

Novel Results for Global Exponential Stability of Uncertain Systems with Interval Time-varying Delay

YajuanLiu*, Sang-Moon Lee[†], Oh-Min Kwon** and Ju H. Park***

Abstract – This paper presents new results on delay-dependent global exponential stability for uncertain linear systems with interval time-varying delay. Based on Lyapunov-Krasovskii functional approach, some novel delay-dependent stability criteria are derived in terms of linear matrix inequalities (LMIs) involving the minimum and maximum delay bounds. By using delay-partitioning method and the lower bound lemma, less conservative results are obtained with fewer decision variables than the existing ones. Numerical examples are given to illustrate the usefulness and effectiveness of the proposed method.

Keywords: Exponential stability, Uncertain linear system, Interval time-varying delays, Lyapunov method.

1. Introduction

Time delays are frequently encountered in many fields such as chemical engineering system, vehicles, biological modeling, economy and other fields. Thus the stability analysis of linear time delay systems has been received considerable attention during the last decades. Also, it is well known that the existence of time delay can lead oscillation, instability or divergence in system performance [1-2]. Thus, the stability analysis and synthesis of systems with time delay have been one of the hottest topic in control society. For recent trends of the topic, see [3-19] and references therein.

More recently, the stability analysis of linear systems with interval time-varying delay, which the lower bound is not restricted to be zero, has been extensively studied by many researchers [3-11, 18, 19]. A real example of such systems is networked control systems [13]. In general, the stability analysis of time-delay systems can be classified into two categories: delay-dependent [6-8] and delay-independent approach [12, 14]. It is well known that delay-dependent criteria are less conservative than the delay-independent ones when the size of time-delay is small. Hence, more attentions have been paid to the study for checking the conservatism of delay-dependent conditions. In the field of delay-dependent stability analysis, an important issue is to enlarge the feasibility region of stability criteria or to obtain a maximum allowable upper bound of time delays as large as possible. In this regard,

Shao [7] provided a new delay-dependent stability criterion for linear systems with interval time-varying delay by utilizing the convex combination method. In [9], less conservative results were derived with much fewer decision variables by introducing the lower bound lemma. Liu [11] constructed a new Lyapunov functional which makes the results less conservative than the results of [7, 9]. It should be noted that these results only focused their effort on asymptotic stability.

In practice, some uncertainties in the systems are unavoidable because it is very difficult to obtain an exact mathematical model due to the environmental noise, uncertain or slowly varying parameters. Thus, it is natural to consider the parameter uncertainties in a mathematical model. In addition, fast convergence of a system is essential for real-time computation, and the exponential convergence rate is generally used to determine the speed of computations. Therefore, the exponential stability analysis for systems with time delays has received deep concern in very recent years [14-19]. However, to the best of authors' knowledge, there are few results about the exponential stability of uncertain linear system with interval time varying delays [18, 19]. In [18], some delay-dependent sufficient conditions for the exponential stabilization of the systems are established in terms of LMIs by the construction of improved Lyapunov-Krasovskii functional combined with Leibniz Newton's formula. In [19], the authors dealt with the same problem using two novel integral equalities. But, these results have two drawbacks. One is that the results cannot be applied when the time delay is differentiable [18, 19], the other is that it involves many matrix variables [19], which increase computation burden.

With this motivation, we revisit the problem of exponential stability of uncertain linear systems with interval time-varying delay. By constructing a new

[†] Corresponding Author: Dept. of Electronic Engineering, Daegu University, Korea. (moony@daegu.ac.kr)

* Dept. of Electronic Engineering, Daegu University, Korea. (yajuan_liu@126.com)

** Dept. of Electrical Engineering, Chungbuk National University, Korea. (madwind@chungbuk.ac.kr)

*** Dept. of Electrical Engineering, Yeungnam University, Korea. (jessie@ynu.ac.kr)

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Lyapunov-Krasovskii functional, novel delay-dependent stability criteria with an exponential convergence rate are derived. Our results can be applied when the delay is differentiable. Less conservative results are obtained with fewer matrix variables than [19] by using delay-partitioning technique and the lower bound lemma. Finally, three numerical examples are shown to confirm the superiority of our results

Notations: Throughout this paper, I denotes the identity matrix with appropriate dimensions, R^n denotes the n dimensional Euclidean space, and $R^{m \times n}$ is the set of all $m \times n$ real matrices, $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. For symmetric matrices A and B , the notation $A > B$ (respectively, $A \geq B$) means that the matrix $A - B$ is positive definite (respectively, nonnegative), $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ stand for the largest and smallest eigenvalue of given square matrix, respectively. $\text{diag}\{\dots\}$ denotes the block diagonal matrix. $\|\phi\| = \sup_{-h_M \leq s \leq 0} \{\|x(t+s)\|, \|\dot{x}(t+s)\|\}$, where $h_M > 0$ is some constant.

2. Problem Statement

Construction the following uncertain linear systems with time varying delay:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + (A_1 + \Delta A_1(t))x(t-h(t)) \quad (1) \\ x(t) &= \varphi(t), t \in [-h_M, 0], \end{aligned}$$

where $x(t) \in R^n$ is the state, A and A_1 are known real constant matrices with appropriate dimensions; $\varphi(t)$ is the initial condition of the system. The time varying delay $h(t)$ is differentiable function satisfying

$$0 \leq h_m \leq h(t) \leq h_M, \quad (2)$$

$$\dot{h}(t) \leq \mu, \quad (3)$$

where the bounds h_m, h_M, μ are known positive scalars.

The uncertainties satisfy the following conditions:

$$[\Delta A(t) \ \Delta A_1(t)] = DF(t)[E \ E_1], \quad (4)$$

where D, E, E_1 are known constant matrices, $F(t) \in R^{n \times n}$ is the unknown real time varying matrices with Lebesgue measurable elements bounded by

$$F^T(t)F(t) \leq I, \forall t \geq 0. \quad (5)$$

Therefore, system (1) with uncertainties satisfying (4) and (5) can be written in the following form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_1x(t-h(t)) + Dp(t), \\ p(t) &= F(t)q(t), \\ q(t) &= Ex(t) + E_1x(t-h(t)). \end{aligned} \quad (6)$$

We need the following definition and Lemmas for deriving the main results.

