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A Target Tracking Based on Bearing and Range Measurement With Unknown Noise Statistics

Jaechan Lim†

Abstract – In this paper, we propose and assess the performance of "H infinity filter (H_∞ , HIF)" and "cost reference particle filter (CRPF)" in the problem of tracking a target based on the measurements of the range and the bearing of the target. HIF and CRPF have the common advantageous feature that we do not need to know the noise statistics of the problem in their applications. The performance of the extended Kalman filter (EKF) is also compared with that of the proposed filters, but the noise information is perfectly known for the applications of the EKF. Simulation results show that CRPF outperforms HIF, and is more robust because the tracking of HIF diverges sometimes, particularly when the target track is highly nonlinear. Interestingly, when the tracking of HIF diverges, the tracking of the EKF also tends to deviate significantly from the true track for the same target track. Therefore, CRPF is very effective and appropriate approach to the problems of highly nonlinear model, especially when the noise statistics are unknown. Nonetheless, HIF also can be applied to the problem of timevarying state estimation as the EKF, particularly for the case when the noise statistcs are unknown. This paper provides a good example of how to apply CRPF and HIF to the estimation of dynamically varying and nonlinearly modeled states with unknown noise statistics.

Keywords: H_∞ (H infinity) filter, Cost reference particle filter, Extended Kalman filter (EKF), Target tracking

1. Introduction

For many problems in engineering or statistical science, we can model and describe them by the dynamic state system (DSS) where the states of interest are correlated in time or space. In most cases, the dynamically varying states are correlated with time, while the other case can be found usually in the estimation problems of consecutive image pixels. Based on the DSS model, there are a number of approaches for estimating the states sequentially in time or space. The Kalman filter is an outstanding approach, and is the optimal method when the model is linear and the noise of the problem is the Gaussian. Its extended version employing Taylor series, i.e. the extended Kalman filter is also successfully applied to non-linear problems [1-4]. However, our concern in this paper is to solve the problem with *unknown* noise statistics regardless of weather the noise is Gaussian or not, particularly for a non-linear model. There are two approaches related with this concern, i.e. "H infinity filter (H_∞ , HIF) [5]" and "cost reference particle filter (CRPF) [6, 7]." CRPF has been newly developed in particle filtering framework [8], and easily adopted to non-linear problems like standard particle filter (SPF) because we do not have to compute the Jacobian, as opposed to the cases of the EKF and HIF. The computational complexity of HIF is very similar to that of the EKF whereas CRPF is much more computationally complex.

In this paper, we track a single target's location and velocity in two dimensional space where a target is moving with random acceleration. We apply the EKF, HIF, and CRPF based on observations, i.e. the range and the bearing of the target measured at the origin of the coordinate system. Fig. 1 describes the range and the bearing of the target measured at the origin of the coordinate. When we apply the EKF, the noise information of the problem is perfectly known. We consider two scenarios when we apply noises for the simulations. In scenario I, we apply

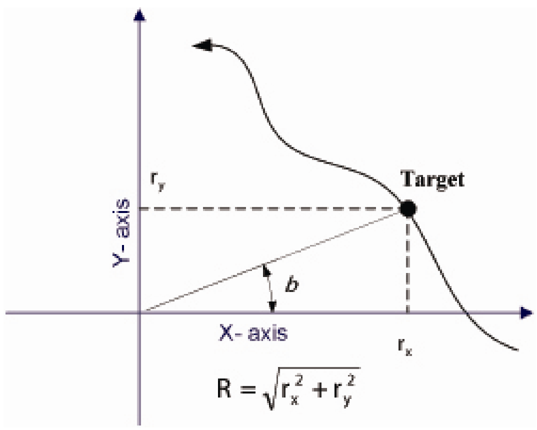

Fig. 1. The range (R) and the bearing (β) measurement of a target in motion

† Corresponding Author: Dept. of Creative IT Engineering and Future IT Innovative Laboratory, Pohang University of Science and Technology, Korea. (jclim@postech.ac.kr)
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only a single Gaussian noise, and apply a mixture Gaussian noise in scenario II. The tracking of HIF tends to diverge when a target track is highly non-linear (i.e. the acceleration is large), and its performance becomes worse in the scenario II. Interestingly, the EKF also shows a bad performance or its tracking deviates significantly from the true target track when HIF's tracking diverges even though the noise information is given for the EKF. The tracking of CRPF never diverges nor deviates significantly from the true track in any scenarios. When we track a same identical track 300 times repeatedly without any diverging tracking, the EKF shows the best result to which CRPF is the second, and HIF shows the worst performance. Overall, CRPF is more robust than the other two methods, and outperforms HIF in any circumstances. Therefore, as the SPF usually outperforms the EKF in significantly non-linear problems, CRPF outperforms HIF for the problems with unknown noise statistics. In other words, SPF is to the EKF what CRPF is to HIF with the difference of whether the noise statistics are known or not.

In summary, CRPF was initially developed in [6], and is still not well known to many researchers even in related areas. Therefore, we propose this approach to the highly nonlinear problem that is investigated in this paper, and show outperforming result over the EKF even with a disadvantage of unknown noise statistics. This superior result of CRPF may not occur in all nonlinear problems; therefore, CRPF is robust particularly in highly nonlinear problems. We also compare the performance of CRPF with that of HIF which has the same feature of non-necessity of the knowledge of noise statistics in its applications, and show superior performance of CRPF. In the future, beyond the result of this paper, we will be able to obtain outperforming results of CRPF in many nonlinear problems where HIF is applied as the state of the art approach

For readability facilitation of the paper, the list of abbreviations used in this paper is as follows:

2. System Model

We want to track a single target which is moving in a two dimensional space with random acceleration. The direction of the target is subject to the acceleration which is determined by the process noise in the DSS equation. The observed measurements are the range and the bearing of the target measured at the origin of the coordinate system. The DSS equation is expressed as follows.

