi_r, \quad D_b\Psi_r = \lambda\Psi_{r-1}, \quad \|\Psi_{r-1}\| = \|\Psi_r\| \tag{2.21}$$

are satisfied, where $\ll \Psi, \Phi \gg = \int_M \langle\Psi, \Phi\rangle$.

From the second equation in (2.14), we have the relation

$$E_{n-r+1}^\lambda(D_b) = jE_r^\lambda(D_b). \tag{2.22}$$

Hence the decomposition (2.20) in Theorem 2.2 can be written as

$$E^\lambda(D_b) \quad = \quad \oplus_{r=1}^{[(n+1)/2]}\{E_r^\lambda(D_b) + jE_r^\lambda(D_b)\}. \tag{2.23}$$

Definition 2.3. An eigenvalue λ of the basic Dirac operator is called of *type* r ($r \in \{1, \cdots, [\frac{n+1}{2}]\}$) if $E_r^\lambda(D_b) \neq \{0\}$.

Proposition 2.4([6]). *Let $\mathcal{F}$ be a Kähler spin foliation. For any $\Psi = \Psi_{r-1} + \Psi_r \in E_r^\lambda(D_b)$, we have*

$$\tilde{D}_b\Psi_{r-1} = iD_b\Psi_{r-1} = i\lambda\Psi_r, \quad \tilde{D}_b\Psi_r = -iD_b\Psi_r = -i\lambda\Psi_{r-1}.$$

Lemma 2.5(6). Let $\Psi \in E_r^\lambda(D_b)$ or $\Gamma_B S_r$. For any vector field $X \in \Gamma Q$, we have

$$\langle JX \cdot \Psi, \tilde{D}_b\Psi\rangle = \langle X \cdot \Psi, D_b\Psi\rangle.$$

3. Proof of Theorem 1.1

First, we define $F : \Gamma_B S(\mathcal{F}) \to \mathbb{R}$ by

$$F(\Psi) = \int_M \{\alpha(\Psi) + \beta(\Psi)\}, \tag{3.1}$$

where

$$\alpha(\Psi) = \mathrm{Re}\langle \kappa_B^\sharp \cdot \Psi, D_b\Psi\rangle \text{ and } \beta(\Psi) = \mathrm{Re}\langle J\kappa_B^\sharp \cdot \Psi, \tilde{D}_b\Psi\rangle. \tag{3.2}$$

Trivially, if $\mathcal{F}$ is minimal, then $F(\Psi) = 0$ for all spinor fields $\Psi \in \Gamma_B S(\mathcal{F})$. From Lemma 2.5, we have the following.

Proposition 3.1([6]). *For any spinor field $\Psi \in \Gamma_B S(\mathcal{F})$, we have*

$$F(\Psi) = 2\sum_{r=0}^{n}\int_M \alpha(\Psi_r) = 2\sum_{r=0}^{n}\int_M \beta(\Psi_r) = \sum_r F(\Psi_r),$$

where $\Psi_r = p_r\Psi$. In particular, for any spinor field $\Psi \in E_r^\lambda(D_b)$, $F(\Psi_{r-1}) = -F(\Psi_r) = -F(j\Psi_r)$ and so $F(\Psi) = 0$.

By the Cauchy-Schwartz inequality, we have the following lemma.

Lemma 3.2. *Let $(M, g_M, \mathcal{F})$ be a compact Riemannian manifold with a Kähler spin foliation $\mathcal{F}$ and a bundle-like metric g_M. Then, for any spinor field $\Psi \in \Gamma_B S_r$, we have*

$$|F(\Psi)| \le 2\sup_M |\kappa_B| \Big(\int_M |\Psi|^2\Big)^{1/2} \Big(\int_M |D_b\Psi|^2\Big)^{1/2}. \tag{3.3}$$

The equality holds if and only if κ_B vanishes.

Proof. From Lemma 2.5 and the Cauchy-Schwartz inequality, we have

$$|F(\Psi)| = 2|\int_M \mathrm{Re}\langle \kappa_B^\sharp \cdot \Psi, D_b\Psi\rangle| \\ \leq 2\Big(\int_M |\kappa_B^\sharp \cdot \Psi|^2\Big)^{1/2}\Big(\int_M |D_b\Psi|^2\Big)^{1/2},$$

which proves (3.3). □

For any $r = 1, \cdots, n$ and vector field X, the transversal Kählerian twistor operator P_{tr}^r of type r is given [6] by

$$P_X^r\Psi = \nabla_X\Psi + \frac{1}{4r}\{\pi(X)\cdot D_b\Psi + J\pi(X)\cdot \tilde{D}_b\Psi\}. \tag{3.4}$$

