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ANALYTIC APPROACH FOR THE STUDY OF AIR AND/OR LIQUID FILLED GEOMEMBRANE TUBE SECTIONS ON A HORIZONTAL

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ABSTRACT. This study considers an air and liquid-filled geomembrane tube section resting on a horizontal foundation. All quantities are normalized to obtain geometrically similar solutions in the static equilibrium condition. Analytic solutions are expressed in closed form. The solution for the air or liquid-filled tube section is derived systematically as an extreme case of the air and liquid-filled tube section. The validity of these solutions is confirmed by comparing to previous study, and some results are shown for the characteristic parameters and shapes of air and/or liquid-filled cases. Using the result of present study, one can estimate the shape and characteristic parameters of a tube section without numerical integrations or iterations.

1. INTRODUCTION

Geomembrane structures are composed of thin flexible sheets, and can be inflated with air, liquid, grain, slurry, or sand. Geomembrane structures are widely utilized in various applications because they are inexpensive and easy to install and dismantle. These structures can be used to store and transport fluids. In the field of civil engineering, geomembrane structures are employed for purposes such as increasing the height of existing dams or spillways, temporary dykes for flood control, breakwaters in beaches, foundations of emergency bridges, and coastal erosion prevention.

Inflated geomembrane structures are generally tube-shaped, so structural analysis is commonly performed about the tube section. Analytic approaches have been focused on the analysis of the structures resting on the horizontal foundations. Demiray and Levinson [1] formulated the governing equations according to the equilibrium of force and the geometric boundary conditions, and obtained the analytic solution expressed in terms of elliptic integrals. Wang and Watson [2] studied the shapes of the structures in lightly or heavily pressurized conditions. Namias [3] found the approximate solutions, which covered a wider range of the low and high pressures than that of Wang and Watson [2]. Plaut and Suherman [4] reviewed the analytic and approximate solutions in terms of the pressure at the bottom of the tube and at the top. Meanwhile, Plaut and Cotton [5] numerically studied the statics

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and vibration of the tube section of air-filled structures using the shooting method. Antman and Schagerl [6] analyzed extensible elastic membranes containing incompressible liquids and compressible gases, and inextensible membrane as a limit case of extensible membrane. However, the authors neglected the weight of membranes. Recently, Ghavanloo and Daneshmand [7] carried out semi-analytic approaches for the air and/or liquid-filled tube sections (that is, the solutions were expressed in integral form). Choi [8] derived the analytic solution in closed form for an air-filled, heavy membrane tube section on an incline.

In the present study, an analytic approach is applied to an air and liquid-filled geomembrane tube resting on a horizontal foundation. Based on the semi-analytic approach by Ghavanloo and Daneshmand [7], the integral terms of their results are evaluated analytically. The problem is divided into two parts: liquid-filled and air-filled. Each part is solved analytically and then matched to obtain the complete solution. The analytic solution for the air or liquid-filled tube is derived systematically using the solution of the air and liquid-filled tube. A comparison with a semi-analytic approach is performed, and characteristic parameters and shapes are shown for the air and/or liquid-filled tube sections.

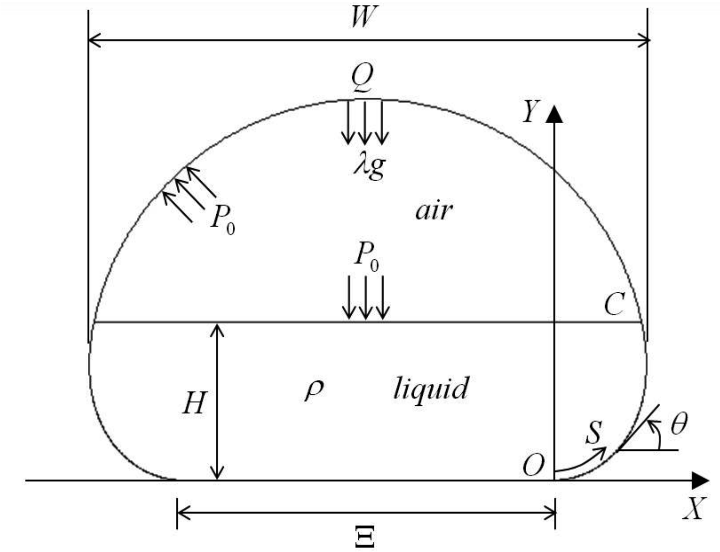


FIGURE 1. Cross section of the air and liquid-filled geomembrane tube.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Figure 1 shows the cross section of an air and liquid-filled geomembrane tube resting on a rigid horizontal foundation. The geomembrane material is assumed to be inextensible and perfectly flexible. Both the friction between the geomembrane tube and the liquid, and that between the geomembrane tube and the rigid foundation are neglected. The origin of the coordinate system O is located at the right separation point. X and Y are the horizontal and vertical axes, respectively, θ is the tangential angle with respect to the horizontal, and S is the arc length from the origin. The contact length is Ξ; the mass per unit length is λ; the density of the liquid is ρ; the height of the liquid level is H; the maximum width is W; the transmural pressure difference in the air-filled part is P_0; the intersection point of the air, liquid, and tube section is C; and the top point of the tube is Q. The analysis can be performed in the right half section due to the geometrical symmetry.