$$\underbrace{\begin{bmatrix} r_{x,k} \\ r_{y,k} \\ v_{x,k} \\ v_{y,k} \end{bmatrix}}_{s_k} = \underbrace{\begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} r_{x,k-1} \\ r_{y,k-1} \\ v_{x,k-1} \\ v_{y,k-1} \end{bmatrix}}_{s_{k-1}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ u_{x,k} \\ u_{y,k} \end{bmatrix}}_{u_k} \quad (1)$$

where r, v, u, and (x, y) denote the location, the velocity, the acceleration, and coordinates, respectively. T is the sampling period, and k is the time index. Therefore, the dynamically time-varying state is composed of four elements, i.e. 2-D location and 2-D acceleration coordinates. The state noise of the location coordinate is zero whereas that of the acceleration is subjected to a random process of u_k. The range and the bearing compose the measurement equation, which is highly non-linear, and described as follows:

$$z_k = h[s_k] + w_k = [R_k \ \beta_k]^T + w_k \quad (2)$$

where the range

$$R_k = \sqrt{r_{x,k}^2 + r_{y,k}^2}, \quad (3)$$

the bearing

$$\beta_k = \arctan\frac{r_{y,k}}{r_{x,k}} \quad (4)$$

and the measurement noise is, $w_k = [w_{R,k} \ w_{\beta,k}]^T$. Therefore, the state s_k is sequentially estimated based on the observed information z_k by applying the approaches.

3. Filtering Methods

We investigate three approaches for tracking a target in this non-linear problem. When we apply the EKF and extended HIF, we have to compute the Jacobian of the measurement equation to approximate it into linear form whereas we do not need to compute in CRPF.

3.1 Extended kalman filtering

In many statistical estimation problems of scientific engineering, parameters of interest to be estimated are dynamically varying in time or space with some statistical features. In this case, the signal is not stationary anymore and the parameter needs to be estimated sequentially. If it is assumed that the parameter is varying with some statistical features, the state of varying parameters is modeled by DSS. Autoregressive or/and moving average models also can be included in this dynamic model, but DSS model is not limited to these stationary processes. Wiener filter might be optimal for this stationary scalar parameter estimation, but, in practice we encounter non-stationary scenario cases. Although classical maximum likelihood approach is asymptotically optimal estimator for the static parameter estimation, it is not pertinent for dynamically varying parameters, particularly in non-stationary mode. The Kalman filter is optimal sequential minimum mean square estimator for the estimation of non-stationary vector signals if the signals are linear jointly Gaussian. Unfortunately, the most estimation problems we encounter in practice are not simply linear; therefore, usually the Kalman filter is extended by using Taylor expansion for the second order linear approximation of any functions. Furthermore, the computation of Jacobian that causes relatively high computational complexity will be required if the estimated state is a vector rather than a scalar as in the case of the problem in this paper.

3.2 Extended H_∞ filtering

HIF has an advantageous feature: it does not require the noise statistics in its application. The filtering scheme of HIF is similar to that of the EKF. However, whereas the mean square error is minimized in the EKF (EKF is a kind of minimum mean square error estimator), the worst case error (or maximized error) is minimized in the EHF. This is why the Kalman filter may also be called H2 filter. More specifically, the norm or the cost function is defined in the H_∞ filtering, and the maximum norm, which is specifically called the H_∞ norm, is minimized. Based on the system Eqs. (1) and (2), H_∞ filter estimates s_k with uniformly small errors, given arbitrary w_k, u_k, and s_0. This idea is very similar to the case of the zero-sum game where maximum benefit-loss is minimized. Therefore, the cost function related to the zero-sum game is defined as follows [5] based on the system Eqs. (1) and (2):

$$J = \frac{\sum_{k=0}^{N-1} \left\| s_k - \hat{s}_k \right\|^2}{\left\| s_0 - \hat{s}_0 \right\|^2_{P_0^{-1}} + \sum_{k=0}^{N-1} \left(\left\| u_k \right\|^2_{W_k^{-1}} + \left\| w_k \right\|^2_{V_k^{-1}} \right)} \tag{5}$$

where P_k, W_k and V_k are the weight parameters that are positive definite matrices; N is the number of total time steps. The vector norm is denoted by $\| \cdot \|$, and $\| u_k \|^2_{W_k^{-1}}$ is defined $u_k^T W^{-1} u_k$. If it is known that the second element of $w(k)$ is small, then $V_k(2,2)$ is chosen to be small compared to other elements.

Direct minimization of J is not tractable; therefore, the performance bound is introduced, and it satisfies

$$J < \gamma^{-1} \tag{6}$$

Then, J′ is defined as

$$J' = -\gamma^{-1} \left\| s_0 - \hat{s}_0 \right\|^2_{P_0^{-1}} + \sum_{k=0}^{N-1} \left[\left\| s_k - \hat{s}_k \right\|^2 - \gamma^{-1} \left(\left\| u_k \right\|^2_{W_k^{-1}} + \left\| w_k \right\|^2_{V_k^{-1}} \right) \right] \tag{7}$$

and, the problem becomes a matter of solving the following minimax problem:

$$\min_{\hat{s}_k} \left(\max_{w_k, u_k, s_0} J' \right) \tag{8}$$

The H_∞ filter reduces to the KF when: $\gamma = \infty$, and the true covariance matrices of the parameters are selected for P_0, W_k and V_k; therefore, the Kalman filter does not guarantee any bound for the cost function of (6) from the H_∞ filter point of view. We describe the essential steps of the extended H_∞ filtering for the problem here, and more details and genera descriptions about HIF can be found in [5, 9, 10]. The following is the summarized steps when HIF is applied to track a target:

1) Initialize the noise attenuation level (γ), the initial estimate ($\hat{s}_0$), and the weight parameters.
2) For n = 1: N−1, recursively take the following steps.
   - Compute $S_k = \left\{ I - \gamma P_k + \left[h'(\hat{s}_{k-1}) \right]^T V_k^{-1} h'(\hat{s}_{k-1}) P_k \right\}^{-1}$, where I is the identity matrix, and $h'(\hat{s}_{k-1}) = \left. \frac{\partial h(s_k)}{\partial s_k} \right|_{s_k = \hat{s}_{k-1}}$
   - Compute $P_{k+1} = A_k P_k S_k A_k^T + W_k$ where $A_k = A$, time invariant in the problem.
   - Compute $H_k = A P_k S_k \left[h'(\hat{s}_{k-1}) \right]^T V_k^{-1}$.
   - Update the estimate, $\hat{s}_k = A\hat{s}_{k-1} + H_k \left[z_k - h(\hat{s}_{k-1}) \right]$