Definition 2.1 [16] For a given positive scalar k , the zero solution of (6) is exponentially stable if there exist a positive scalar γ such that every solution $x(t)$ of (6) satisfies the following condition

$$\|x(t)\| \leq \gamma e^{-kt} \|\phi\|.$$

Lemma 2.1 [7] For any constant positive definite matrix $M \in R^n$ and $\beta \leq s \leq \alpha$, the following inequalities hold

$$\begin{aligned} -(\alpha - \beta) \int_{\beta}^{\alpha} \dot{x}^T(s) M \dot{x}(s) ds \leq \\ -[x(\alpha) - x(\beta)]^T M [x(\alpha) - x(\beta)]. \end{aligned}$$

Lemma 2.2 [9] (Lower bounds lemma) Let $f_1, f_2, \dots, f_N : R^m \rightarrow R$ have positive values in an open subset D of R^m . Then, the reciprocally convex combination of f_i over D satisfies

$$\min_{\{\alpha_i | \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{\{g_{i,j}\}} \sum_{i \neq j} g_{i,j}(t)$$

Subject to

$$\left\{ g_{i,j} : R^m \rightarrow R, g_{i,j}(t) \triangleq g_{j,i}, \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} > 0 \right\}.$$

3. Main Results

In this section, new stability criteria for system (1) will be derived by use of Lyapunov method and LMI framework

3.1 Exponential stability for nominal systems with interval time-varying delay

First, we present a delay dependent exponential stability condition for the following nominal interval time-varying delay systems with $\Delta A(t) = 0, \Delta A_1(t) = 0$.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_1x(t-h(t)) \\ x(t) &= \varphi(t), t \in [-h_M, 0]. \end{aligned} \quad (7)$$

By introducing augmented Lyapunov-Krasovskii func-

tional, a less conservative delay-dependent stability criterion for system (7) will be proposed. In convenience, let us define

$$h_1 = \frac{h_m}{2}, h_2 = \frac{h_M + h_m}{2},$$

$$\xi^T(t) = \begin{bmatrix} x^T(t), x^T(t-h(t)), x^T(t-h_m), \\ x^T(t-h_M), x^T(t-h_1), x^T(t-h_2) \end{bmatrix}.$$

The corresponding block entry matrices are defined as $e_i \in R^{6 \times n}$ ($i = 1, 2, \dots, 6$) e.g., $e_4^T = [0 \ 0 \ 0 \ I \ 0 \ 0]^T \in R^{6 \times n}$. Defining $e_7 = (Ae_1^T + A_1e_2^T)^T$, then (7) can be written as $\dot{x}(t) = e_7^T \xi(t)$.

Denote

$$\delta = \frac{h_M - h_m}{2},$$

$$\Sigma_1 = 2ke_1Pe_1^T + e_1Pe_7^T + e_7Pe_1^T,$$

$$\Sigma_2 = e^{-2kh_m}e_3Re_3^T - (1-\mu)e^{-2kh_m}e_2Re_2^T,$$

$$\Sigma_3 = h_m^2e_7Ue_7^T - e^{-2kh_m}[e_1 - e_3]U[e_1 - e_3]^T + (h_M - h_m)^2e_7Ue_7^T - e^{-2kh_m}[e_3 - e_2 \ e_2 - e_4] \begin{bmatrix} U & T \\ T^T & U \end{bmatrix} [e_3 - e_2 \ e_2 - e_4]^T,$$

$$\Sigma_4 = (h_M - h_m)e_1Se_1^T + e^{-2kh_m}e_3Ne_3^T + (h_M - h_m)e_7Se_7^T - e^{-2kh_m}e_4Ne_4^T,$$

$$\Sigma_5 = [e_1 \ e_5]Q_1[e_1 \ e_5]^T - e^{-2kh_1}[e_5 \ e_3]Q_1[e_5 \ e_3]^T + [e_3 \ e_6]Q_1[e_3 \ e_6]^T - e^{-2kh_2}[e_6 \ e_4]Q_1[e_6 \ e_4]^T,$$

$$\Sigma_6 = h_1^2e_7Z_1e_7^T - e^{-2kh_1}[e_1 - e_5]Z_1[e_1 - e_5]^T + (h_M - h_2)^2e_7Z_2e_7^T - e^{-2kh_m}[e_6 - e_4]Z_2[e_6 - e_4]^T + (h_2 - h_m)^2e_7Z_2e_7^T - e^{-2kh_2}[e_3 - e_2 \ e_2 - e_6] \begin{bmatrix} Z_2 & T_1 \\ T_1^T & Z_2 \end{bmatrix} [e_3 - e_2 \ e_2 - e_6]^T,$$

$$\Sigma_7 = h_1^2e_7Z_1e_7^T - e^{-2kh_1}[e_1 - e_5]Z_1[e_1 - e_5]^T + (h_2 - h_m)^2e_7Z_2e_7^T - e^{-2kh_2}[e_3 - e_6]Z_2[e_3 - e_6]^T + (h_M - h_2)^2e_7Z_2e_7^T - e^{-2kh_m}[e_6 - e_2 \ e_2 - e_4] \begin{bmatrix} Z_2 & T_1 \\ T_1^T & Z_2 \end{bmatrix} [e_6 - e_2 \ e_2 - e_4]^T,$$

$$\Upsilon_1 = \Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4 + \Sigma_5 + \Sigma_6,$$

$$\Upsilon_2 = \Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4 + \Sigma_5 + \Sigma_7.$$