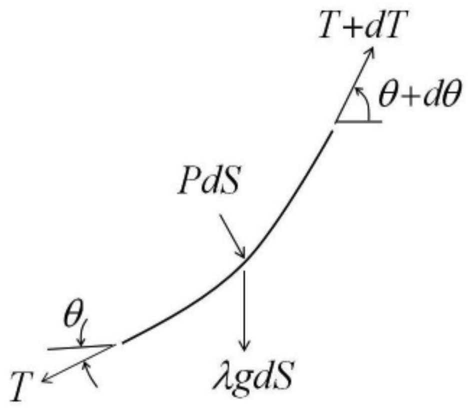


FIGURE 2. Free-body diagram for a differential element.

From the free-body diagram of Figure 2, the static equilibrium gives

$$\frac{dT}{dS} = \lambda g \sin\theta, \tag{2.1}$$

$$T\frac{d\theta}{dS} = P + \lambda g \cos\theta, \tag{2.2}$$

where P is the transmural pressure difference. P in the air- and liquid-filled parts is expressed as

$$P = \begin{cases} P_0 + \rho g(H - Y) & if \quad 0 \le S \le S_C, \\ P_0 & if \quad S_C \le S \le \dfrac{L-\Xi}{2}, \end{cases} \tag{2.3}$$

where L is the perimeter of the tube section.

To simplify the analysis, dimensional quantities are nondimensionalized using L and ρ as follows:

$$x = \frac{X}{L}, \quad y = \frac{Y}{L}, \quad s = \frac{S}{L}, \quad \xi = \frac{\Xi}{L}, \quad h = \frac{H}{L},$$

$$w = \frac{W}{L}, \quad v = \frac{V}{L^2}, \quad t = \frac{T}{\rho g L^2}, \quad p = \frac{P_0}{\rho g L}, \quad \mu = \frac{\lambda}{\rho L}, \tag{2.4}$$

where V is the sectional area of the tube. In (2.4), μ indicates the weight ratio of geomembrane structure to the liquid. Using these quantities and (2.3), (2.1) and (2.2) can be rewritten as follows:

$$\frac{dt}{ds} = 0, \quad t\frac{d\theta}{ds} = p + h - y \qquad \textit{if} \quad 0 \le s \le s_C, \tag{2.5a,b}$$

$$\frac{dt}{ds} = \mu \sin\theta, \quad t\frac{d\theta}{ds} = p + \mu\cos\theta \qquad \textit{if} \quad s_C \le s \le \frac{1-\xi}{2}. \tag{2.6a,b}$$

In (2.5), the effect of the tube weight is neglected in the liquid-filled part because μ is generally very small. Hence, it can be noted that the tension is constant in this part.

The geometric relations are

$$\frac{dx}{ds} = \cos\theta, \quad \frac{dy}{ds} = \sin\theta. \tag{2.7a,b}$$

The same equations can be found in Ghavanloo and Daneshmand [7].

This problem is governed by the system of non-linear differential equations in (2.5)-(2.7). The boundary conditions of this problem at points O, C, and Q are as follows:

$$x = y = \theta = 0 \quad at \quad s = 0, \tag{2.8}$$

$$x = x_C, \quad y = h, \quad \theta = \theta_C \quad at \quad s = s_C, \tag{2.9}$$

$$x = -\frac{\xi}{2}, \quad \theta = \pi \quad at \quad s = \frac{1-\xi}{2}. \tag{2.10}$$

3. Analytic Solutions

As can be seen in (2.5) and (2.6), this problem can be broken into two problems: the liquid-filled part problem and the air-filled part problem. Therefore, each of the two problems may be analyzed independently. After solving the two problems with unknowns x_C, θ_C, and s_C at point C, matching the two solutions at point C yields the complete solution.