In the steps, we can compute the Jacobian of the measurement equation as

$$\frac{\partial h(s_k)}{\partial s_k} = \begin{bmatrix} \frac{r_{x.k}}{R_k} & \frac{r_{y.k}}{R_k} & 0 & 0 \\ \frac{-r_{y,k}}{R_k^2} & \frac{-r_{x,k}}{R_k^2} & 0 & 0 \end{bmatrix} \tag{9}$$

and the same Jacobian is used when we apply the EKF to the problem. When the noise attenuation level γ is selected, we have to be very careful, especially it has to satisfy the condition that $\gamma < P_k^{-1} + \left[\left. \frac{\partial h(s_k)}{\partial s_k} \right|_{s_k = \hat{s}_{k-1}} \right]^T V_k^{-1} \left[\left. \frac{\partial h(s_k)}{\partial s_k} \right|_{s_k = \hat{s}_{k-1}} \right]$ to maintain $P_k > 0$ [9].

3.3 Cost reference particle filter

The Cost reference particle filter (CRPF) also has the same feature as HIF that it does not require the prior information about the noise distributions of the “state” and the “measurement” equations [11]. The dynamic state system that describes the hidden state s and observed measurement z with zero mean and additive noise processes of u and w at time k is expressed as follows.

$$s_k = g(s_{k-1}) + u_k, \tag{10}$$

$$z_k = h(s_k) + w_k \tag{11}$$

where g(·) and h(·) are the given state transition and the observation function, respectively. In CRPF algorithm, we need to define a couple of important functions, i.e. the cost function and the risk function. These functions are adopted as the measure of the quality of particles in the algorithm. The cost function needs to satisfy strictly convex with respect to s_k to avoid the ambiguities in estimates and in the resampling step. The risk function needs to be simple and highly tractable in computation for practical implementation of the algorithm. The cost function in CRPF, which is of the recursive additive structure and corresponds to the "weight" in the SPF is defined as

$$C(s_{0:k} \mid z_{1:k}, \lambda) = \lambda C(s_{0:k-1} \mid z_{1:k-1}, \lambda) + \Delta C(s_k \mid z_k) \tag{12}$$

where λ is the forgetting factor $(0 \le \lambda \le 1)$ which makes it possible to adaptively change the amount of contributions of past particles in evaluating the cost function, and ΔC is the "incremental cost function" which gives the accuracy of the estimate of s_k given z_k. The cost function is a measure of "estimate quality" like the weight as in PF. Similarly to PF, the cost-based random measure is represented by a set of *particles* and associated *costs* as,

$$\Xi_k = \left\{ s_{0:k}^{(i)}, C_k^{(i)} \right\}_{i=1}^{M}, \tag{13}$$

Where

$$C_k^{(i)} = C(\mathrm{s}_{0:\mathrm{k}}^{(i)} \mid z_{1:k}, \lambda), \tag{14}$$

i is the particle index, and M is the number of particles. The "risk function" is defined in CRPF as

$$R(s_{k-1} \mid z_k) = \Delta C(E[\mathrm{s}_k] \mid z_k) = \Delta C(g(s_{k-1}) \mid z_k). \tag{15}$$

where $E[s_k] = g(\mathrm{s}_{k-1})$. This can be computed by $\left\| z_k - h[\mathrm{g}(\mathrm{s}_{k-1})] \right\|^q$ where q ≥ 1. The risk function measures the adequacy of the estimate, s_{k-1} given the observation z_k. Also, the risk function is a prediction of the cost increment, $\Delta C(s_k, z_k)$ (can be computed by $\left\| z_k - h(s_k) \right\|^q$). Based on these definitions, the sequential algorithm proceeds recursively repeating the steps of "risk evaluation", "resampling", "particle propagation", and updating the cost with time. The steps are summarized in Table 1. CRPF can be easily adopted for the target tracking problem based on (1) and (2). The function g(·) is linear, and is equal to A in the problem.

Table 1. Cost reference particle filter algorithm

Initialization for *i=1, ..., M,* generate $s_0^{(i)} \sim p_0(s_0)$, and assign the cost $C_0^{(i)} = 0$, and initialize $\sigma_0^{2,(i)}$.

Recursive update for *k=1, ..., K*

(1) Compute, (for *i= 1, ..., K*)

$R_k^{(i)} = \lambda C_{k-1}^{(i)} + \left\| z_k - h[\mathrm{g}(\mathrm{s}_{k-1}^{(i)})] \right\|^q$ for $q \ge 1$, and PMF,

$$\hat{\pi}_k^{(i)} \propto \mu(R_k^{(i)}) = \frac{1}{(R_k^{(i)} - \min\left\{R_k^{(i)}\right\}_{i=1}^{M} + \delta)^{\beta}}$$

(2) Selection, or resampling $\hat{\Xi}_{k-1} = \left\{ \hat{s}_{k-1}^{(i)}, \hat{C}_{k-1}^{(i)} \right\}_{i=1}^{M}$ according to $\hat{\pi}_k^{(i)}$ where "^" denotes resampled version of the particle set.