Theorem 3.1 For given scalars h_m, h_M ($h_M > h_m$),

$\mu, k \geq 0$, system (7) is globally exponentially stable if there exist symmetric positive definite matrices $Q_1 \in R^{2n \times 2n}, Q_2 \in R^{2n \times 2n}, n \times n$ dimensional positive symmetric matrices P, R, U, S, Z_1, Z_2 , and appropriate dimension matrices T, T_1, N satisfying the following LMIs:

$$\Upsilon_1 < 0, \Upsilon_2 < 0, \tag{8}$$

$$\begin{bmatrix} U & T \\ T^T & U \end{bmatrix} \geq 0, \begin{bmatrix} Z_2 & T_1 \\ T_1^T & Z_2 \end{bmatrix} \geq 0, \begin{bmatrix} S & N \\ N^T & S \end{bmatrix} \geq 0. \tag{9}$$

Proof. Consider the following Lyapunov-Krasoskii functional

$$V(t) = \sum_{i=1}^6 V_i \tag{10}$$

where

$$V_1 = e^{2kt}x^T(t)Px(t),$$

$$V_2 = \int_{t-h(t)}^{t-h_m} e^{2ks}x^T(s)Rx(s)ds,$$

$$V_3 = h_m \int_{-h_m}^0 \int_{t+\alpha}^t e^{2ks}\dot{x}^T(s)U\dot{x}(s)d\alpha ds + (h_M - h_m) \int_{-h_M}^{-h_m} \int_{t+\alpha}^t e^{2ks}\dot{x}^T(s)U\dot{x}(s)d\alpha ds,$$

$$V_4 = \int_{-h_M}^{-h_m} \int_{t+\alpha}^t e^{2ks}(x^T(s)Sx(s) + \dot{x}^T(s)S\dot{x}(s))d\alpha ds,$$

$$V_5 = \int_{t-h_1}^t e^{2ks} \begin{bmatrix} x(s) \\ x(s-h_1) \end{bmatrix}^T Q_1 \begin{bmatrix} x(s) \\ x(s-h_1) \end{bmatrix} ds + \int_{t-h_2}^{t-h_m} e^{2ks} \begin{bmatrix} x(s) \\ x(s-\delta) \end{bmatrix}^T Q_2 \begin{bmatrix} x(s) \\ x(s-\delta) \end{bmatrix} ds,$$

$$V_6 = h_1 \int_{-h_1}^0 \int_{t+\alpha}^t e^{2ks}\dot{x}^T(s)Z_1\dot{x}(s)d\alpha ds + (h_M - h_2) \int_{-h_M}^{-h_2} \int_{t+\alpha}^t e^{2ks}\dot{x}^T(s)Z_2\dot{x}(s)d\alpha ds + (h_2 - h_m) \int_{-h_2}^{-h_m} \int_{t+\alpha}^t e^{2ks}\dot{x}^T(s)Z_2\dot{x}(s)d\alpha ds,$$

Calculating the time-derivative of $V(t)$, we have

$$\dot{V}_1 = 2ke^{2kt}x^T(t)Px(t) + 2e^{2kt}x^T(t)P\dot{x}(t) = e^{2kt}\xi^T(t)\Sigma_1\xi(t), \tag{11}$$

$$\dot{V}_2 = e^{2kt} \left\{ e^{-2kh_m}x^T(t-h_m)Rx(t-h_m) - (1-h'(t))e^{-2kh(t)}x^T(t-h(t))Rx(t-h(t)) \right\} \leq e^{2kt} \left\{ e^{-2kh_m}x^T(t-h_m)Rx(t-h_m) - (1-\mu)e^{-2kh_m}x^T(t-h(t))Rx(t-h(t)) \right\}$$

$$= e^{2kt} \xi^T(t) \Sigma_2 \xi(t), \tag{12}$$

$$\begin{aligned} \dot{V}_3 &= e^{2kt} h_m^2 \dot{x}^T(t) U \dot{x}(t) - h_m \int_{t-h_m}^t e^{2ks} \dot{x}^T(s) U \dot{x}(s) ds \\ &+ e^{2kt} (h_M - h_m)^2 \dot{x}^T(t) U \dot{x}(t) \\ &- (h_M - h_m) \int_{t-h_M}^{t-h_m} e^{2ks} \dot{x}^T(s) U \dot{x}(s) ds \\ &\leq e^{2kt} \left\{ h_m^2 \dot{x}^T(t) U \dot{x}(t) - h_m e^{-2kh_m} \int_{t-h_m}^t \dot{x}^T(s) U \dot{x}(s) ds \right. \\ &+ (h_M - h_m)^2 \dot{x}^T(t) U \dot{x}(t) \\ &\left. - (h_M - h_m) e^{-2kh_M} \int_{t-h_M}^{t-h_m} e^{2ks} \dot{x}^T(s) U \dot{x}(s) ds \right\}, \tag{13} \end{aligned}$$

Here, using Lemma 2.1, it can be obtained that

$$\begin{aligned} &-h_m \int_{t-h_m}^t \dot{x}^T(s) U \dot{x}(s) ds \\ &\leq -[x(t) - x(t-h_m)]^T U [x(t) - x(t-h_m)], \tag{14} \end{aligned}$$

Also, the following inequality is obtained from Lemma 2.2,

$$\begin{aligned} &- (h_M - h_m) \int_{t-h_M}^{t-h_m} \dot{x}^T(s) U \dot{x}(s) ds \\ &= - (h_M - h_m) \int_{t-h_M}^{t-h(t)} \dot{x}^T(s) U \dot{x}(s) ds \\ &- (h_M - h_m) \int_{t-h(t)}^{t-h_m} \dot{x}^T(s) U \dot{x}(s) ds \\ &\leq - \left(\frac{h_M - h_m}{h(t) - h_m} \right) \left(\int_{t-h(t)}^{t-h_m} \dot{x}(s) ds \right)^T U \left(\int_{t-h(t)}^{t-h_m} \dot{x}(s) ds \right) \\ &- \left(\frac{h_M - h_m}{h_M - h(t)} \right) \left(\int_{t-h_M}^{t-h(t)} \dot{x}(s) ds \right)^T U \left(\int_{t-h_M}^{t-h(t)} \dot{x}(s) ds \right) \\ &\leq - \begin{bmatrix} x(t-h_m) - x(t-h(t)) \\ x(t-h(t)) - x(t-h_M) \end{bmatrix}^T \begin{bmatrix} U & T \\ T^T & U \end{bmatrix} \\ &\begin{bmatrix} x(t-h_m) - x(t-h(t)) \\ x(t-h(t)) - x(t-h_M) \end{bmatrix}. \tag{15} \end{aligned}$$

It should be noted that when $h(t) = h_m$ or $h(t) = h_M$, we have $\int_{t-h(t)}^{t-h_m} \dot{x}(s) ds = 0$ or $\int_{t-h_M}^{t-h(t)} \dot{x}(s) ds = 0$, respectively. So the relation (15) still holds.