3.1. **Solution of the liquid-filled part.** This problem is composed of the governing equations (2.5) and (2.7) with the boundary conditions (2.8) and (2.9). From (2.5a), tension in this part is constant and denoted as $t = t_0$. Differentiation of (2.5b) with respect to s yields an integrable equation as follows:

$$\frac{d^2\theta}{ds^2} = -\frac{\sin\theta}{t_0}. \tag{3.1}$$

Integration of the above equation results in

$$\frac{d\theta}{ds} = \sqrt{\frac{2}{t_0}}\sqrt{a+\cos\theta}\,, \tag{3.2}$$

where a is defined as follows:

$$a = \frac{1}{2}\frac{p^2}{t_0} - \cos\theta_C = \frac{1}{2}\frac{(p+h)^2}{t_0} - 1. \tag{3.3}$$

The relation in (3.3) is provided by the values in (2.5b) at points O and C. Using (3.2), analytic integration can be performed with respect to θ (Gradshteyn and Ryzhik [9]).

$$s = \begin{cases} \sqrt{t_0}\sqrt{\dfrac{2}{a+1}}F\left(\dfrac{\theta}{2}, \sqrt{\dfrac{2}{a+1}}\right) & if \quad a > 1, \\ \sqrt{t_0}F\left(\sin^{-1}\sqrt{\dfrac{1-\cos\theta}{a+1}}, \sqrt{\dfrac{a+1}{2}}\right) & if \quad -\cos\theta_C \le a \le 1. \end{cases} \tag{3.4}$$

$$x = \begin{cases} \sqrt{t_0}\sqrt{\dfrac{2}{a+1}}\left[(a+1)E\left(\dfrac{\theta}{2}, \sqrt{\dfrac{2}{a+1}}\right) - aF\left(\dfrac{\theta}{2}, \sqrt{\dfrac{2}{a+1}}\right)\right] & if \quad a > 1, \\ \sqrt{t_0}\left[2E\left(\sin^{-1}\sqrt{\dfrac{1-\cos\theta}{2}}, \sqrt{\dfrac{a+1}{2}}\right) - F\left(\sin^{-1}\sqrt{\dfrac{1-\cos\theta}{2}}, \sqrt{\dfrac{a+1}{2}}\right)\right] & if \quad -\cos\theta_C \le a \le 1. \end{cases} \tag{3.5}$$

$$y = \sqrt{2t_0}\left[\sqrt{a+1} - \sqrt{a+\cos\theta}\right]. \tag{3.6}$$

In the above equations, F and E are the incomplete elliptic integrals of the first and second kind, respectively, which are defined in Gradshteyn and Ryzhik [9], and can be evaluated without numerical integrations (Press et al. [10]).

The sectional area of this part (v_l) is derived by integrating (2.5b).

$$v_l = 2t_0\sin\theta_C + h\xi - 2px_C. \tag{3.7}$$

3.2. **Solution of the air-filled part.** This problem is composed of the governing equations (2.6) and (2.7) with the boundary conditions (2.9) and (2.10). To inflate the tube, the air pressure should be $p > \mu$. Elimination of s from (2.6a) and (2.6b) yields

$$t = t_0 \frac{p + \mu \cos\theta_C}{p + \mu\cos\theta}. \tag{3.8}$$

Using this result and the boundary condition (2.9), equations (2.7a), (2.7b), and (2.6b) can be integrated and expressed as functions of θ (Gradshteyn and Ryzhik [9]).

$$x = x_C + t_0 \frac{p + \mu\cos\theta_C}{p^2 - \mu^2} \left[\begin{array}{l} \dfrac{p\sin\theta}{p + \mu\cos\theta} - \dfrac{p\sin\theta_C}{p + \mu\cos\theta_C} \\ -\dfrac{2\mu}{\sqrt{p^2-\mu^2}} \left\{ \tan^{-1}\left(\sqrt{\dfrac{p-\mu}{p+\mu}} \tan\dfrac{\theta}{2} \right) - \tan^{-1}\left(\sqrt{\dfrac{p-\mu}{p+\mu}} \tan\dfrac{\theta_C}{2} \right) \right\} \end{array} \right]. \tag{3.9}$$

$$y = h + t_0 \frac{\cos\theta_C - \cos\theta}{p + \mu\cos\theta}. \tag{3.10}$$

$$s = s_C - t_0 \frac{p + \mu\cos\theta_C}{p^2 - \mu^2} \left[\begin{array}{l} \dfrac{\mu\sin\theta}{p + \mu\cos\theta} - \dfrac{\mu\sin\theta_C}{p + \mu\cos\theta_C} \\ -\dfrac{2p}{\sqrt{p^2-\mu^2}} \left\{ \tan^{-1}\left(\sqrt{\dfrac{p-\mu}{p+\mu}} \tan\dfrac{\theta}{2} \right) - \tan^{-1}\left(\sqrt{\dfrac{p-\mu}{p+\mu}} \tan\dfrac{\theta_C}{2} \right) \right\} \end{array} \right]. \tag{3.11}$$