(3) Particle propagation (for *i=1, ..., M*)

$s_k^{(i)} \sim p_k(s_k \mid \hat{s}_{k-1}^{(i)}) = N\left(\mathrm{g}(\mathrm{s}_{k-1}^{(i)}), \sigma_{k-1}^{2,(i)} I_{[s]} \right)$

where $N(a, \mathrm{b}^2)$ denotes a Gaussian distribution with the mean of a and the variance of b^2,

$$\sigma_k^{2,(i)} = \frac{k}{k-1}\sigma_{k-1}^{2,(i)} + \frac{\left\| s_k^{(i)} - g(\hat{s}_{k-1}^{(i)}) \right\|^2}{k \times dim[x]}.$$

(4) Compute the cost (for *i=1, ..., M*)

$C_k^{(i)} = \lambda C_{k-1}^{(i)} + \left\| z_k - h(\mathrm{s}_k^{(i)}) \right\|^q$ and normalized PMF,

$$\pi_k^i \propto \mu_2\left(C_k^{(i)}\right) = \frac{1}{\left(C_k^{(i)} - \min\left\{C_k^{(i)}\right\}_{i=1}^{M} + \delta\right)^{\beta}}$$

where $\alpha, \beta > 0$.

(5) Estimation $s_k = s_k^{mean} = \sum_{i=1}^{M} \pi_k^{(i)} s_k^{(i)}$.

3.4 Discussion

While the information of the noise statistics of the state and the measurement is not required for HIF and CRPF, we need to tune a number of parameters. Particularly, the performance bound γ and the initial variance of the propagation density $\sigma_0^{2,(i)}$ are crucial factors for the performance of HIF [9] and CRPF, respectively. Whereas $\sigma_k^{2,(i)}$ is adjusted online unless an unduly large or small $\sigma_0^{2,(i)}$ is selected, a static value of γ needs to be carefully selected for enhanced performance of HIF. Before we assess the performance of the proposed approaches, we perform extensive preliminary simulations of tuning process for parameters setting. On the other hand, tuning process is not required for the EKF that is applied with exactly known noise statistics.

4. Simulations

We apply the proposed methods and the EKF for tracking a target in a two dimensional space based on the model of (1) and (2). Sampling period, $T=1$, and the initial true state of the target is $s_0=[10\ -5\ -0.2\ 0.2]^T$. The given initial estimate of the state for all methods is $\hat{s}_0=[5\ 5\ 0\ 0]^T$. Diverse variance scenarios of uncorrelated noises for the acceleration (in both directions), the range, and the bearing are investigated when a single Gaussian is applied. When a mixture Gaussian is applied, $u_{x,y}\sim 0.1\cdot N(0,1)+4\times10^{-4}\cdot$

$N(0,\ 1)\ +2.5\times10^{-5}N(0,1)$, $w_R\sim\sqrt{0.1}\cdot N(0,1)+0.04\cdot N(0,1)+2.5\times10^{-5}\cdot N(0,1)$ and $w_\beta\sim 0.1\cdot N(0,1)+4\times 10^{-4}\cdot\ N(0,1)+1.6\times10^{-7}\cdot N(0,1)$ where $N(a,b^2)$ denotes the Gaussian distribution with the mean of a and the variance of b^2. The given initial covariance for the EKF is diag(1 1 1 1) where "diag" denotes the diagonal matrix with the diagonal elements in the parenthesis, and the noise statistics are perfectly known when we apply the EKF. The parameters for HIF are; $\gamma=0.01$, $\mathrm{V}=diag(1\ 1)$, and $\mathrm{P}_0=diag(0\,0\,1\,1)$, and $W=diag(0\,0\,1\,1)$. The value for the performance bound γ is selected after extensive tuning process. We run 10,000 simulations with various values of γ and diverse noise scenarios: $\gamma=0,10^{-100},10^{-3},10^{-2}$; variances of $w_\beta=0.01,0.1,1$; variances of $w_R=0.1,1,5$; variances of $u_{x,y}=0.001,\ 0.01,\ 0.1$. The results of tuning process are shown in Figs. 2-3. Overall, we obtain similar results such as Fig. 2 regardless of the scenarios except for the scenario that results in Fig. 3 where the noise variance scenario is $(0.1,0.1,0.1)$ for $(\mathrm{w}_\beta,w_R,u_{x,y})$. HIF shows significantly poor performance when 0.01 is employed for γ as shown in Fig. 3. Except for that scenario, HIF shows similar or better performance when 0.001 is selected for γ compared to that when the other values are employed. Therefore, we select 0.001 for γ for entire performance assessment.

We also perform preliminary tuning process for CRPF. The performance of CRPF highly relies on the initial variance of propagation density $\sigma_0^{2,(i)}=\left[\sigma_{0,(1)}^{2,(i)}\ \sigma_{0,(2)}^{2,(i)}\ \sigma_{0,(3)}^{2,(i)}\ \sigma_{0,(4)}^{2,(i)}\right]^T$. We run 500 simulations with various values of $\sigma_0^{2,(i)}$ and diverse noise scenarios: $\sigma_{0,(1,2)}^{2,(i)}=1,5,10$; $\sigma_{0,(3,4)}^{2,(i)}=0.1,1,5$; variances of $w_\beta=0.01,0.1$; variances of $w_R=0.1,1$; variances of $u_{x,y}=0.001,0.01$. The result for various scenarios are summarized in Table 2. In the table, we specifies values of $\sigma_{0,(1,2)}^{2,(i)}$ that result in competitive performance with respect to the variance set of $\left(w_\beta,w_R,\sigma_{0,(3,4)}^{2,(i)}\right)$ in addition to the optimal combination of the elements of $\sigma_0^{2,(i)}$. We can note that 5 can be used for

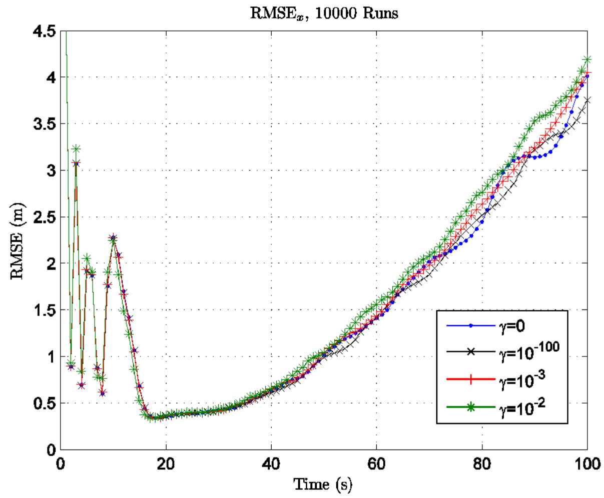


Fig. 2. Tuning γ. Performance of H infinity filter with various values of γ.

