A Probabilistic Approach to Small Signal Stability Analysis of Power Systems with Correlated Wind Sources

Hao Yue[†], Gengyin Li* and Ming Zhou*

Abstract – This paper presents a probabilistic methodology for small signal stability analysis of power system with correlated wind sources. The approach considers not only the stochastic characteristics of wind speeds which are treated as random variables with Weibull distributions, while also the wind speed spatial correlations which are characterized by a correlation matrix. The approach based on the $2m+1$ point estimate method and Cornish Fisher expansion, the orthogonal transformation technique is used to deal with the correlation of wind farms. A case study is carried out on IEEE New England system and the probabilistic indexes for eigenvalue analysis are computed from the statistical processing of the obtained results. The accuracy and efficiency of the proposed method are confirmed by comparing with the results of Monte Carlo simulation. The numerical results indicate that the proposed method can actually capture the probabilistic characteristics of mode properties of the power systems with correlated wind sources and the consideration of spatial correlation has influence on the probability of system small signal stability.

Keywords: Small signal stability, Probabilistic analysis, Correlated wind sources, Point estimate method

1. Introduction

With the increase in penetration of wind power which is essentially intermittent and random, the dynamic performance of the power system will be affected [1]. It is necessary and imperative for the power system engineers to understand in essential how the wind power penetration affects an existing interconnected large-scale power system, especially for the power system small signal stability [2-4].

Deterministic strategies used for small signal stability analysis of power systems as affected by penetration of large scale wind generation are limited since they are carried out based on a specified operating condition (e.g. mean wind generation scenario). However, since the wind generation is primarily determined by wind speed and thus fluctuating constantly, the operating conditions of the system are stochastically uncertain. The results obtained by deterministic strategies are too conservative, in other words, though the system is stable deterministically, there exists the certain probability that the system can lose stability due to the stochastic fluctuations caused by wind generation [5]. The behavior of these probabilistic characteristics of the power system can be described only in statistical terms. Moreover, wind sources are spatially correlated within a given geographical area in a very significant manner, as

they are influenced often by the same physical phenomena [6]. This correlation can have a significant impact on the power flow, voltage stability and reliability of power systems [6-8]. Thus, there is a clear need to develop an algorithm of probabilistic small signal stability analysis (PSSSA) which includes the uncertainty of wind generation and the dependencies of the wind sources.

The probabilistic analysis was firstly applied in power system by Borkowska in [9] for power flow study and was firstly introduced into investigating small signal stability of power system by Burchett and Heydt in [10]. In the previous published literatures, a number of different probabilistic theory-based approaches have been proposed to deal with the uncertainty problems in power system. These approaches could be divided into three main categories: Monte Carlo simulation, analytical methods, and approximate methods.

Monte Carlo simulation (MCS) which has been used in reliability assessment for many years is a repetitive procedure which generates a large number of random computational scenarios according to the distribution density of the input variables. In [11-13], MCS is introduced to study the influence of uncertainties of wind generation on power system small signal stability. Although MCS can provide accurate results, thousands of simulations are usually required to attain convergence and high computational burden makes this method unattractive. Most of researchers only use it for comparison purpose.

The advantage of analytical methods such as fast Fourier transform (FFT), cumulant-based method is computational efficiency, but these methods rely on complex mathematical

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approximations and extensive modifications of the original model. The method of combined cumulants and Gram-Charlier expansion with considering the spatial correlations of wind generation is employed in [5] to determine the probabilistic small signal stability of power systems penetrated by multiple wind sources. This method utilizes first order eigenvalue sensitivity with respect to wind power generation, and the statement is made that this provides accurate results for the system under consideration. However, there are a number of situations where first order approximation may not be sufficient accurate [14].

Approximate methods provide an approximate description of the statistical properties of output random variables with reduced computational efforts compared to MCS. First-order second-moment method (FOSMM) and point estimate methods (PEMs) fit into the family of approximate methods. Since Rosenblueth's two point estimate method [15] needs a large number of simulations if the number of input random variables is high and Harr's $2m$ PEM [16] is constrained to symmetric variables, $2m+1$ scheme based on Hong's point estimate method [17] is used in this paper. The main advantages are as following:

1) Compared with MCS, PEMs can calculate the statistical properties of output random variables with satisfactory accuracy and much less computation effort.

2) As cumulant-based method, PEMs also utilize the statistical features of input random variables to provide (i.e., first few statistical moments), overcoming the difficulties associated with the lack of analytical expression of the probability functions of inputs; however different approaches are applied to obtain the statistical features of outputs, which is not based on the first order eigenvalue sensitivity terms.

3) Hong's PEM can deal with the case in which the variables are skewed, since the wind speed is generally treated as a random variable assumed to have a Weibull distribution which is not symmetrical.

4) Among different schemes based on Hong's PEM, the $2m+1$ scheme has been shown to have good performance in terms of both accuracy and computational time and is the most efficient scheme in dealing with non-normal distribution [18, 19].

The aim of PEMs is to compute the first moments of a random variable z that is a function of m random input variables x_i, i.e., $z = F(x_1, \cdots, x_i, \cdots, x_m)$. Knowing the first few statistical moments, it is possible to obtain the probability density functions (PDFs) or the cumulative density functions (CDFs) of the output variables by using the analytical expressions, such as Cornish-Fisher expansion method [20], Gram-Charlier expansion series. However, the original Hong's PEM can only be applied when the input random variables are uncorrelated. Thus, a suitable adjustment has to be introduced to deal with the correlation. The well-known orthogonal (rotational) transformation technique is applied in this paper.