Form (13)-(15), we obtain

$$\begin{aligned} \dot{V}_3 &\leq e^{2kt} \xi^T(t) \Sigma_3 \xi(t), \\ \dot{V}_4 &\leq e^{2kt} \left\{ (h_M - h_m)^2 x^T(t) S x(t) + (h_M - h_m)^2 \dot{x}^T(t) S \dot{x}(t) \right. \\ &\left. - (h_M - h_m) e^{-2kh_M} \int_{t-h_M}^{t-h_m} e^{2ks} x^T(s) S x(s) ds \right\} \end{aligned}$$

$$- (h_M - h_m) e^{-2kh_M} \int_{t-h_M}^{t-h_m} e^{2ks} \dot{x}^T(s) S \dot{x}(s) ds \Big\},$$

Inspired by the work of [3], the following two zero inequalities hold for any appropriate dimension matrix N ,

$$\begin{aligned} &e^{2k(t-h_M)} \left[x^T(t-h_M) N x(t-h_M) - x^T(t-h(t)) N x(t-h(t)) \right. \\ &\quad \left. - 2 \int_{t-h(t)}^{t-h_M} x^T(s) N \dot{x}(s) ds \right] = 0, \\ &e^{2k(t-h_M)} \left[x^T(t-h(t)) N x(t-h(t)) - x^T(t-h_M) N x(t-h_M) \right. \\ &\quad \left. - 2 \int_{t-h_M}^{t-h(t)} x^T(s) N \dot{x}(s) ds \right] = 0, \end{aligned}$$

From the above zero equalities, one can obtain

$$\begin{aligned} \dot{V}_4 &\leq e^{2kt} \left\{ \xi^T(t) \Sigma_4 \xi(t) \right. \\ &- e^{-2kh_M} \int_{t-h(t)}^{t-h_m} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} S & N \\ N^T & S \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\ &\left. - e^{-2kh_M} \int_{t-h_M}^{t-h(t)} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} S & N \\ N^T & S \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \right\}, \tag{16} \end{aligned}$$

$$\begin{aligned} \dot{V}_5 &\leq e^{2kt} \left\{ \begin{bmatrix} x(t) \\ x(t-h_1) \end{bmatrix}^T \mathbb{Q}_1 \begin{bmatrix} x(t) \\ x(t-h_1) \end{bmatrix} \right. \\ &- e^{-2kh_1} \begin{bmatrix} x(t-h_1) \\ x(t-h_m) \end{bmatrix}^T \mathbb{Q}_1 \begin{bmatrix} x(t-h_1) \\ x(t-h_m) \end{bmatrix} \\ &+ \begin{bmatrix} x(t-h_m) \\ x(t-h_2) \end{bmatrix}^T \mathbb{Q}_2 \begin{bmatrix} x(t-h_m) \\ x(t-h_2) \end{bmatrix} \\ &\left. - e^{-2kh_2} \begin{bmatrix} x(t-h_2) \\ x(t-h_M) \end{bmatrix}^T \mathbb{Q}_2 \begin{bmatrix} x(t-h_2) \\ x(t-h_M) \end{bmatrix} \right\} \\ &= e^{2kt} \xi^T(t) \Sigma_5 \xi(t), \tag{17} \end{aligned}$$

$$\begin{aligned} \dot{V}_6 &\leq e^{2kt} \left\{ h_1^2 \dot{x}^T(t) Z_1 \dot{x}(t) - h_1 e^{-2kh_1} \int_{t-h_1}^t \dot{x}^T(s) Z_1 \dot{x}(s) ds \right. \\ &+ (h_M - h_2)^2 \dot{x}^T(t) Z_2 \dot{x}(t) \\ &- (h_M - h_2) e^{-2kh_M} \int_{t-h_M}^{t-h_2} e^{2ks} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\ &+ (h_2 - h_m)^2 \dot{x}^T(t) Z_2 \dot{x}(t) \\ &\left. - (h_2 - h_m) e^{-2kh_2} \int_{t-h_2}^{t-h_m} e^{2ks} \dot{x}^T(s) Z_2 \dot{x}(s) ds \right\}. \tag{18} \end{aligned}$$

Here, using Lemma 2.1, we have

$$\begin{aligned} &-h_1 \int_{t-h_1}^t \dot{x}^T(s) Z_1 \dot{x}(s) ds \\ &\leq -[x(t) - x(t-h_1)]^T Z_1 [x(t) - x(t-h_1)], \tag{19} \end{aligned}$$

In the following discussions, the upper bound of \dot{V}_6 is derived by considering two different cases for (i) $h_m \leq h(t) \leq h_2$ and (ii) $h_2 \leq h(t) \leq h_M$.