Table 2. Tuning initial variance of propagation density, $\sigma_0^{2,(i)}$. The variance of each scenario is in the order of $(w_\beta,w_R,u_{x,y},\sigma_{0,(3,4)}^{2,(i)})$.

				Optimal combination of $\sigma_0^{2,(i)}$
Noise scenario	1. (0.01 0.1 0.001 0.1)	2. (0.01 0.1 0.001 1)	3. (0.01 0.1 0.001 5)	$[1\ 1\ 0.1\ 0.1]^T$
$\sigma_{0,(1,2)}^{2,(i)}$	1, 5 & 10	1, 5 & 10	1, 5 & 10	
Noise scenario	4. (0.01 0.1 0.01 0.1)	5. (0.01 0.1 0.01 1)	5. (0.01 0.1 0.01 5)	$[(5\ or\ 10)\ (5\ or\ 10)\ 0.1\ 0.1]^T$
$\sigma_{0,(1,2)}^{2,(i)}$	5 & 10	5 & 10	5 & 10	
Noise scenario	7. (0.01 1 0.001 0.1)	8. (0.01 1 0.001 1)	9. (0.01 1 0.001 5)	$[(5\ or\ 10)\ (5\ or\ 10)\ 5\ 5]^T$
$\sigma_{0,(1,2)}^{2,(i)}$	1, 5 & 10	1, 5 & 10	1, 5 & 10	
Noise scenario	10. (0.01 1 0.01 0.1)	11. (0.01 1 0.01 1)	12. (0.01 1 0.01 5)	$[10\ 10\ 1\ 1]^T$
$\sigma_{0,(1,2)}^{2,(i)}$	5 & 10	5 & 10	5 & 10	
Noise scenario	13. (0.1 0.1 0.001 0.1)	14. (0.1 0.1 0.001 1)	15. (0.1 0.1 0.001 5)	$[1\ 1\ 1\ 1]^T$
$\sigma_{0,(1,2)}^{2,(i)}$	1, 5 & 10	1, 5 & 10	1, 5 & 10	
Noise scenario	16. (0.1 0.1 0.001 0.1)	17. (0.1 0.1 0.001 1)	18. (0.1 0.1 0.001 5)	$[(5\ or\ 10)\ (5\ or\ 10)\ 0.1\ 0.1]^T$
$\sigma_{0,(1,2)}^{2,(i)}$	5 & 10	5 & 10	5 & 10	
Noise scenario	19. (0.1 1 0.001 0.1)	20. (0.1 1 0.001 1)	21. (0.1 1 0.001 5)	$[1\ 1\ 0.1\ 0.1]^T$
$\sigma_{0,(1,2)}^{2,(i)}$	1, 5 & 10	1, 5 & 10	1, 5 & 10	
Noise scenario	22. (0.1 1 0.01 0.1)	23. (0.1 1 0.01 1)	24. (0.1 1 0.01 5)	$[10\ 10\ 1\ 1]^T$
$\sigma_{0,(1,2)}^{2,(i)}$	5 & 10	5 & 10	5 & 10	

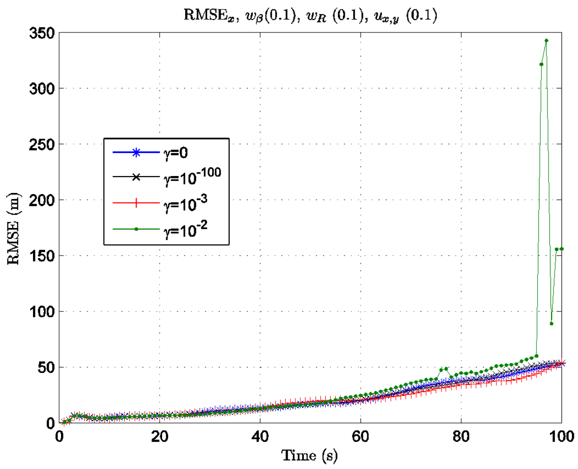
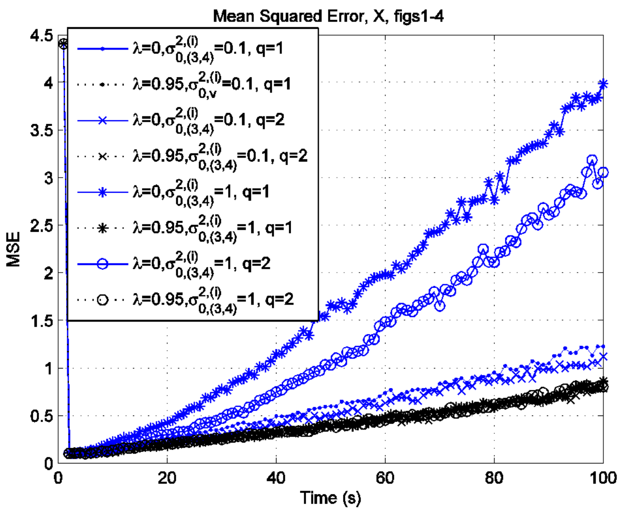
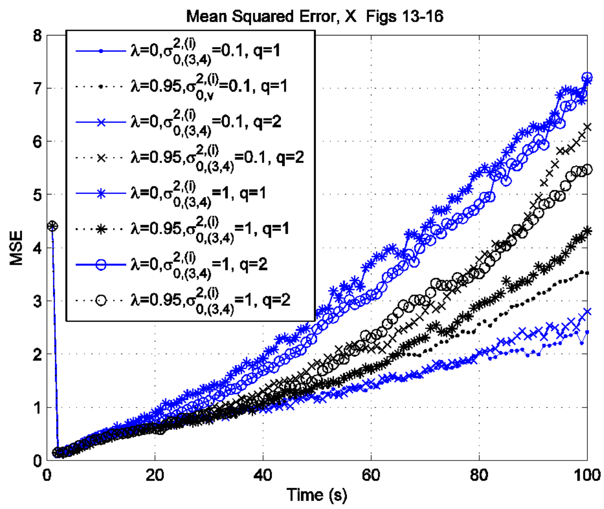


Fig. 3. Example of poor performance by H infinity filter with a bad selection of $\gamma = 0.01$.