This paper presents a probabilistic methodology based on $2m+1$ PEM for small signal stability analysis of power system with correlated wind sources. The approach considers not only the stochastic characteristics of wind speeds which are treated as random variables with Weibull distributions, while also the wind speed spatial correlations which are characterized by a correlation matrix. The rest of the paper is organized as follows. Section 2 briefly describes wind speed correlation and its impact on power system. Section 3 gives a short review of the probabilistic small signal stability and presents the dynamic model of wind turbine. Section 4 explains the theoretical foundations of the $2m+1$ PEM, together with Cornish Fisher expansion, and the orthogonal transformation technique. In section 5, a case study on IEEE New England system is carried out and the probabilistic indexes for PSSSA are computed from the statistical processing of the obtained results. The results are compared with those obtained by MCS to validate the accuracy and efficiency of the proposed method. Section 6 summarizes some relevant conclusions based on the numerical results.

2. Wind Speed Correlation

Wind is a highly variable and site-specific energy source with instantaneous, hourly, diurnal and seasonal variations of wind speed. Wind speeds at different wind sites can be assumed to be independent if they are far away from each other. However, the wind farms are correlated to some degree if the distances between the wind sites are not very large. This correlation can have a significant impact on the power flow, voltage stability and reliability of power systems. Therefore, it should be considered in PSSSA of power system integrated with wind power generation.

The wind speed correlation between two wind sites can be calculated using cross-correlation. The cross-correlation coefficient ρ_{xy} is a measure of how well two time series follow each other [21], as shown in (1).

$$\rho_{xy} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x)(y_i - \mu_y)}{\sigma_x \sigma_y} \tag{1}$$

where x_i and y_i are elements of the first and second time series, respectively, μ denotes the mean value, σ the standard deviation, and n the number of points of the time series. The value of ρ_{xy} is near the maximum value of 1.0, if the two time series totally dependent. The value is close to zero, if the two time series are basically uncorrelated.

The cross-correlation coefficients were calculated for 89 wind sites in Nebraska of US for one year 2004. The wind speed data was collected from the wind integration datasets of National Renewable Energy Laboratory's website [22]. The Results is presented in figure Fig. 1, which shows the relationship between the cross-correlation coefficient and
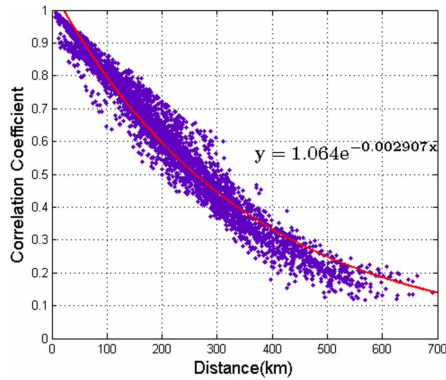
Fig. 1. Cross-correlation coefficients of the wind sites in Nebraska of US for one year

Fig. 3. Frequency distribution of aggregated wind power of wind-site-pairs with different correlation level

the distance of two wind sites. The coefficient was fitted by exponential function with a damping ratio of 0.002907 km^{-1}. It can be seen from the figure that there is a decrease tendency for cross-correlation of two different wind sites as the distance increases.

The wind speed time series during 1000 hour period for different wind-site-pairs with a maximum value (ρ_{xy}= 0.9920), middle value (ρ_{xy}=0.500) and minimum (ρ_{xy}= 0.1168) value of cross-correlation coefficients are shown in Fig. 2. This figure shows that in the high correlation level scenario, the up and down movements of the two wind time series occur in the same direction, in the low correlation level scenario, the two wind time series do not follow each other and are complementary in most of the time. Thus, high level correlation of wind speed will strength the synchronization (increase and decrease simultaneously) of different wind farms' power output and increase the fluctuation of aggregated wind power output and low level correlation will smooth out the wind power variation. Fig. 3 shows the frequency distribution of hourly aggregated wind power of wind-site-pairs for one year with different correlation level. It can be seen in Fig. 3 that high correlation level increase the occasions with near zero and peak power output.
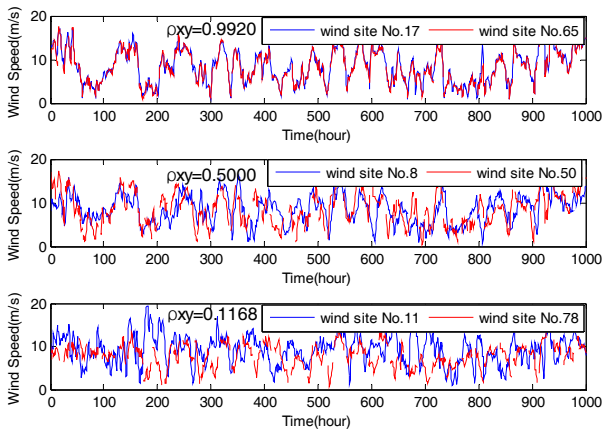
Variable speed wind generators which are widely used today will not themselves cause electromechanical modes of oscillation. However, as illustrated above, the time-varying and correlated wind speeds will change the aggregated wind generation and hence have the potential to indirectly change the damping performance of the system by [4, 23] : (i) significantly altering the dispatch of synchronous generation in order to accommodate wind generation; (ii) significantly altering the power flows in the transmission network; and (iii) interacting with synchronous machines to change the damping torques induced on their shafts. So it is necessary to consider the uncertainty and correlation of wind speeds in PSSSA of power system integrated with wind power generation.