When $h_m \leq h(t) \leq h_2$, the following inequality is satisfied by Lemma 2.1

$$\begin{aligned}
 & -(h_M - h_2) \int_{t-h_M}^{t-h_2} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
 & \leq -[x(t-h_2) - x(t-h_M)]^T Z_2 [x(t-h_2) - x(t-h_M)],
 \end{aligned} \tag{20}$$

By using the similar methods in (15), we obtain

$$\begin{aligned}
 & -(h_2 - h_m) \int_{t-h_2}^{t-h_m} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
 & \leq - \begin{bmatrix} x(t-h_m) - x(t-h(t)) \\ x(t-h(t)) - x(t-h_2) \end{bmatrix}^T \begin{bmatrix} Z_2 & T_1 \\ T_1^T & Z_2 \end{bmatrix} \\
 & \quad \begin{bmatrix} x(t-h_m) - x(t-h(t)) \\ x(t-h(t)) - x(t-h_2) \end{bmatrix}
 \end{aligned} \tag{21}$$

From (18)-(21), in this cases

$$\dot{V}_6 \leq e^{2kt} \xi^T(t) \Sigma_6 \xi(t), \tag{22}$$

If (9) hold, the following inequality is satisfied with (11)-(22) by S-procedure [1],

$$\dot{V}(t) \leq e^{2kt} \xi^T(t) \Upsilon_1 \xi(t). \tag{23}$$

When $h_2 \leq h(t) \leq h_M$, the following inequality is satisfied by Lemma 2.1

$$\begin{aligned}
 & -(h_2 - h_m) \int_{t-h_2}^{t-h_m} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
 & \leq -[x(t-h_m) - x(t-h_2)]^T Z_2 [x(t-h_m) - x(t-h_2)],
 \end{aligned} \tag{24}$$

By using the similar methods in (15), we obtain

$$\begin{aligned}
 & -(h_M - h_2) \int_{t-h_M}^{t-h_2} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
 & \leq - \begin{bmatrix} x(t-h_2) - x(t-h(t)) \\ x(t-h(t)) - x(t-h_M) \end{bmatrix}^T \begin{bmatrix} Z_2 & T_1 \\ T_1^T & Z_2 \end{bmatrix} \\
 & \quad \begin{bmatrix} x(t-h_2) - x(t-h(t)) \\ x(t-h(t)) - x(t-h_M) \end{bmatrix}.
 \end{aligned} \tag{25}$$

From (18)-(21), in this cases

$$\dot{V}_6 \leq e^{2kt} \xi^T(t) \Sigma_7 \xi(t), \tag{26}$$

If (9) hold, the following inequality is satisfied with (11)-(19) and (26) by using S-procedure [1],

$$\dot{V}(t) \leq e^{2kt} \xi^T(t) \Upsilon_2 \xi(t). \tag{27}$$

Now we can conclude that if condition (8) and (9) are satisfied, then $\dot{V}(x(t)) \leq 0$. Thus, $V(x(t)) \leq V(x(0))$. Furthermore, from the definition of $V(x(t))$ and (10), we can derive the following inequalities:

$$V(x(0)) \leq a \|\phi\|^2,$$

where

$$\begin{aligned}
 a = & \lambda_{\max}(P) + (h_M - h_m) \lambda_{\max}(R) \\
 & + \left(\frac{h_m^3}{2} + \frac{(h_M - h_m)(h_M^2 - h_m^2)}{2} \right) \lambda_{\max}(U) \\
 & + (h_M^2 - h_m^2) \lambda_{\max}(S) + 2h_1 \lambda_{\max}(Q_1) \\
 & + 2h_2 \lambda_{\max}(Q_2) + \frac{h_m^3}{2} \lambda_{\max}(Z_1) + \\
 & \left(\frac{(h_M - h_2)(h_M^2 - h_2^2)}{2} + \frac{(h_2 - h_m)(h_2^2 - h_m^2)}{2} \right) \lambda_{\max}(Z_2).
 \end{aligned}$$

On the other hand, we have

$$V(x(t)) \geq e^{2kt} \lambda_{\min}(P) \|x(t)\|^2.$$

Hence,

$$\|x(t)\|^2 \leq \sqrt{\frac{a}{\lambda_{\min}(P)}} e^{-kt} \|\phi\|.$$

Then, the proof is completed by the Lyapunov stability theorem.

Remark 3.1 When $0 \leq h_m \leq h(t) \leq h_M$, the most attractive contribution is that we have made the best use of the lower bound (network induced delay) of the interval time-varying delay. In fact, in order to derive the less conservative stability criterion, we employ a new Lyapunov-Krasovskii functional (10), which is mainly based on the information about $h_1 = \frac{h_m}{2}, h_2 = \frac{h_m + h_M}{2}$.

Remark 3.2 When μ is known, Theorem 3.1 can be applied while [8, 16, 17] fails to work. If μ is unknown or $h(t)$ is not differentiable, then the following result can be obtained from Theorem 3.1 by setting $R = 0$, which will be introduced as Corollary 3.1.

Corollary 3.1 For given scalars $h_m, h_M (h_M > h_m), k \geq 0$, system (7) is globally exponentially stable if there exist symmetric positive definite matrices $Q_1 \in \mathbb{R}^{2n \times 2n}, Q_2 \in \mathbb{R}^{2n \times 2n}, n \times n$ dimensional positive symmetric matrices P, U, S, Z_1, Z_2 , and appropriate dimension

matrices T, T_1, N satisfying the following LMIs:

$$\tilde{Y}_1 < 0, \tilde{Y}_2 < 0, \tag{28}$$

$$\begin{bmatrix} U & T \\ T^T & U \end{bmatrix} \geq 0, \begin{bmatrix} Z_2 & T_1 \\ T_1^T & Z_2 \end{bmatrix} \geq 0, \begin{bmatrix} S & N \\ N^T & S \end{bmatrix} \geq 0, \tag{29}$$

where

$$\begin{aligned} \tilde{Y}_1 &= \Sigma_1 + \Sigma_3 + \Sigma_4 + \Sigma_5 + \Sigma_6, \\ \tilde{Y}_2 &= \Sigma_1 + \Sigma_3 + \Sigma_4 + \Sigma_5 + \Sigma_7. \end{aligned}$$

3.2 Robust exponential stability for uncertain systems with interval time varying delay

Based on the result of Theorem 3.1, the following theorem provides a robust exponential stability condition of uncertain linear systems with interval time-varying delay (6).