By a straightforward calculation, we have that, for any spinor field $\Psi \in \Gamma_B S(\mathcal{F})$

$$|P_{tr}^r\Psi|^2 = |\nabla_{tr}\Psi|^2 + \frac{n-4r}{8r^2}(|D_b\Psi|^2 + |\tilde{D}_b\Psi|^2) - \frac{1}{4r}\{\alpha(\Psi)+\beta(\Psi)\} \\ + \frac{1}{8r^2}\{\langle D_b\Psi, \Omega\tilde{D}_b\Psi\rangle + \langle \Omega\tilde{D}_b\Psi, D_b\Psi\rangle\}. \tag{3.5}$$

For any spinor field $\Psi \in \Gamma_B S(\mathcal{F})$ on a Kähler spin foliation, we have

$$G(\Psi) := \int_M \langle D_b\Psi, \Omega\tilde{D}_b\Psi\rangle = \int_M \langle \Omega\tilde{D}_b\Psi, D_b\Psi\rangle, \tag{3.6}$$

i.e., $G(\Psi)$ is real [6].

Lemma 3.3([6]). *For any spinor field $\Psi = \Psi_{r-1} + \Psi_r \in E_r^\lambda(D_b)$, we have*

$$G(\Psi) = \lambda^2\int_M |\Psi|^2 = \int_M |D_b\Psi|^2, \tag{3.7}$$

$$G(\Psi_{r-1}) = -\lambda^2\mu_r\int_M |\Psi_r|^2 = -\lambda^2\mu_r\int_M |\Psi_{r-1}|^2, \tag{3.8}$$

$$G(j\Psi_r) = \lambda^2\mu_{r-1}\int_M |\Psi_{r-1}|^2 = \lambda^2\mu_{r-1}\int_M |j\Psi_r|^2. \tag{3.9}$$

From (3.5) and (3.6), we have the following proposition.

Proposition 3.4([6]). *On a Kähler spin foliation, we have*

$$4r^2\int_M |P_{tr}^r\Psi|^2 = \int_M \{f(r)|D_b\Psi|^2 - r^2K^\sigma|\Psi|^2\} + G(\Psi) - rF(\Psi),$$

where $f(r) = 4r^2 - 2r + \mu_r$.

On the other hand, if there exists a spinor field $\Psi \in \mathrm{Ker} P_{\mathrm{tr}}^r \cap (\mathrm{Ker} D_b)^\perp$, then $\Psi \in \Gamma_B S_{r-1}$ or $\Gamma_B S_{n-r+1}$ [6]. Hence, from Lemma 3.2, Lemma 3.3 and Proposition 3.4, for any eigenspinor $\Psi = \Psi_{r-1} + \Psi_r \in E_r^\lambda(D_b)$ corresponding to the eigenvalue λ of D_b, we have

$$\begin{aligned}
&4r\int_M |P_{tr}^r \Psi_{r-1}|^2 \\
&= (4r-2)\int_M (\lambda^2 - \frac{r}{4r-2}K^\sigma)|\Psi_{r-1}|^2 - F(\Psi_{r-1}) \\
&\le (4r-2)\int_M \Big(\lambda^2 + \frac{\sup_M|\kappa_B|}{2r-1}|\lambda| - \frac{r}{4r-2}\inf_M K^\sigma\Big)|\Psi_r|^2 \\
&= (4r-2)\int_M \Big((|\lambda| + \frac{\sup_M|\kappa_B|}{4r-2})^2 - (\frac{\sup_M|\kappa_B|}{4r-2})^2 - \frac{r}{4r-2}\inf_M K^\sigma\Big)|\Psi_r|^2.
\end{aligned}$$

Hence we have

$$(|\lambda| + \frac{\sup_M|\kappa_B|}{4r-2})^2 \ge \frac{r}{4r-2}\inf_M K^\sigma + (\frac{\sup_M|\kappa_B|}{4r-2})^2. \tag{3.10}$$

If we put $A = \frac{\sup_M|\kappa_B|}{4r-2} \ge 0$, then we have

$$\lambda^2 \ge \frac{r}{4r-2}\inf_M K^\sigma + 2C, \tag{3.11}$$

where $C = A(A - \sqrt{\frac{r}{4r-2}\inf_M K^\sigma + A^2}) \le 0$. If the equality of (3,10) holds, then from the equation above, $P_{tr}^r\Psi_{r-1} = 0$. Hence κ_B vanishes [6, Theorem 5.1]. So, $C = 0$. Hence any eigenvalue λ of type $r(r \in \{1, \cdots, \frac{[n+1]}{2}\})$ of the basic Dirac operator D_b satisfies

$$\lambda^2 \ge \frac{r}{4r-2}\inf_M \sigma^\nabla. \tag{3.12}$$

Hence the proof is completed.

Acknowledgements. This work was supported by the Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology(2010-0021005) and NRF-2011-616-C00040.

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