3. Probabilistic Small Signal Stability Analysis

3.1 Modal analysis

Small signal stability is the ability of a power system to maintain synchronism when subjected to small disturbances. The dynamic behavior of power system can be described by a set of nonlinear differential algebraic equations (DAEs):

$$\dot{x} = f(x, y)$$
$$0 = g(x, y) \tag{2}$$

where, $x \in \mathbf{R}^n$ is the vector of state variables, i.e. synchronous and asynchronous machine rotor speeds, synchronous machine power angles, magnetic flux linkages, controller state variables, etc. $y \in \mathbf{R}^m$ is the vector of algebraic variables, i.e. voltage amplitudes and phases at the network buses and all other algebraic variables such as generator field voltages, AVR reference voltages, etc. f are the vectors of non-linear functions defining the states and g consists of the stator algebraic equations and the power flow equations in the power-balance form. The most adequate tool to perform small signal stability studies is

Fig. 2. Wind speed time series of wind-site-pairs with different correlation level

modal analysis which is performed by using the linearization of the DAEs (2) around a system operating point:

$$\begin{bmatrix} \Delta \dot{\boldsymbol{x}} \\ 0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_x & \boldsymbol{F}_y \\ \boldsymbol{G}_x & \boldsymbol{G}_y \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{x} \\ \Delta \boldsymbol{y} \end{bmatrix} \qquad (3)$$

Eliminating the algebraic variables $\Delta \boldsymbol{y}$ from (3), we get $\Delta \dot{\boldsymbol{x}} = \boldsymbol{A}_s \Delta \boldsymbol{x}$, where $\boldsymbol{A}_s$ is called state matrix implicitly assuming that $\boldsymbol{G}_y$ is non-singular:

$$\boldsymbol{A}_s = \boldsymbol{F}_x - \boldsymbol{F}_y \boldsymbol{G}_y^{-1} \boldsymbol{G}_x \qquad (4)$$

Analysis of the eigenproperties of $\boldsymbol{A}_s$, such as eigenvalues, eigenvector, participation factor, provides valuable information regarding the stability of the system. According to Lyapunov's first method, the small signal stability of a power system is given by the eigenvalues of $\boldsymbol{A}_s$. If all eigenvalues have a negative real part, all oscillation modes (OM) decay with time and the system is said to be stable. The critical eigenvalues which determine the small signal stability of the power system are characterized by being complex and by being located near the imaginary axis of the complex plane. The damping ratio determines the rate of decay of the amplitude of the oscillation. The mode shape given by the right eigenvector helps to distinguish the various types of oscillation. Besides, the participation factor indicates the relative contribution of each state variable to a certain mode. The electro-mechanical (EM) oscillation mode is recognized according to the electro-mechanical relative coefficient ρ_{EM} and the frequency of oscillation f, i.e., $\rho_{EM} > 1$ and $0.2 < f < 2.5 \, \text{Hz}$.

When power system uncertainties are considered, the system's equilibrium is no longer deterministic. In particular, the DAEs will contain non-deterministic system parameters which have known statistics. The presence of these random variables will cause the eigenvalues of $\boldsymbol{A}_s$ to be non-deterministic. It is the purpose of PSSSA to determine the probability density of the real part of the eigenvalues of $\boldsymbol{A}_s$ and characterize the stochastic nature of power system stability.

In this paper, the wind speed is chosen as the uncertain parameter and other uncertainties are neglected. A widely used Matlab-based power system analysis and simulation tool — Power System Analysis Toolbox (PSAT) [24], is used to run the power flow and calculate the eigenvalues and other eigenproperties of state matrix $\boldsymbol{A}_s$ of the investigated scenarios.

3.2 DAEs of doubly-fed induction generator system

Nowadays, wind turbines of variable speed type have become more common than traditional fixed speed turbines. Especially, the Doubly-Fed Induction Generator (DFIG) -

based wind turbine is gaining prominence in the power industry due to its characteristics of high energy transfer efficiency, low investment and flexible control. Therefore, wind farms represented by DFIG-based wind turbines will be used in PSSSA in this paper.

The dynamic model of DFIG system contains the several components: driven train, pitch controller, generator, and converter controller. The DAEs of DFIG system is presented in Appendix A and the following assumptions and strategies are adopted.

The drive train comprising turbine, gearbox, shafts and other transmission components is represented by a two-mass model.

The dynamic model of the grid-side converter controller is neglected as it is noted that the dynamics of rotor-side converter controller has more significant impact on the power system small signal stability than grid-side converter controller. The decoupling control strategy developed in [25] is used for the active power and reactive power of rotor-side converter. The stator voltage-oriented control scheme is adopted, which makes the stator voltage line in accordance with q-axis of d-q reference frame, then u_{ds} becomes zero and u_{qs} is equal to the magnitude of the terminal voltage.