$\sigma^{2,(i)}_{0,(1,2)}$ in all scenarios although it does not result in the optimal performance for all scenarios. Therefore, in the tuning process of q and the forgetting factor λ, we use $\sigma^{2,(i)}_{0,(1,2)} = 5$ and $\sigma^{2,(i)}_{0,(3,4)} = 0.1, 1$. We do not use 5 for $\sigma^{2,(i)}_{0,(3,4)}$ because we obtain the optimal performance when $\sigma^{2,(i)}_{0,(3,4)} = 0.1$ or 1 except for the scenarios of 7, 8, and 9 as shown in Table 2. We run 500 preliminary simulations for tuning q and λ with diverse scenarios: variances of $w_\beta = 0.01, 0.1$; variances of $w_R = 0.1, 1$; variances of $u_{x,y} = 0.001, 0.01$; $q = 1, 2$; $\sigma^{2,(i)}_{0,(1,2)} = 5$; $\sigma^{2,(i)}_{0,(3,4)} = 0.1, 1$; $\lambda = 0, 0.95$. The values of λ and q need to satisfy: $0 \le \lambda \le 1$ and $q \ge 1$. The value of zero is also considered for λ because the mean state estimate (s_k^{mean}) becomes asymptotically optimal in terms of its incremental cost when $\lambda = 0$ [6]. The selections of q and λ affect the risk and the cost functions that consequently affect the performance of CRPF. Fig. 4(a) shows the result when the



(a)



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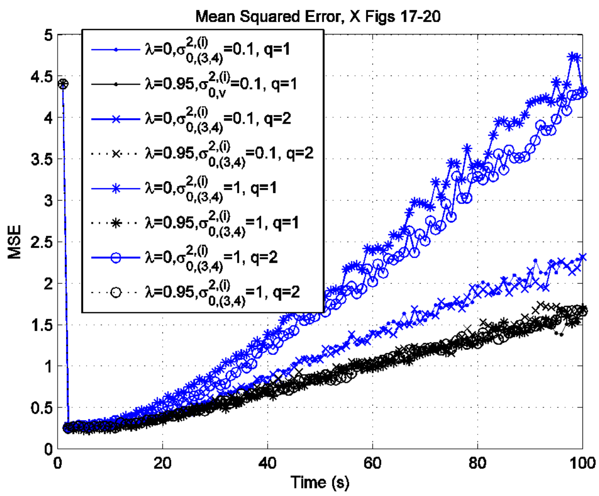
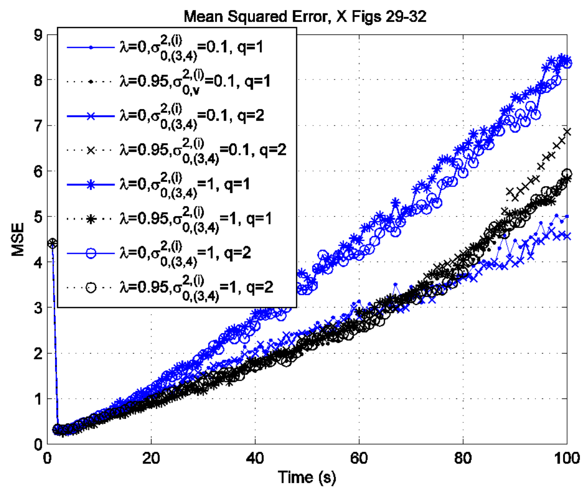


(c)



(d)

Fig. 4. Turing q and λ for CRPF: (a) Variances of $w_\beta = 0.01$, $w_R = 0.1$, $u_{x,y} = 0.001$; (b) Variances of $w_\beta = 0.01$, $w_R = 1$, $u_{x,y} = 0.01$; (c) Variances of $w_\beta = 0.1$, $w_R = 0.1$, $u_{x,y} = 0.001$; (d) Variances of $w_\beta = 0.01$, $w_R = 1$, $u_{x,y} = 0.01$.

Table 3. Tuning λ for CRPF. Noise variance scenarios when λ=0 needs to be selected rather than λ= 0.95 for better performance. 5 is used for $\sigma^{2,(i)}_{0,(1,2)}$.

w_β	w_R	$u_{x,y}$	
0.01	1	0.01	$\lambda = 0, q = 1$ *or* $2, \sigma^{2,(i)}_{0,(3,4)} = 0.1$
0.01	1	0.01	$\lambda = 0, q = 1, \sigma^{2,(i)}_{0,(3,4)} = 0.1$

noise variance scenario is 0.01, 0.1, and 0.001 for w_β, w_R and $u_{x,y}$, respectively. We have similarly better performance when 0.95 is employed than the case when 0 is employed for λ regardless of the values of q and $\sigma^{2,(i)}_{0,(3,4)}$. We obtained similar results to Fig. 4(a) when the noise scenarios are: variances of $(w_\beta, w_R, u_{x,y})$ are (0.01, 0.1, 0.01), (0.001, 1, 0.01). Fig. 4(b) shows the result when the noise variance scenario is (0.01, 1, 0.01). In this scenario, we need to select $\lambda = 0$, $q = 1$, $\sigma^{2,(i)}_{0,(3,4)} = 0.1$ for the best performance as shown in Fig. 4(b). Fig. 4(c) shows the result when the noise variance scenario is (0.1, 0.1, 0.001), and we obtained similar results to Fig. 4(c) for the scenarios of (0.1, 0.1, 0.01) and (0.1, 1, 0.001). Fig. 4(d) shows the result when the noise variance scenario is (0.1, 1, 0.01). In this scenario, we had better using $\lambda = 0$ for better performance. We summarized the scenarios when we need to use $\lambda = 0$ in Table 3. According to the result of preliminary tuning process, we need to employ $\lambda = 0.95$ in most cases; furthermore, the selection of 1 or 2 for q did not make considerable difference except for the specified case in Table 3. Therefore, the parameters for CRPF are selected based on the result of preliminary tuning process; besides, the number of particles is 500; $\delta = 0.1$; $\beta = 2$, respectively.