For simplicity in Theorem 3.2, $e_i \in R^{7n \times n} (i=1, 2, \dots, 7)$ are defined as block matrices. (for example, $\bar{e}_4^T = [0 \ 0 \ 0 \ I \ 0 \ 0 \ 0]^T \in R^{7n \times n}$). The other notations for some vectors and matrices are defined as:

$$\begin{aligned} \bar{\xi}^T(t) &= \begin{bmatrix} x^T(t), x^T(t-h(t)), x^T(t-h_m), \\ x^T(t-h_M), x^T(t-h_1), x^T(t-h_2), p^T(t) \end{bmatrix} \\ \mathbb{E} &= [E \ E_1 \ 0 \ 0 \ 0 \ 0 \ 0], \\ \mathbb{A} &= [A \ A_1 \ 0 \ 0 \ 0 \ 0 \ D], \\ \Phi &= \varepsilon \mathbb{E}^T \mathbb{E} - \varepsilon \bar{e}_7 \bar{e}_7^T, \\ \Sigma_1 &= 2k\bar{e}_1 P \bar{e}_1^T + \bar{e}_1 P \mathbb{A} + \mathbb{A}^T P \bar{e}_1^T, \\ \Sigma_2 &= e^{-2kh_m} \bar{e}_3 R \bar{e}_3^T - (1-\mu)e^{-2kh_M} \bar{e}_2 R \bar{e}_2^T, \\ \Sigma_3 &= h_m^2 \mathbb{A}^T U \mathbb{A} - e^{-2kh_1} [\bar{e}_1 - \bar{e}_3] U [\bar{e}_1 - \bar{e}_3]^T \\ &+ (h_M - h_m)^2 \mathbb{A}^T U \mathbb{A} \\ &- e^{-2kh_M} [\bar{e}_3 - \bar{e}_2 \ \bar{e}_2 - \bar{e}_4] \begin{bmatrix} U & T \\ T^T & U \end{bmatrix} [\bar{e}_3 - \bar{e}_2 \ \bar{e}_2 - \bar{e}_4]^T, \\ \Sigma_4 &= (h_M - h_m) \bar{e}_1 S \bar{e}_1^T + e^{-2kh_M} \bar{e}_3 N \bar{e}_3^T \\ &+ (h_M - h_m) \mathbb{A}^T S \mathbb{A} - e^{-2kh_M} \bar{e}_4 N \bar{e}_4^T, \\ \Sigma_5 &= [\bar{e}_1 \ \bar{e}_5] Q_1 [\bar{e}_1 \ \bar{e}_5]^T - e^{-2kh_1} [\bar{e}_5 \ \bar{e}_3] Q_1 [\bar{e}_5 \ \bar{e}_3]^T \\ &+ [\bar{e}_3 \ \bar{e}_6] Q_1 [\bar{e}_3 \ \bar{e}_6]^T - e^{-2kh_1} [\bar{e}_6 \ \bar{e}_4] Q_1 [\bar{e}_6 \ \bar{e}_4]^T, \\ \Sigma_6 &= h_1^2 \mathbb{A}^T Z_1 \mathbb{A} - e^{-2kh_1} [\bar{e}_1 - \bar{e}_5] Z_1 [\bar{e}_1 - \bar{e}_5]^T \\ &+ (h_M - h_2)^2 \mathbb{A}^T Z_2 \mathbb{A} - e^{-2kh_M} [\bar{e}_6 - \bar{e}_4] Z_2 [\bar{e}_6 - \bar{e}_4]^T \\ &+ (h_2 - h_m)^2 \mathbb{A}^T Z_2 \mathbb{A} \\ &- e^{-2kh_2} [\bar{e}_3 - \bar{e}_2 \ \bar{e}_2 - \bar{e}_6] \begin{bmatrix} Z_2 & T_1 \\ T_1^T & Z_2 \end{bmatrix} [\bar{e}_3 - \bar{e}_2 \ \bar{e}_2 - \bar{e}_6]^T, \end{aligned}$$

$$\begin{aligned} \Sigma_7 &= h_1^2 \mathbb{A}^T Z_1 \mathbb{A} - e^{-2kh_1} [\bar{e}_1 - \bar{e}_5] Z_1 [\bar{e}_1 - \bar{e}_5]^T \\ &+ (h_2 - h_m)^2 \mathbb{A}^T Z_2 \mathbb{A} - e^{-2kh_2} [\bar{e}_3 - \bar{e}_6] Z_2 [\bar{e}_3 - \bar{e}_6]^T \\ &+ (h_M - h_2)^2 \mathbb{A}^T Z_2 \mathbb{A} \\ &- e^{-2kh_M} [\bar{e}_6 - \bar{e}_2 \ \bar{e}_2 - \bar{e}_4] \begin{bmatrix} Z_2 & T_1 \\ T_1^T & Z_2 \end{bmatrix} [\bar{e}_6 - \bar{e}_2 \ \bar{e}_2 - \bar{e}_4]^T, \\ \bar{Y}_1 &= \bar{\Sigma}_1 + \bar{\Sigma}_2 + \bar{\Sigma}_3 + \bar{\Sigma}_4 + \bar{\Sigma}_5 + \bar{\Sigma}_6 + \Phi, \\ \bar{Y}_2 &= \bar{\Sigma}_1 + \bar{\Sigma}_2 + \bar{\Sigma}_3 + \bar{\Sigma}_4 + \bar{\Sigma}_5 + \bar{\Sigma}_7 + \Phi. \end{aligned}$$

Theorem 3.2 For given scalars $h_m, h_M (h_M > h_m), \mu, k \geq 0$, system (6) is globally exponentially stable if there exist symmetric positive definite matrices $Q_1 \in \mathbb{R}^{2n \times 2n}, Q_2 \in \mathbb{R}^{2n \times 2n}, n \times n$ dimensional positive symmetric matrices P, R, U, S, Z_1, Z_2 , and appropriate dimension matrices T, T_1, N satisfying the following LMIs:

$$\bar{Y}_1 < 0, \bar{Y}_2 < 0, \tag{30}$$

$$\begin{bmatrix} U & T \\ T^T & U \end{bmatrix} \geq 0, \begin{bmatrix} Z_2 & T_1 \\ T_1^T & Z_2 \end{bmatrix} \geq 0, \begin{bmatrix} S & N \\ N^T & S \end{bmatrix} \geq 0. \tag{31}$$