4. Solution Method

4.1 2m+1 PEM

The 2m+1 PEM developed in [17] is applied in this paper to solve the problem of PSSSA, where m is the number of input random variable. This method uses deterministic routines for solving probabilistic problems; however, it generally requires a lower computational burden compared with MCS.

Random variables z_j is function F_j of m input random variables ($x_1, x_2, \cdots, x_m$):

$$z_j = F_j(\boldsymbol{x}) = F_j(x_1, x_2, \cdots, x_m) \qquad (5)$$

The 2m+1 PEM is used to obtain the first few moments of the output random variables of interest only required few statistical moments of the input random variables. To obtain these moments, the function F_j has to be calculated 2m+1 times. For each input random variable x_i, the function F_j is calculated using two input variable vectors $\boldsymbol{x}_{i,1}$, $\boldsymbol{x}_{i,2}$:

$$\boldsymbol{x}_{i,k} = (\mu_{x_1}, \mu_{x_2}, \cdots, \hat{x}_{i,k}, \cdots, \mu_{x_m})^{\text{T}} \quad k=1, 2 \quad i=1,2,\cdots,m \qquad (6)$$

Where $\hat{x}_{i,k}$ is called the location of x_i, μ_{x_i} is the means of the m-1 remaining input variables.

After 2m calculations are carried out, one addition evaluation of the function F is required at the point $\boldsymbol{x}_\mu$

constituted by the means of all of the input random variables:

$$\boldsymbol{x}_\mu = (\mu_{x_1}, \mu_{x_2}, \cdots, \mu_{xi}, \cdots, \mu_{x_m})^{\mathrm{T}} \tag{7}$$

Once the solution of the $2m+1$ functions $F_j(\cdot)$ is known, the moments of the output random variables can be obtained by using the weighting factors $w_{i,k}$, w_0 associated with $\boldsymbol{x}_{i,k}$ and $\boldsymbol{x}_\mu$, respectively.

The pair $(\hat{x}_{i,k}, w_{i,k})$ composed by a location $\hat{x}_{i,k}$ at which function $F_j(\cdot)$ is to be evaluated and a weighting factor $w_{i,k}$ measuring the impact of this evaluation on the random behavior of output variable z_j is called the *k*th ($k=1, 2$) concentration of the random input variable x_i.

For each input random variable x_i, the location $\hat{x}_{i,k}$ is depend on the first four central moments and expressed as

$$\hat{x}_{i,k} = \mu_{x_i} + \xi_{i,k}\sigma_{x_i} \quad k=1, 2 \tag{8}$$

where μ_{x_i} and σ_{x_i} are the mean and standard deviation of x_i, respectively, $\xi_{i,k}$ is the standard location:

$$\xi_{i,k} = \frac{\lambda_{x_i,3}}{2} + (-1)^{3-k}\sqrt{\lambda_{x_i,4} - \frac{3}{4}\lambda_{x_i,3}^2} \quad k=1, 2 \tag{9}$$

where $\lambda_{x_i,3}$ and $\lambda_{x_i,4}$ denote the third and fourth standardized central moments of x_i with probability density function $f_i(x)$, are also the skewness and kurtosis of x_i.

$$\lambda_{x_i,r} = \frac{\mu_{x_i,r}}{\sigma_{x_i}^r} \quad r=3, 4 \tag{10}$$

$$\mu_{x_i,r} = \int_{-\infty}^{+\infty}(x - \mu_{x_i})^r f_i(x)dx \quad r=3, 4 \tag{11}$$

where $\mu_{x_i,r}$ is the *r*th central moments of x_i.

Each location $\hat{x}_{i,k}$ is coupled with a weighting factor $w_{i,k}$ computed as

$$w_{i,k} = \frac{(-1)^{3-k}}{\xi_{i,k}\left(\xi_{i,1} - \xi_{i,2}\right)} \quad k=1, 2 \tag{12}$$

$$w_0 = 1 - \sum_{i=1}^{m}\frac{1}{(\lambda_{x_i,4} - \lambda_{x_i,3}^2)} \tag{13}$$

Once all the concentrations of input random variables are determined by using (8)-(13), the $2m+1$ evaluations of function $F_j(\cdot)$ is then calculated as

$$z_j(i,k) = F_j(\boldsymbol{x}_{i,k}) = F_j(\mu_{x_1}, \mu_{x_2}, \cdots, \hat{x}_{i,k}, \cdots, \mu_{x_m}) \\ i=1,2,\cdots,m, \quad k=1,2 \tag{14}$$

$$z_j(2m+1) = F_j(\boldsymbol{x}_\mu) = F_j\left(\mu_{x_1}, \mu_{x_2}, \cdots, \mu_{xi}, \cdots, \mu_{x_m}\right) \tag{15}$$

The *n*th raw moment of the output random variable z_j, denoted by $m_{z_j,n}$ is estimated as

$$m_{z_j,n} = E(z_j^n) \approx \sum_{i=1}^{m}\sum_{k=1}^{2} w_{i,k}[F_j(\boldsymbol{x}_{i,k})]^n + w_0[F_j(\boldsymbol{x}_\mu)]^n \tag{16}$$

where $E(\cdot)$ denotes the expectation operator. The mean value and the standard deviation of z_j, denoted by μ_{z_j} and σ_{z_j}, can be estimated according to (16).