The sectional area of this part (v_a) is calculated as follows:

$$v_a = -2\int_{x_C}^{-\xi/2} y\,dx - 2h\left(x_C + \frac{\xi}{2} \right) = -2t_0^2 \left(p + \mu\cos\theta_C \right) \int_{\theta_C}^{\pi} \frac{\left(\cos\theta_C - \cos\theta \right)\cos\theta}{\left(p + \mu\cos\theta \right)^3} d\theta. \tag{3.12}$$

The last integral term in (3.12) can be evaluated analytically (after some manipulations of the integration by parts). The result is then given as follows:

$$\begin{aligned} v_a = {} & 2t_0^2 \left(p + \mu\cos\theta_C \right) \frac{p^2 + 3p\mu\cos\theta_C + 2\mu^2}{\left(p^2 - \mu^2 \right)^2 \sqrt{p^2 - \mu^2}} \left[\frac{\pi}{2} - \tan^{-1}\left(\sqrt{\frac{p-\mu}{p+\mu}} \tan\frac{\theta_C}{2} \right) \right] \\ & + t_0^2 \frac{3p\mu + \left(p^2 + 2\mu^2 \right)\cos\theta_C}{\left(p^2 - \mu^2 \right)^2} \sin\theta_C \end{aligned}. \tag{3.13}$$

3.3. **Matching of two solutions.** For the liquid-filled part, the application of boundary conditions (2.9) to (3.4) and (3.5) results in the following relation.

$$s_C - x_C = \begin{cases} 2\sqrt{t_0}\sqrt{\frac{a+1}{2}}\left[F\left(\frac{\theta_C}{2},\sqrt{\frac{2}{a+1}}\right)-E\left(\frac{\theta_C}{2},\sqrt{\frac{2}{a+1}}\right)\right] & if \quad a>1, \\ 2\sqrt{t_0}\left[F\left(\sin^{-1}\sqrt{\frac{1-\cos\theta_C}{a+1}},\sqrt{\frac{a+1}{2}}\right)-E\left(\sin^{-1}\sqrt{\frac{1-\cos\theta_C}{a+1}},\sqrt{\frac{a+1}{2}}\right)\right] & if \quad -\cos\theta_C \le a \le 1. \end{cases} \tag{3.14}$$

On the other hand, the application of boundary conditions (2.10) to (3.9) and (3.11) results in the following relation for the air-filled part.

$$\frac{1}{2} = s_C - x_C + t_0\frac{p+\mu\cos\theta_C}{p-\mu}\left[\frac{\sin\theta_C}{p+\mu\cos\theta_C}+\frac{2}{\sqrt{p^2-\mu^2}}\left\{\frac{\pi}{2}-\tan^{-1}\left(\sqrt{\frac{p-\mu}{p+\mu}}\tan\frac{\theta_C}{2}\right)\right\}\right]. \tag{3.15}$$

From (3.3), t_0 can be expressed as follows:

$$t_0 = \frac{h(2p+h)}{2(1-\cos\theta_C)}. \tag{3.16}$$

Because t_0 and a are expressed in terms of the parameters p, h, and θ_C as can be seen in (3.3) and (3.16), elimination of $s_C - x_C$ from (3.14) and (3.15) yields an equation that represents the relation among those parameters. For given values of p and h, one can find θ_C as the root of this combined equation using various numerical schemes, such as the bisection method. The monotonic property of the combined equation to θ_C confirms the uniqueness of the solution. The upper bound of h for a given p corresponds to the h of the fully liquid-filled case, which is described in the next section. After determining θ_C, the shape and all characteristic parameters can be determined for the given p and h.

$$x_C = x(\theta=\theta_C), \qquad x_{max} = x\left(\theta=\frac{\pi}{2}\right), \qquad y_{max} = y(\theta=\pi),$$

$$-\frac{\xi}{2} = x(\theta=\pi), \quad w = \xi + 2x_{max}, \quad t_{max} = t(\theta=\pi). \tag{3.17}$$

All solutions solved in this study are geometrically similar solutions, because all quantities are normalized by the perimeter of the tube section and the density of the contained liquid.

4. TWO EXTREME CASES: LIQUID-FILLED AND AIR-FILLED

Using the solution of the air and liquid-filled geomembrane tube, the solution for the liquid or air-filled tube can be derived.