In Fig. 5, the root mean squared error (RMSE) of the methods are depicted with respect to time when we run 300 times for an identical track with a single Gaussian noise, and there was no diverging tracking for any methods during the simulations for this particular track. The applied noise variances are 0.01, 0.1, and 0.001 for w_β, w_R and $u_{x,y}$, respectively. The result shows that the EKF outperforms the other two methods, but CRPF has very similar performance to that of the EKF even though the noise statistics are known only for the EKF. Fig. 5 shows that CRPF converges faster than the other two methods. When we apply the mixture Gaussian noise for the identical track over 300 simulations, the result is very similar to Fig. 5 although we do not show the result here. To obtain more various simulation results with a single Gaussian noise, we perform simulations with various noise variance scenarios: applied variances of w_β, w_R, and $u_{x,y}$ are (0.01, 0.1), (0.1,1), (0.001, 0.01), respectively. In Figs. 6-7, the RMSEs are depicted when a single Gaussian was applied over 300 simulations where the true target tracks are randomly generated at each run, and some tracking of highly nonlinear tracks are diverging. In these cases, CRPF outperforms even the EKF which takes advantage of the

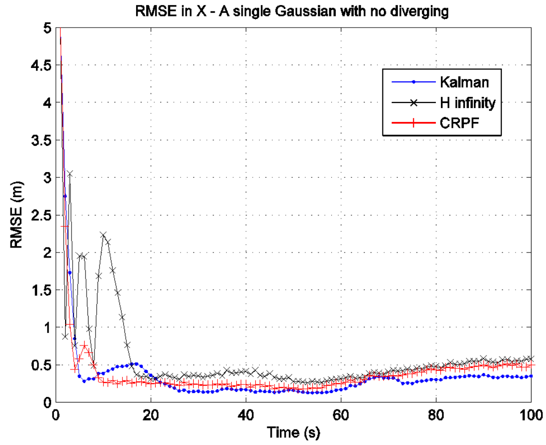


Fig. 5. RMSE in X coordinate by 300 runs for the same track when a single Gaussian noise was applied

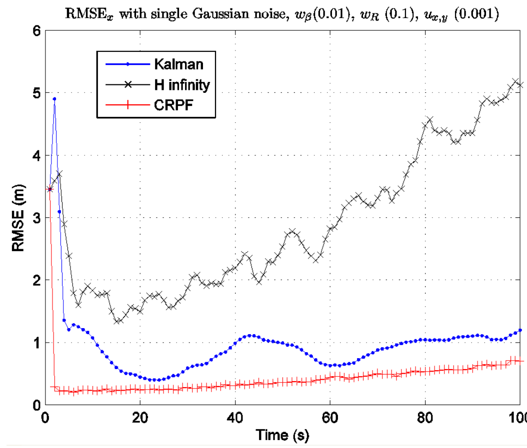


Fig. 6. RMSE in X coordinate by 300 runs of randomly different tracks when a single Gaussian noise was applied. The noise variance scenario is (0.01, 0.1, 0.001) in the order of w_β, w_R, and $u_{x,y}$.

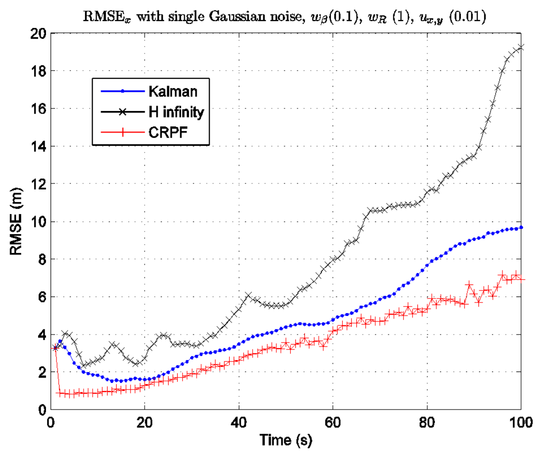
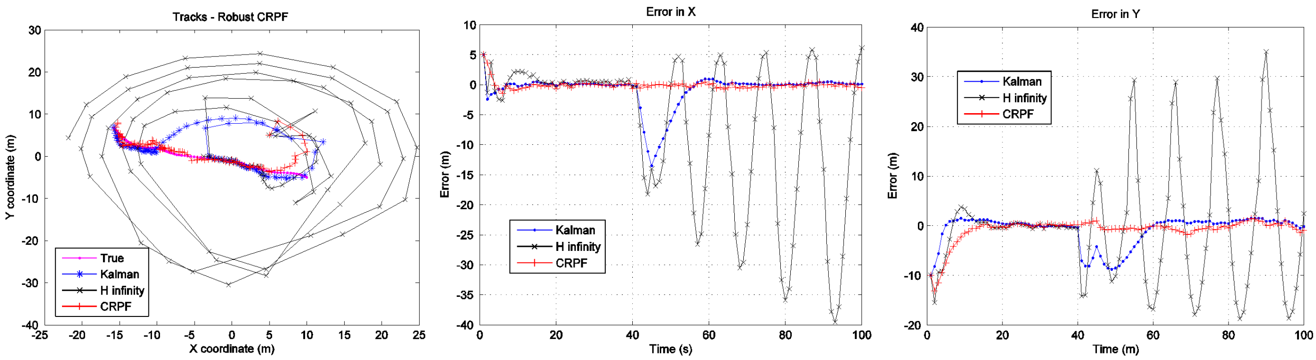


Fig. 7. RMSE in X coordinate by 300 runs of randomly different tracks when a single Gaussian noise was applied. The noise variance scenario is (0.01, 0.1, 0.001) in the order of w_β, w_R, and $u_{x,y}$



(a) Diverging by H_∞ (b) Error in X coordinate (c) Error in Y coordinate

Fig. 8. Example of diverging tracking by H_∞ filter.