$$\mu_{z_j} = m_{z_j,1} = E(z_j) \approx \sum_{i=1}^{m}\sum_{k=1}^{2} w_{i,k}F_j(\boldsymbol{x}_{i,k}) + w_0 F_j(\boldsymbol{x}_\mu) \tag{17}$$

$$m_{z_j,2} = E(z_j^2) \approx \sum_{i=1}^{m}\sum_{k=1}^{2} w_{i,k}[F_j(\boldsymbol{x}_{i,k})]^2 + w_0[F_j(\boldsymbol{x}_\mu)]^2 \tag{18}$$

$$\sigma_{z_j} = \sqrt{E(z_j^2) - [E(z_j)]^2} = \sqrt{m_{z_j,2} - m_{z_j,1}^2} \tag{19}$$

4.2 Cornish fisher expansion

Knowing the statistical moments, it is possible to obtain the PDFs or the CDFs of the output variables by using the Cornish-Fisher expansion method or Gram-Charlier expansion series [20]. Ref.[26] proved that Cornish-Fisher expansion is more adequate for the problem conditions (non-Gaussian PDF of the wind power uncertainties), instead of the Gram-Charlier expansion series. So in this paper, the Cornish-Fisher expansion approach is applied to compute the PDFs and the CDFs of the output random variables. This approach provides the approximation of the normalized quantiles α of any cumulative distribution function $F(x)$ in terms of the quantile of the standard normal N(0,1) distribution Φ and the cumulants of $F(x)$.

Using the first five cumulants, the expansion is given by (20).

$$x(\alpha) \approx \Phi^{-1}(\alpha) + \frac{1}{6}(\Phi^{-1}(\alpha)^2 - 1)\kappa_3 + \frac{1}{24}(\Phi^{-1}(\alpha)^3 - 3\Phi^{-1}(\alpha))\kappa_4 \\ - \frac{1}{36}(2\Phi^{-1}(\alpha)^3 - 5\Phi^{-1}(\alpha))\kappa_3^2 + \frac{1}{120}(\Phi^{-1}(\alpha)^4 - 6\Phi^{-1}(\alpha)^2 + 3)\kappa_5 \\ - \frac{1}{24}(\Phi^{-1}(\alpha)^4 - 5\Phi^{-1}(\alpha)^2 + 2)\kappa_3\kappa_4 + \frac{1}{324}(12\Phi^{-1}(\alpha)^4 - 53\Phi^{-1}(\alpha)^2 + 17)\kappa_3^3 \tag{20}$$

where $x(\alpha) = F^{-1}(x)$. κ_i is the *i*th cumlants, which can be obtained from its raw moments as follows:

$$\kappa_1 = m_1 \\ \kappa_i = m_i - \sum_{k=1}^{i-1}\binom{i-1}{k} \cdot m_k \cdot \kappa_{i-k} \quad (i \geq 2) \tag{21}$$

4.3 Managing the correlations of input variables

The procedure mentioned above can only be applied when the input random variables are uncorrelated. In the presence of correlation among random input variables, the well-known orthogonal (rotational) transformation technique

is used to transform the set of input correlated random variables x into an uncorrelated set of random variable y. Once the set of uncorrelated variables is obtained, the $2m+1$ PEM described by (8)-(13) can be applied and then the concentrations of y is determined. Finally, these points are untransformed to the original correlated variable space. The details about the procedure can be found in [6].

4.4 Computational procedure of PSSSA

The computational procedure to solve a PSSSA problem with correlated wind sources using the $2m+1$ PEM is summarized below.

(1) The random wind speeds of m correlated wind farms are considered as the input random variables $x = (x_1, x_2, \cdots, x_i, \cdots, x_m)^T$. Known the PDF of x_i and the correlation coefficient matrix ρ, determine the covariance matrix C_x and calculate the first four central moment of each variable x_i.

(2) Transform the first four central moments of x into uncorrelated space by applying the orthogonal transformation.

(3) Determine all the concentrations $(\hat{y}_{i,k}, w_{i,k})$ according to (8)-(13) and formed the $2m+1$ transformed input points $y_{i,k}$, y_μ.

(4) Obtain $x_{i,k}$, x_μ which are in the original space by using inverse transformation.

(5) Run the deterministic power flow for $2m+1$ input point (wind speed) $x_{i,k}$ and x_μ.

(6) Solve z_j which denote the eigenvalues and the eigenvectors of state matrix, damping ratio and participation factors at each deterministic operating point determined by power flow. Estimate the raw moments of z_j as expressed in (16)-(19).

(7) After obtaining the moments of the eigenvalues, find the CDFs of real part of critical modes(CMs) by applying the Cornish-Fisher expansion as explained in (20), (21), and then determine the probability of power system small signal stability.

All the steps described above have been implemented in MATLAB with the help of PSAT.

5. Case Study

5.1 Simulation conditions and assumptions

In this section, the proposed algorithm will be applied in the PSSSA of the IEEE New England (10-machine 39-bus) system [27], modified to include two wind farms, to demonstrate its validity.

In this system, two 375MW wind farms having 250 1.5MW DFIG-type wind turbines are located at bus 40 and 41 which connected with bus 33 and 34 via transformers.