4.1. **Liquid-filled case.** $\theta_C = \pi$, $h = y_{\max}$, $x_C = -\xi/2$, and $s_C = (1-\xi)/2$ in the solutions of the liquid-filled part, and the tension is constant (t_0). In this case, p is the top pressure in the tube section. Noting (3.3), a is defined as follows, and should be $a > 1$.

$$a = \frac{1}{2}\frac{p^2}{t_0} + 1 = \frac{1}{2}\frac{(p+h)^2}{t_0} - 1. \tag{4.1}$$

Equation (3.16) is then rewritten as

$$t_0 = \frac{1}{4}h(2p+h). \tag{4.2}$$

Substitution of (4.1) and (4.2) into (3.14) yields

$$\frac{1}{2} = (p+h)\left[F\left(\frac{\pi}{2}, \sqrt{1-\left(\frac{p}{p+h}\right)^2}\right) - E\left(\frac{\pi}{2}, \sqrt{1-\left(\frac{p}{p+h}\right)^2}\right)\right]. \tag{4.3}$$

h is determined as the root of (4.3) for a given p. Therefore, h is dependent on p. This value provides the upper bound of h in the air and liquid-filled case for the given p. After determining h, the shape and characteristic parameters are calculated by (3.4)-(3.6) and (3.17). Equation (3.15) is automatically satisfied. These results are identical to those of previous studies (Demiray and Levinson [1] and Namias [3]).

From (3.7), the sectional area is

$$v = v_l = \xi(h+p). \tag{4.4}$$

4.2. **Air-filled case.** This case corresponds to $\theta_C = 0$, $h = 0$, $x_C = 0$, and $s_C = 0$ in the solutions of the air-filled part. In this case, the liquid density ρ is meaningless. Thus, new nondimensional parameters for the air pressure and tension are introduced as follows:

$$\tilde{p} = \frac{p}{\mu} = \frac{P_0}{\lambda g}, \qquad \tilde{t} = \frac{t}{\mu} = \frac{T}{\lambda g L}, \tag{4.5}$$

where $\tilde{p} > 1$. Using the above definition, (3.15) yields

$$\frac{1}{2} = \tilde{t}_0 \frac{\pi(\tilde{p}+1)}{(\tilde{p}-1)\sqrt{\tilde{p}^2 - 1}}. \tag{4.6}$$

Hence, $\tilde{t}_0$ is determined for a given $\tilde{p}$ as follows:

$$\tilde{t}_0 = \frac{\tilde{p}-1}{2\pi}\sqrt{\frac{\tilde{p}-1}{\tilde{p}+1}}\,. \tag{4.7}$$

Equation (3.14) is automatically satisfied. Using (4.5) and (4.7), the solutions of the air-filled part (3.8)-(3.11) yield the following results.

$$\tilde{t} = \frac{\tilde{p}-1}{2\pi}\frac{\sqrt{\tilde{p}^2-1}}{\tilde{p}+\cos\theta}\,. \tag{4.8}$$

$$x = \frac{1}{2\pi}\left[\sqrt{\frac{\tilde{p}-1}{\tilde{p}+1}}\frac{\tilde{p}\sin\theta}{\tilde{p}+\cos\theta} - \frac{2}{\tilde{p}+1}\tan^{-1}\left(\sqrt{\frac{\tilde{p}-1}{\tilde{p}+1}}\tan\frac{\theta}{2}\right)\right]. \tag{4.9}$$

$$y = \frac{\tilde{p}-1}{2\pi}\sqrt{\frac{\tilde{p}-1}{\tilde{p}+1}}\frac{1-\cos\theta}{\tilde{p}+\cos\theta}\,. \tag{4.10}$$

$$s = \frac{1}{2\pi}\left[\frac{2\tilde{p}}{\tilde{p}+1}\tan^{-1}\left(\sqrt{\frac{\tilde{p}-1}{\tilde{p}+1}}\tan\frac{\theta}{2}\right) - \sqrt{\frac{\tilde{p}-1}{\tilde{p}+1}}\frac{\sin\theta}{\tilde{p}+\cos\theta}\right]. \tag{4.11}$$

The characteristic parameters are also derived as follows:

$$\tilde{t}_{\max} = \frac{\sqrt{\tilde{p}^2-1}}{2\pi}\,, \tag{4.12}$$

$$\xi = \frac{1}{\tilde{p}+1}\,, \tag{4.13}$$

$$x_{\max} = \frac{1}{2\pi}\left[\sqrt{\frac{\tilde{p}-1}{\tilde{p}+1}} - \frac{2}{\tilde{p}+1}\tan^{-1}\left(\sqrt{\frac{\tilde{p}-1}{\tilde{p}+1}}\right)\right], \tag{4.14}$$

$$y_{\max} = \frac{1}{\pi}\sqrt{\frac{\tilde{p}-1}{\tilde{p}+1}}\,. \tag{4.15}$$

These closed-form solutions correspond to those of Choi [8] in the case of a horizontal foundation.