Proof. By considering the same Lyapunov-Krasovskii functional in Theorem 3.1, the upper bounds of $\dot{V}(t)$ are obtained as

$$\begin{cases} \dot{V}(t) \leq e^{2kt} \left\{ \bar{\xi}^T(t) \left(\sum_{i=1}^5 \bar{\Sigma}_i + \bar{\Sigma}_6 \right) \bar{\xi}(t) \right\}, \text{ when } h_m \leq h(t) \leq h_2, \\ \dot{V}(t) \leq e^{2kt} \left\{ \bar{\xi}^T(t) \left(\sum_{i=1}^5 \bar{\Sigma}_i + \bar{\Sigma}_7 \right) \bar{\xi}(t) \right\}, \text{ when } h_2 \leq h(t) \leq h_m. \end{cases}$$

On the other hand, the following inequality holds From (4)-(6)

$$e^{2kt} p^T(t) p(t) \leq q^T(t) q(t). \tag{32}$$

By S-procedure [1], we can derive inequalities (32) with a positive scalar ε such that

$$\begin{aligned} &e^{2kt} \left\{ \varepsilon \left[p^T(t) p(t) \leq q^T(t) q(t) \right] \right\} \\ &= e^{2kt} \bar{\xi}^T(t) \Phi \bar{\xi}(t) \geq 0. \end{aligned} \tag{33}$$

Therefore, the uncertain system (6) is exponentially stable, if LMIs (30-31) hold. This completes our proof.

The following result is obtained from Theorem 3.2 when μ is unknown or $h(t)$ is not differentiable.

Corollary 3.2 For given scalars $h_m, h_M (h_M > h_m), k \geq 0$, system (7) is globally exponentially stable if there

exist symmetric positive definite matrices $\mathbb{Q}_1 \in \mathbb{R}^{2n \times 2n}$, $\mathbb{Q}_2 \in \mathbb{R}^{2n \times 2n}$, $n \times n$ dimensional positive symmetric matrices P, U, S, Z_1, Z_2 , and appropriate dimension matrices T, T_1, N satisfying the following LMIs:

$$\hat{Y}_1 < 0, \hat{Y}_2 < 0, \tag{34}$$

$$\begin{bmatrix} U & T \\ T^T & U \end{bmatrix} \geq 0, \begin{bmatrix} Z_2 & T_1 \\ T_1^T & Z_2 \end{bmatrix} \geq 0, \begin{bmatrix} S & N \\ N^T & S \end{bmatrix} \geq 0, \tag{35}$$

where

$$\begin{aligned} \hat{Y}_1 &= \bar{\Sigma}_1 + \bar{\Sigma}_3 + \bar{\Sigma}_4 + \bar{\Sigma}_5 + \bar{\Sigma}_6 + \Phi, \\ \hat{Y}_2 &= \bar{\Sigma}_1 + \bar{\Sigma}_3 + \bar{\Sigma}_4 + \bar{\Sigma}_5 + \bar{\Sigma}_7 + \Phi, \end{aligned}$$

Remark 3.3 For system (7) with the routine delay case described by $0 \leq h(t) \leq h_M$ ($h'(t) \leq \mu$), the corresponding Lyapunov-Krasovskii reduces to

$$\begin{aligned} V &= e^{2kt} x^T(t) P x(t) + \int_{t-h(t)}^t e^{2ks} x^T(s) R x(s) ds \\ &+ h_M \int_{-h_M}^0 \int_{t+\alpha}^t e^{2ks} \dot{x}^T(s) U \dot{x}(s) ds d\alpha \\ &+ \int_{-h_M}^0 \int_{t+\alpha}^t e^{2ks} (x^T(s) S x(s) + \dot{x}^T(s) S \dot{x}(s)) ds d\alpha \\ &+ e^{2ks} \begin{bmatrix} x(s) \\ x(s-\delta) \end{bmatrix}^T \mathbb{Q}_2 \begin{bmatrix} x(s) \\ x(s-\delta) \end{bmatrix} ds \\ &+ (h_M - h_2) \int_{-h_M}^{-h_2} \int_{t+\alpha}^t e^{2ks} \dot{x}^T(s) Z_2 \dot{x}(s) ds d\alpha \\ &+ h_2 \int_{-h_2}^0 \int_{t+\alpha}^t e^{2ks} \dot{x}^T(s) Z_2 \dot{x}(s) ds d\alpha. \end{aligned}$$

Similar to the proof of Theorem 3.1(Corollary 3.1) and Theorem 3.2(Corollary 3.2), one can easily derive less conservative results than some existing ones.

Remark 3.4 In most of the applications of neural networks there is a shared requirement of raising the networks convergence speed in order to cut down the time of neural computing. Since the exponential convergence rate could be used to determine the speed of neural computation [20, 21]. On the other hand, from the Dynamic simulation systems [22] and Real-time computing [23], it can be confirmed that, fast convergence of a system is essential for real-time computation, and the exponential convergence rate is generally used to determine the speed of computations.

Remark 3.5 By constructing a new augmented Lyapunov-Krasovskii functional and using lower bound lemma, exponential stability criteria have been obtained which are expected to be less conservative than the results discussed in the recent literature [5, 8, 11]. The effectiveness

of the proposed methods has been shown elaborately through the following numerical examples.

4. Numerical examples

Example 4.1 Consider the system given in (7) with following parameters

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

Case 1: When $h_m = 0.5, h_M = 1$. Table 1 gives the allowable of the maximum exponential convergence rate k for different μ . For this case, the exponential stability criteria in [8, 11] are not applicable because the criteria are only for asymptotic stability.

Case 2: For various μ and unknown μ , the allowable bound h_M , which guarantee the asymptotic stability of system for given lower bounds h_m are provided in Table 2. It is easy to see that our method gives improved results than the existing ones.