Each wind farm is regarded as an aggregated wind turbine which is represented by the dynamic model of DFIG described in section 3.2 and the parameters of the DFIG system is presented in [28]. Synchronous generator G2 considered as the swing bus is modeled by the classic electro-mechanical model (the 2nd-order model). The other 6 synchronous generators are all modeled by the 4th-order models, with magnetic saturation neglected, and extended by 3rd-order exciter models.

The same Weibull distribution of wind speed v_w as stated in (22), with scale and shape parameters, c and k, equal to 7.65 and 2.06, respectively, is used to model wind speed at both sites.

$$f(v_w, c, k) = \frac{k}{c}\left(\frac{v_w}{k}\right)^{k-1} e^{-\left(\frac{v_w}{c}\right)^k} \tag{22}$$

5.2 Performance evaluation

In order to demonstrate the accuracy and efficiency of the proposed method, a comparison of the results obtained by $2m+1$ PEM are compared with those obtained by a MCS with 5000 trials. Inverse Nataf transformation is adopted in this paper to generate the random samples of correlated wind speed in the MCS.

Table 1 shows the mean value μ and standard deviation σ of the real part of critical eigenvalue *Eigreal* with different wind speed correlations (ρ_{xy}=0.1 to ρ_{xy}=0.9 with a step of 0.1). The results indicate that the proposed method provides a good approximation by comparing the results from MC method, both for the mean value and the standard deviation. However, MC requires 5000 simulations while the proposed $2m+1$ PEM method requires only 5 simulations. Therefore the conclusion is made that $2m+1$ PEM method can provide accurate results and is computationally much more efficient than MCS.

5.3 Impact of wind speed correlation on PSSSA

Two situations are presented using the proposed method to illustrate the importance of modeling wind farms with considering wind speed correlation: weak correlated

Table 1. Mean and standard deviation of *Eigreal* with different wind speed correlations

ρ_{xy}	PEM		MCS	
	μ	σ	μ	σ
0.1	-0.0861	0.0184	-0.0861	0.0184
0.2	-0.0853	0.0220	-0.0852	0.0219
0.3	-0.0847	0.0221	-0.0844	0.0221
0.4	-0.0839	0.0228	-0.0839	0.0224
0.5	-0.0829	0.0231	-0.0831	0.0231
0.6	-0.0818	0.0262	-0.0818	0.0260
0.7	-0.0811	0.0263	-0.0811	0.0263
0.8	-0.0790	0.0279	-0.0789	0.0274
0.9	-0.0781	0.0322	-0.0781	0.0322

(a) Weak correlated situation

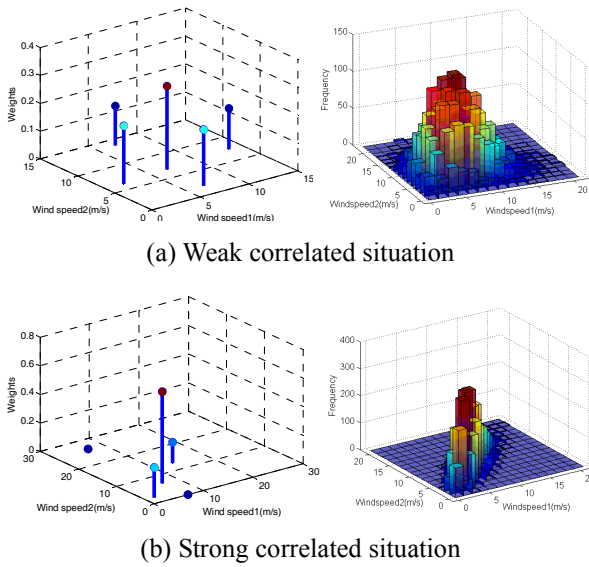(b) Strong correlated situation

Fig. 5. Wind speed joint distribution

situation (ρ_{xy}=0.1) and strong correlated situation (ρ_{xy}=0.9). Fig. 5 shows the wind speed joint distribution using $2m$+1 PEM and MCS respectively, under two situations. By comparing of (a) and (b), wind speed joint distribution tends to concentrate more on the diagonal in the strong correlated case than the weak correlated case, which will strength the synchronization of two wind farms' power output and may change the probability of small signal stability.

Based on the computational procedure described in section 4, the PSSSA of the power system concluding two wind farms is conducted and the statistic information of eigenvalues is then evaluated. By using Cornish-Fisher expansion, the CDF of the real part of critical eigenvalue which determine the small signal stability of the system can be obtained, as shown in Fig. 6, assuming weak correlated and strong correlated respectively. Obviously, the results indicate a difference in the probability of small signal stability between the two situations. It can be observed from Fig. 6 that the critical eigenvalue of the system has a probability of 98.90% to remain in the left half-plane in weak correlated situation, in other words, the system has a probability of 1.1% to be unstable. However, in the strong correlated situation, the probability of small signal stability of the system decreases to 96.98%.