From (3.13), the sectional area is

$$v = v_a = \frac{1}{4\pi} \frac{\tilde{p}+2}{\tilde{p}+1} \sqrt{\frac{\tilde{p}-1}{\tilde{p}+1}}. \tag{4.16}$$

5. NUMERICAL RESULTS AND DISCUSSION

Figure 3 shows the dependence of θ_C on the air pressure p for several liquid levels h. To compare the results of present study to those of Ghavanloo and Daneshmand [7], the parameter μ is assumed hereafter to be 0.0035. $\theta_C = 180°$ indicates the liquid-filled case. θ_C approaches the value of the perfectly circular shape for the given h as p increases. There is a minimum pressure required to keep a certain liquid level, which corresponds to the liquid-filled case. This minimum pressure increases as h increases. The computing burden to find θ_C using the bisection method was negligibly small.

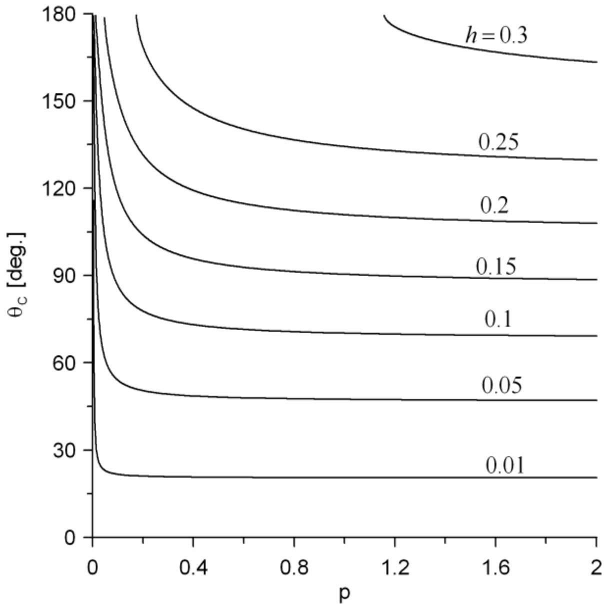


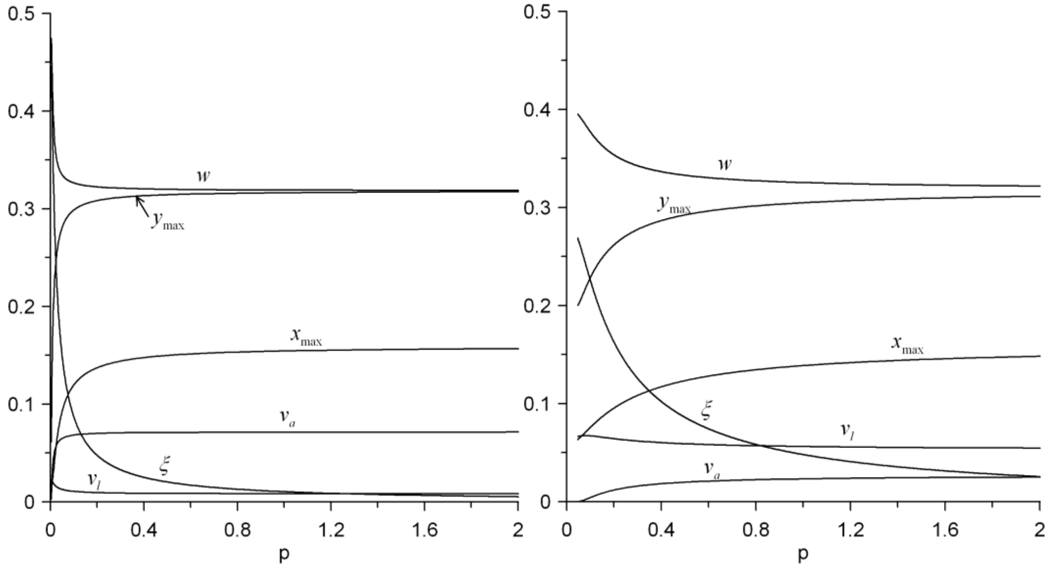
FIGURE 3. θ_c related to the nondimensional air pressure p.