Example 4.2 Consider the system given in (6) with following parameters

$$\begin{aligned} A &= \begin{bmatrix} -0.5 & -2 \\ 1 & -1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.5 & -1 \\ 0 & 0.6 \end{bmatrix}, \quad D = I, \\ E &= E_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}. \end{aligned}$$

For $0 < h_m \leq h(t) \leq h_M$, Table 3 presents the allowance

Table 1. The maximum exponential convergence rate k for various μ

μ	0	0.1	0.3	0.9	Unknown
Theorem 3.1	0.5190	0.4998	0.4498	0.3331	0.3331

Table 2. The maximum bound h_M with given h_m for different μ

h_m	Methods	$\mu = 0.1$	$\mu = 0.3$	$\mu = 0.5$	$\mu = 0.9$	unknown
1	[8]	4.1935	3.0538	2.3058	2.3058	2.3058
	[11]	4.4045	3.1208	3.1208	2.3513	2.3513
	Ours	4.4052	3.1655	3.1655	2.4594	2.4594
2	[8]	4.4932	3.0129	2.5663	2.5663	2.5663
	[11]	4.5729	3.1092	2.6987	2.6987	2.6987
	Ours	4.6160	3.1773	2.7509	2.7509	2.7509
3	[8]	4.3979	3.3408	3.3408	3.3408	3.3408
	[11]	4.5406	3.4186	3.4186	3.4186	3.4186
	Ours	4.6508	3.4626	3.4626	3.4626	3.4626
4	[8]	4.1978	4.1690	4.1690	4.1690	4.1690
	[11]	4.2367	4.2097	4.2097	4.2097	4.2097
	Ours	4.5255	4.2576	4.2576	4.2576	4.2576
5	[8]	5.0275	5.0275	5.0275	5.0275	5.0275
	[11]	5.0440	5.0440	5.0440	5.0440	5.0440
	Ours	5.0976	5.0976	5.0976	5.0976	5.0976

Table 3. The maximum bound h_M with given h_m for unknown μ

h_m	0.1	0.2	0.3	0.4	0.5	0.6	0.9
[5]	0.383	0.452	0.519	0.584	0.648	0.712	0.904
Ours	0.483	0.531	0.581	0.634	0.688	0.742	0.905

Table 4. $h_m = 0.5, \mu = 0.5$, The maximum bound h_M for various k

k	0	0.05	0.1	0.3	0.4
Ours	0.9272	0.8568	0.7997	0.6510	0.5768

of the upper bound when μ is unknown and the convergence $k = 0$. It is obvious that our results are better than [5].

Example 4.3 Consider the system given in (6) with following parameters

$$A = \begin{bmatrix} 0.5 & 1 \\ -1 & -1 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & 0.3 \\ 0.2 & -0.5 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.04 & -0.001 \\ 0.002 & -0.05 \end{bmatrix},$$

$$E = \begin{bmatrix} -0.07 & 0.004 \\ 0.005 & 0.075 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} -0.0045 & 0.002 \\ 0.001 & 0.04 \end{bmatrix}.$$

Given $h_m = 0.5, \mu = 0.5$, Table 4 gives the maximum allowable value of h_M for different convergence rate k . For this cases, it is noted that the criteria in [5, 8, 11] are not applicable for exponential stability. Therefore, our work is more general cases than in the existing results

5. Conclusion

In this paper, the problem of delay-dependent exponential stability of time-delay systems has been investigated. We have considered the time-varying delay in a range for which the lower bound is not restricted to be zero. By introducing a different Lyapunov functional, new delay-dependent criteria have been derived in terms of LMIs. It is shown via numerical examples that our proposed criteria are less conservative than existing ones.

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Yajuan Liu received her B.S degree in mathematics and applied mathematics from shanxi nominal university, Linfen, China, in 2010, and M.S degree in applied mathematics from University of Science and Technology Beijing, Beijing, China, in 2012. She is currently working toward the PH.D degree

in Electronic Engineering from Daegu University, Korea. Her current research interests include time delay systems, robust control and filtering, model predictive control.



Sang-Moon Lee received the B.S. degree in Electronic Engineering from Gyungbuk National University, and M. S and PH. D degrees at Department of Electronic Engineering from POSTECH, Korea Currently, he is an Assistant Professor at Division of Electronic Engineering in Daegu University.

His main research interests include robust control theory, nonlinear systems, model predictive control and its industrial applications.



Oh-Min Kwon received B.S degree in Electronic Engineering from Gyungbuk National University, Daegu, Korea in 1997, and PH.D degree in Electrical and Engineering from Pohang University of Science and Technology, Pohang, Korea, in 2004. From February 2004 to January 2006, he was a senior Researcher in Mechatronics Center of Samsung Heavy Industries. He is currently working as an Associate Professor in School of Electrical Engineering, Chungbuk National University. His research interests include time-delay systems, cellular neural networks, robust control and filtering, large-scale systems, secure communication through synchronization between two chaotic systems, complex dynamical networks, multiagent systems, and so on. He has presented a number of papers in these areas. He is a member of KIEE, ICROS, and IEEK. He had joined an editorial member of KIEE from 2011 to 2012 and also served as an editorial member in *Nonlinear Analysis: Hybrid Systems* from 2011 and *Scientific World Journal* from 2013.



Ju H. Park received the B.S and M.S degrees in Electronics Engineering from Kyungpook National University, Daegu, Republic of Korea, in 1990 and 1992 and the Ph.D. degree in Electronics and Electrical Engineering from POSTECH, Pohang, Republic of Korea, in 1997. From May 1997 to February

2000, he was a Research Associate in ERC-ARC, POSTECH. In March 2000, he joined Yeungnam University, Kyongsan, Republic of Korea, where he is currently a Full Professor. From December 2006 to December 2007, he was a Visiting Professor in the Department of Mechanical Engineering, Georgia Institute of Technology. Prof Park’s research interests include robust control and filtering, neural networks, complex networks, and chaotic systems. He has published a number of papers in these areas. Prof. Park severs as Editor of *International Journal of Control, Automation and Systems*. He is also an Associate Editor/Editorial Board member for several international journals, including *Applied Mathematics and Computation*, *Journal of The Franklin Institute*, *Journal of Applied Mathematics and Computing*, etc.