Furthermore, in order to study the impact of correlation level of wind speed on PSSSA, a set of wind speed correlation coefficients as shown in Table 1 is used to calculate the probability of small signal stability of the test system. The results are shown in Fig. 7. It can be seen from Fig. 7 that wind speed correlation has a negative impact on the small signal stability as the degree of correlation between the two wind farms increases. When the correlation coefficient changes from 0.1 to 0.9 (weak independent caseec to strong independent case), the probability of small signal stability is decreased from
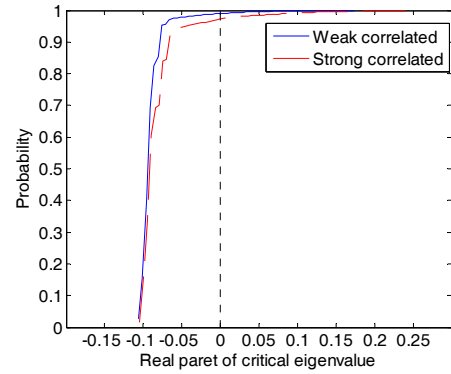

Fig. 6. The CDF comparison of the real part of critical eigenvalue
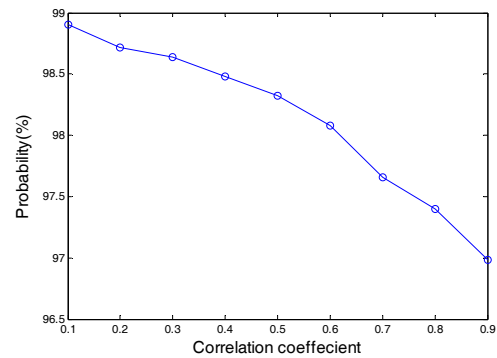

Fig. 7. The probability of small signal stability of the power system with different wind speed correlation coefficient

98.90% to 96.98%.

In addition to the real part of critical mode *Eigreal*, the mean and standard deviation of other important properties: the oscillation frequency *f*, damping ratio ξ and the electro-mechanical relative coefficient ρ_{EM} are also shown in Fig. 8. Fig. 8 shows that along with the increase of correlation coefficient, $\mu_{Eigreal}$ and $\mu_{\rho_{EM}}$ increase with percentage change of 4.78%, 9.29% respectively, μ_f and μ_ξ decreases with percentage change of -0.77%, -19.17% respectively. The standard deviations of all properties increase, the percentage change corresponding to $\sigma_{Eigreal}$, σ_f, σ_ξ and $\sigma_{\rho_{EM}}$ are 75%, 40.49%, 75.65% and 27.94%. The standard deviations can be seen as a measure of the uncertainty level affecting the power system, and therefore the smaller they are, the better. So the conclusion can be drawn that the damping performance of the test system tends to deteriorate and the variation of critical mode properties becomes larger as the correlation level of two wind farms increases.

5.4 EM mode analysis

EM oscillation modes of moderate correlated case (ρ_{xy}=0.5) are recognized by using the criterion of $\rho_{EM}>1$ and $0.2<f<2.5$ Hz. There are nine EM oscillation
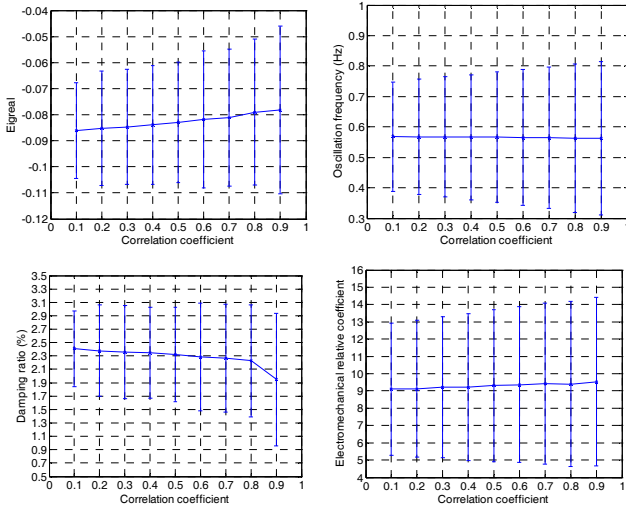
Fig. 8. The mean and standard deviation of critical mode with different wind speed correlation coefficient
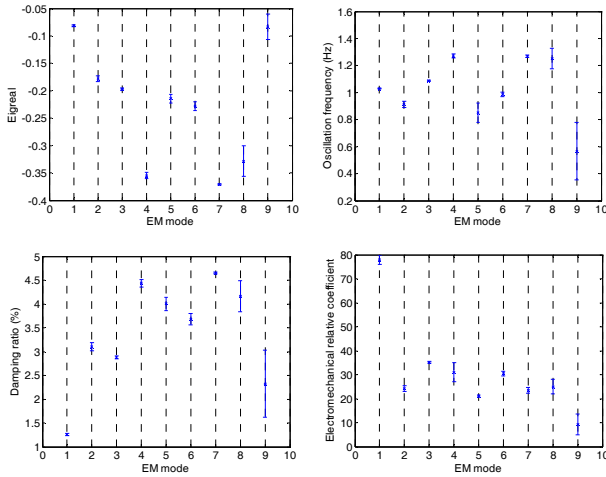

Fig. 9. The mean and standard deviation of EM mode properties

modes in the modified 10-machine 39-bus system, due to the two DFIG based wind turbines do not engage in power system oscillation [2, 4]. The mean and standard deviation of mode properties are depicted in Fig. 9. It can be seen from Fig. 9 that the damping performances of 8th EM (local oscillation) and 9th EM (inter-area oscillation) are greatly influenced by the fluctuation of wind power output.

6. Conclusion

In this paper, a methodology based on the 2*m*+1 PEM combined with Cornish-Fisher expansion and modal analysis is applied to solve the problem of probabilistic small signal stability of power systems with correlated wind sources.