To ensure the validity of the analytic solutions of this study, some characteristic parameters are compared with those of the semi-analytic approach by Ghavanloo and Daneshmand [7]. As shown in Table 1, close agreement is achieved between the two results. The solution of the semi-analytic approach contains the integral terms which should be computed numerically, while that of present study does not contain any integral terms.

TABLE 1. Some characteristic parameters of the air and liquid-filled tube sections.

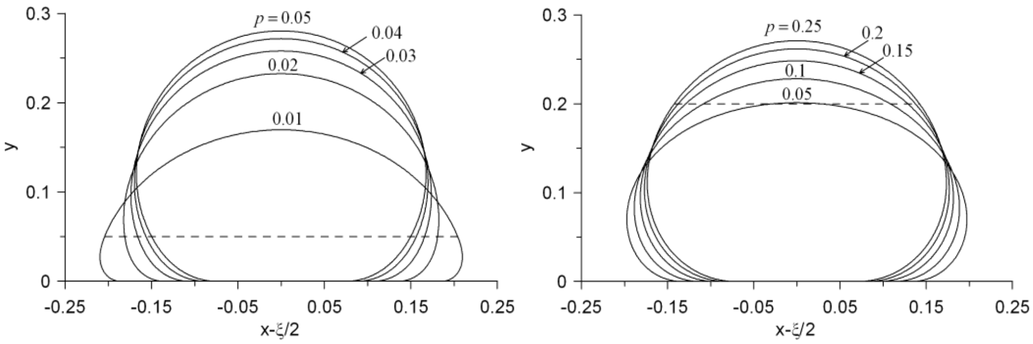
p	h	ξ		θ_C		t_0	
		Present study	Ghavanloo and Daneshmand [7]	Present study	Ghavanloo and Daneshmand [7]	Present study	Ghavanloo and Daneshmand [7]
0.25	0.10	0.0785	0.0785	1.3252	1.3252	0.0396	0.0396
0.25	0.15	0.1156	0.1155	1.7585	1.7587	0.0411	0.0411
0.25	0.175	0.1306	0.1305	1.9810	1.9814	0.0422	0.0422
0.25	0.20	0.1422	0.1421	2.2148	2.2151	0.0437	0.0437
0.05	0.10	0.2415	0.2414	1.7821	1.7825	0.0083	0.0083
0.10	0.10	0.1585	0.1588	1.5111	1.5096	0.0160	0.0160
0.15	0.10	0.1182	0.1183	1.4101	1.4095	0.0238	0.0238
0.20	0.10	0.0943	0.0944	1.3575	1.3567	0.0317	0.0317

Figures 4(a) and 4(b) show the characteristic parameters for $h=0.05$ and 0.2, respectively. As expected, all values approach those of the circle for the given h as p increases. The approaches to those limit values are slower for larger values of h because gravity tends to flatten more for higher liquid level. The values for the minimum air pressure for the given h (that is, the pressure for $h=y_{\max}$) correspond to those of the liquid-filled case. Figures 5(a) and 5(b) depict the shapes for $h=0.05$ and 0.2, respectively. The dashed lines in the figures represent liquid levels h. The shapes of the air-filled part look like circular arcs because the parameter $\mu=0.0035$ is very small (that is, very light geomembrane structure).



(a) $h=0.05$ (b) $h=0.2$

FIGURE 4. Characteristic parameters of the air and liquid-filled tube section.



(a) $h = 0.05$ (b) $h = 0.2$

FIGURE 5. Shapes of the air and liquid-filled tube section.

Figures 6 and 7 plot the characteristic parameters and shapes for the liquid-filled case. All characteristic parameters approach for the parameters of the perfectly folded tube section as $p \to 0$. Asymptotic values for $p \to \infty$ are those of the circle with unit perimeter.

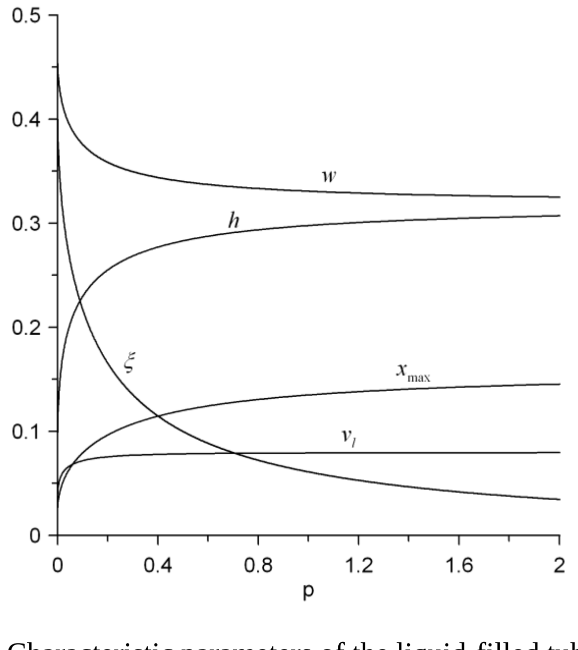


FIGURE 6. Characteristic parameters of the liquid-filled tube section.