Table 4. The number of diverging or considerable deviating tracking from the true target tracks out of 300 runs

Applied Noise	CRPF	EKF	HIF
Single Gaussian	0	37	39
Mixture Gaussian	0	51	57

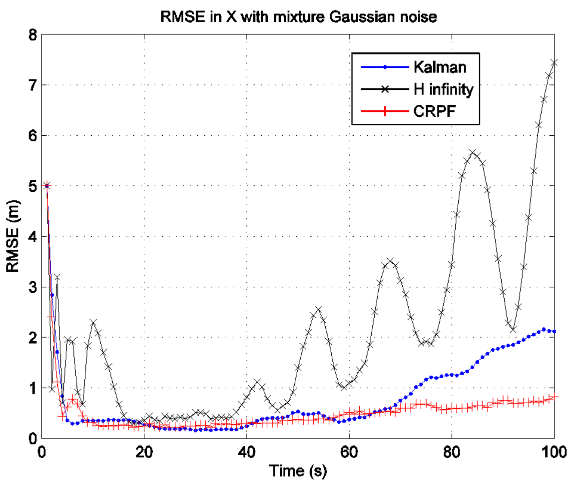


Fig. 9. RMSE in X coordinate by 300 runs of randomly different tracks when a mixture Gaussian noise was applied.

noise information of the system. HIF shows the worst performance, and it may not be robust and appropriate method for the problem. Although we do not show results here, all the other scenarios show similar results with Figs. 6-7. Fig. 8 shows an example of diverging tracking by HIF, and also shows the poor performance of the EKF. The tracking by HIF tends to diverge particularly when the true track is highly nonlinear.

In this figure, only CRPF demonstrates the robust performance. Fig. 9 shows a similar result when we apply the mixture Gaussian noise to the problem. CRPF, is not affected by the mixture Gaussian noise as shown in Fig. 9. Fig. 10 shows an example of target tracking by the three

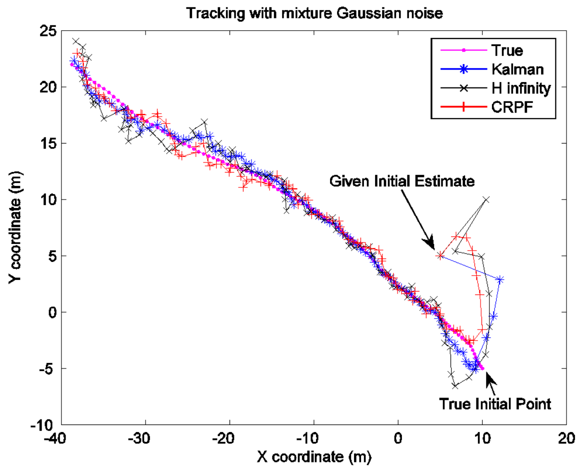


Fig. 10. Target tracking by the methods when a mixture Gaussian noise is applied.

methods when the mixture Gaussian noise is applied without any diverging or deviating tracks.

Under the noise variance scenario of ($w_\beta = 0.01$, $w_R = 0.1$, and $u_{x,y} = 0.001$), the numbers of diverging or deviating tracking from the true target tracks during the simulations are summarized in Table 4. We set the threshold of divergence or deviation to ±10m for HIF, ±5m for both the EKF and CRPF, respectively, in any directions and at any time step when we count the number of times the trackers deviate from the true tracks.

5. Summary and Conclusion

We have proposed and assessed the performance of HIF and CRPF and compared to that of the well-known Kalman filtering for the problem of tracking a target in two dimensional space based on the range and the bearing of the target. The problem is modeled by the dynamic state system and the state is estimated based on the measurement that is a highly nonlinear function of the state of interest.

Classical approaches such as maximum likelihood approaches are not pertinent for this nonlinear and dynamically time-varying parameter estimation. Some adaptive filters such as least mean squares method or recursive least squares method may not show the optimal performance because these filters do not take advantage of the knowledge of the state model. Well-known Wiener filter also has constraints in its usage only for a scalar and stationary signals. Interesting feature of the proposed approaches is that the noise statistics of the problem are not needed in their applications whereas the noise statistics need to be perfectly known for the well-known Kalman filter. The direction of the target motion is subject to the acceleration which can be forced by external diverse factors. Under the condition that any tracking of the methods do not diverge, the EKF shows the best performance, CRPF is the second, and HIF shows the worst performance. CRPF converges fastest even though the EKF shows the best performance under the condition. Sometimes, the tracking of HIF tends to diverge, especially when the target track is highly non-linear, and the performance becomes worse when the noise of the problem is a mixture Gaussian. Interestingly, the EKF also shows degraded tracking performance consistently with HIF when HIF's tracking diverges; nonetheless, the EKF's performance is not as bad as that of HIF. On the contrary, CRPF demonstrates robust tracking performance without any diverging or considerable deviating track even though it requires high computational cost which is a essential for the Monte Carlo method framework such as particle filtering. Regardless of the tracking performance of HIF and CRPF, these filters can be employed for many nonlinear problems that can be modeled by DSS model taking advantage of the feature that the noise statistics are not needed in their applications.

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Jaechan Lim received the B.S. degree in physics from Korea University, Seoul, Korea in 1996 and the M.S. and Ph.D. degrees in electrical engineering from Stony Brook University, Stony Brook, NY, in 1999 and 2007, respectively. His research areas include statistical signal processing and control, Bayesian estimation, particle filtering, channel estimation, channel equalization, operations of wireless access network and communication systems, and biomedical image processing. and biomedical image processing. He was an Adjunct professor with the department of electrical engineering in the University of Bridgeport, Bridgeport, CT. He was a Research Professor with the department of electronic engineering in the Sogang University, Seoul, Korea as a research professor. He was a Senior Staff Research Engineer with GERI, Gumi. He is currently a Senior Staff Researcher with the Department of Creative IT Engineering and Future IT Innovative Laboratory, Pohang University of Science and Technology, Pohang.