A case study is carried out on the modified IEEE New England system with two grid-connected DFIG wind farms

and the wind speeds with Weibull distribution are regarded as the input random variables. The accuracy and efficiency of the proposed method are confirmed by comparing with the results of MCS. The stable probability is assessed based on the statistical information of critical eigenvalues obtained by conducting the proposed method. Comparison of different correlation situations shows that wind speed correlation has a negative impact on the small signal stability as the degree of correlation between the two wind farms increases. Finally, electro-mechanical oscillation modes of the system are picked out; the numerical results indicate that the proposed method can actually capture the probabilistic characteristics of mode properties of the power systems with correlated wind sources.

Acknowledgements

Appendix: DAEs of DFIG system

Nomenclature:

ω_t, ω_r : Wind turbine and generator rotor angle speed
ω : Synchronous angle speed
H_t, H_g : Inertia constants of the turbine and the generator
T_m, T_e : Mechanical and electrical torque
K_s, θ_s : Shaft stiffness coefficient and shaft twist angle
D : Damping coefficient
ρ : Air density
R : Wind turbine blade radius
C_p : Power coefficient
v_w : Wind speed
β : Pitch angle
K_{pp}, K_{pi} : Proportional and integrating gains of the wind turbine speed regulator
ψ_{ds}, ψ_{qs} : Stator flux (dq components)
i_{dr}, i_{qr} : Rotor current (dq components)
u_{qs} : Stator voltage (dq components)
u_{dr}, u_{qr} : Rotor voltage (dq components)
R_s : Stator resistance
L_{rr}, L_{ss} : Rotor and stator self-inductance
L_m : Mutual inductance
x_1, x_2, x_3, x_4 : Intermediate variables
K_{p1}, K_{i1} : Proportional and integrating gains of the active power regulator
K_{p2}, K_{i2} : Proportional and integrating gains of the reactive power regulator
C_p, C_i : Proportional and integrating gains of the rotor-

side converter current regulator

P_{ref}, Q_{ref} : Active and reactive power control reference

P_s, Q_s : Active and reactive power of stator-side

Wind turbine and driven train:

$$\dot{\omega}_t = \frac{1}{2H_t}[T_m - K_s\theta_s - D(\omega_t - \omega_r)]$$

$$\dot{\omega}_r = \frac{1}{2H_g}[K_s\theta_s + D(\omega_t - \omega_r) - T_e]$$

$$\dot{\theta}_s = \omega_t - \omega_r$$

$$T_m = \frac{0.5\rho\pi R^2 C_p v_w^3}{\omega_t}$$

$$T_e = L''(\psi_{ds}i_{qr} - \psi_{qs}i_{dr})$$

Pitch controller:

$$\dot{\beta} = K_{pp}\frac{T_m - K_s\theta_s - D(\omega_t - \omega_r)}{2H_t} + K_{pi}\Delta\omega_t$$

Generator:

$$\dot{\psi}_{ds} = -\frac{R_s}{L_{ss}}\psi_{ds} + L''R_s i_{dr} + \omega\psi_{qs}$$

$$\dot{\psi}_{qs} = -\frac{R_s}{L_{ss}}\psi_{qs} + L''R_s i_{qr} - \omega\psi_{ds} + u_{qs}$$

$$i_{dr} = \frac{1}{L'}[-R_r i_{dr} + u_{dr} + (\omega - \omega_r)L'i_{qr} + (\omega - \omega_r)L''\psi_{qs}$$
$$- L''(-\frac{R_s}{L_{ss}}\psi_{ds} + L''R_s i_{dr} + \omega\psi_{qs})]$$

$$i_{qr} = \frac{1}{L'}[-R_r i_{qr} + u_{qr} - (\omega - \omega_r)L'i_{dr} - (\omega - \omega_r)L''\psi_{ds}$$
$$- L''(-\frac{R_s}{L_{ss}}\psi_{qs} + L''R_s i_{qr} - \omega\psi_{ds} + u_{qs})]$$

$$L' = L_{rr} - \frac{L_m^2}{L_{ss}}, L'' = \frac{L_m}{L_{ss}}$$

Rotor-side converter:

$$\dot{x}_1 = P_{ref} - P_s$$

$$\dot{x}_2 = K_{p1}(P_{ref} - P_s) + K_{i1}x_1 - i_{qr}$$

$$\dot{x}_3 = Q_{ref} - Q_s$$

$$\dot{x}_4 = K_{p2}(Q_{ref} - Q_s) + K_{i2}x_3 - i_{dr}$$

$$u_{dr} = C_p(K_{p2}(Q_{ref} - Q_s) + K_{i2}x_3 - i_{dr}) + C_i x_4 - (\omega - \omega_r)(L'i_{qr} + L''\psi_{qs})$$

$$u_{qr} = C_p(K_{p1}(P_{ref} - P_s) + K_{i1}x_1 - i_{qr}) + C_i x_2 - (\omega - \omega_r)(L'i_{dr} + L''\psi_{ds})$$

$$P_s = u_{qs}(-\frac{\psi_{qs}}{L_{ss}} + L''i_{qr})$$

$$Q_s = u_{qs}(-\frac{\psi_{ds}}{L_{ss}} + L''i_{dr})$$

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