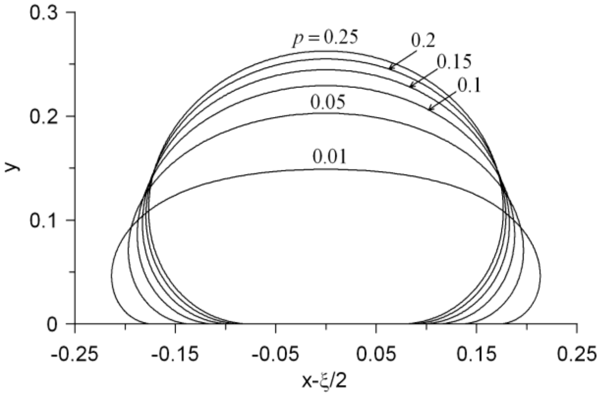


FIGURE 7. Shapes of the liquid-filled tube section.

Figures 8 and 9 show the characteristic parameters and shapes for the air-filled case. The characteristic features are similar to those of the liquid-filled case. The shapes are a little wide and flabby at the bottom section compared to those of the liquid-filled case.

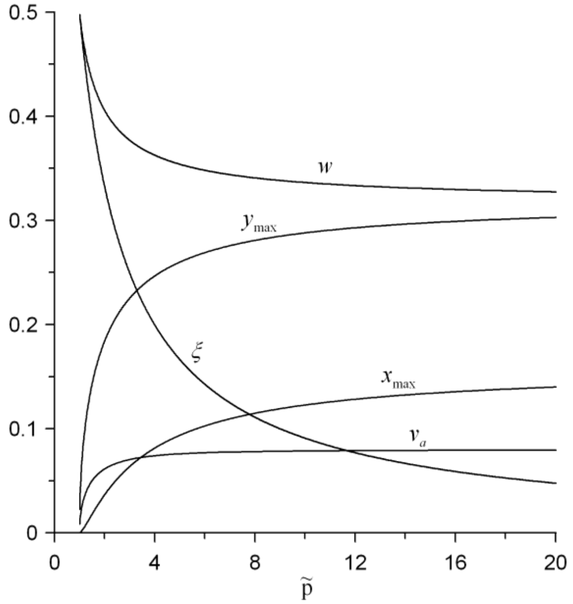


FIGURE 8. Characteristic parameters of the air-filled tube section.

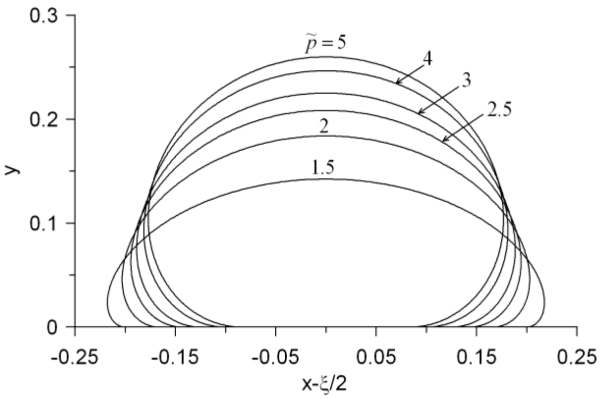


FIGURE 9. Shapes of the air-filled tube section.

6. CONCLUSIONS

The cross section of an air and liquid-filled geomembrane tube resting on a rigid horizontal foundation is analyzed using an analytic approach. Problems are solved separately for the air-filled and liquid-filled parts. The solution of the liquid-filled part is expressed in terms of elliptic integrals, while the solution of the air-filled part is described by trigonometric functions. The matching of two solutions is carried out to obtain the complete solution. The solution for the air or liquid-filled tube section is derived systematically as an extreme case of the air and liquid-filled tube section.

Because all solutions in this study are described in the closed form and are nondimensionalized, it is possible to determine the shapes and characteristic parameters for any size of geomembrane tube section without numerical iterations or integrations required in a direct numerical or semi-analytic approach, respectively. In addition to this, the closed-form solution is more amenable to interpreting physics and parametric study of the problem than a direct numerical or semi-analytic solution.

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