

# 고성능 LDPC 코드를 생성하기 위한 최적화된 알고리즘

서희종\*

An Optimized Algorithm for Constructing LDPC Code with Good Performance

Hee-Jong Suh\*

요 약

본 논문에서, 성능 좋은 LDPC(Low density parity check) 코드를 위한 태너(Tanner) 그래프를 생성하는 알고리즘을 제안한다. 이 알고리즘은 데프스 컨스트레인트(depth constraints)를 유지하면서 태너 그래프의 새로운 가지를 생성한다. 이 알고리즘은 그래프의 스톱핑 셸(stopping set)을 효과적으로 줄이고, 기존의 다른 알고리즘보다도 낮은 계산복잡도를 갖는다. 모의시험을 통해서 이 알고리즘의 개선된 성능을 확인 할 수 있었다.

ABSTRACT

In this paper, an algorithm having new edge growth with depth constraints for constructing Tanner graph of LDPC(Low density parity check) codes is proposed. This algorithm reduces effectively the number of small stopping set in the graph and has lower complexity than other algorithm. The simulation results shows the improved performance of the LDPC codes constructed by this algorithm.

키워드

LDPC, Stopping Set, ACE, PEG  
LDPC, 스톱핑 셸(Stopping Set), ACE, PEG

## 1. Introduction

The Low density parity check(LDPC) code becomes one of the best codes because of their low-complexity iterative decoding algorithm and capacity approaching performance[1], and the code attracts more and more researchers' attention to do work in recent years. To construct the good LDPC code, the bipartite graph was considered as an important part. The performance of low density

parity check codes could be improved by optimizing the Tanner graph of this code.

A variety of LDPC code with Tanner graph optimization algorithm is appeared from 1999, and these proposed algorithm can reduce the graph of the stopping set, and improve the performance of the code.

To improve the performance, there are algorithms which are using the girth-conditioning techniques to increase the girth (shortest cycle

\* 교신저자(corresponding author) : 전남대학교 전자통신공학과(hjsuh@chonnam.ac.kr)  
접수일자 : 2013. 06. 10

심사(수정)일자 : 2013. 07. 23

게재 확정일자 : 2013. 08. 23

length) of LDPC codes. In 2001, D. M. Arnold et al. put forward the PEG(Progressive Edge Growth) algorithm[2], this algorithm is through the heuristic search method, and constructing Tanner graphs with large girth by establishing edges or connections progressively between symbols and check nodes in an edge-by-edge manner.

And in 2004, there was the ACE(Approximate Cycle Extrinsic message degree) algorithm by T. Tian et al. to improve the connectivity of the Tanner graph.[3], this algorithm is different from the previous methods by emphasizing the connectivity as well as the length of cycles. There appeared several improved ACE algorithms [4,5].

But the PEG algorithm is a greedy algorithm, to enlarge the girth, and not to consider about the connectivity of the graph. The ACE algorithm is too complicated, in this algorithm, a list is randomly generated, test cycles and calculate each new cycle with ACE value to determine whether it accord with ACE standard or not. In case of discordance, the list must be generated again.

In this paper, based on ACE and PEG algorithm, a new improved algorithm having edge growth with depth constraints for constructing Tanner graph of LDPC codes is proposed. In this algorithm there is the number of stoping set effectively reduced in the graph and this algorithm has lower complexity than ACE algorithm. With simulation we could see that the performance of LDPC codes which constructed with the new algorithm is better than other's one.

In Section II, definitions related with LDPC are explained. In Section III an algorithm having new edge growth with depth constraints for constructing Tanner graph of LDPC codes is proposed. In Section IV, there are the result of experiments for comparison of the proposed algorithm with PEG and ACE algorithm. Chapter V is conclusions. Next is references.

## II. LDPC Code

The parity check matrix and its bipartite graph of a LDPC code are as shown Fig. 1. The column and row in the parity check matrix correspond to the variable and check node in the bipartite graph. There exists an edge between variable and check node if and only if  $h_{ij}=1$ , where  $h_{ij}$  denotes the element of parity check matrix in the  $i$ th row and  $j$ th column.

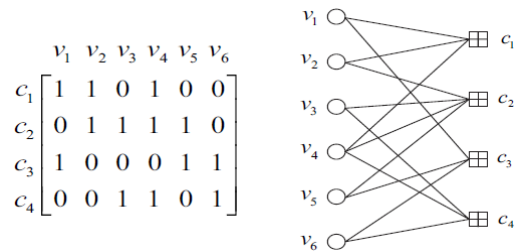


Fig 1. Parity check matrix and its bipartite graph

LDPC code has a stopping sets[6]. A subset  $S$  of variable nodes is said to form a stopping set if all its neighboring check nodes are connected to  $S$  at least twice. In Fig. 1, a stopping set is the subset of variable nodes  $A=[v_3, v_5, v_6]$ . But, if a subset has two neighboring check nodes, the subset is not a stopping set. For an example, a subset  $B=[v_1, v_2, v_3]$  is not stopping set. since the subset  $B$  has two neighboring check nodes,  $c_3$  and  $c_4$  which are connected to  $B$  only once. In parity check matrix, the stopping sets also can be determined by the ordinary(not mod 2) sum of the columns. In sum of some columns of the matrix, when 1 does not appear, a vertex set representing the columns are stopping set. The  $A$  sum of columns is  $[0 \ 2 \ 2 \ 2]^T$  and there is no 1 in it.

It is proved that a small stopping set means small minimum distance[7]. To avoid small stopping sets, we have to make the subset of variable nodes have many extrinsic edges. The quantities of these extrinsic edges defines an extrinsic message degree

(EMD).

The EMD of a variable node set is the number of extrinsic constraint nodes of this variable node set[7]. An extrinsic check node of a variable node set is a check node that is singly connected to this set. The EMD of a variable node set is the number of extrinsic check nodes of this variable node set.

The EMD of a stopping set is zero and the number of 1 in the sum of the columns over subset of variable nodes is equal to EMD. In the above example, the sum of column over  $B$  is  $[2 \ 2 \ 1 \ 1]^T$ , so the EMD of  $B$  is 2.

The approximate cycle extrinsic message degree (ACE) is defined[8]. The ACE of a length  $2d$  cycle is  $\sum_i (d_i - 2)$ , where  $d_i$  is the degree of the  $i$ th variable node in this cycle.

When variable nodes and their neighboring check nodes compose of a single cycle, the EMD of these variable nodes is equal to the ACE. However, when they compose of multi-cycles, the ACE becomes an upper bound of EMD. In Fig. 1, variable nodes  $[v_2, v_3, v_1]$  and their neighboring check nodes compose of 6-cycle and ACE is one. However, EMD of these variable nodes is zero since this 6-cycle contains a 4-cycle.

### III. Algorithm

A proposed algorithm uses cycles girth instead of ACE standard in the ACE algorithm. The algorithm is adjusting the parameters of the ACE algorithm. It needs consider cycle girth to add each edge, but not aims to the largest cycles like PEG algorithm. This algorithm uses each variable node degree assigned a target minimum cycle  $L_{d_i}$ , parameter standard. For each variable node, when it connects line, the cycles with all bigger degrees than the  $L_{d_i}$ , must be generated.

So, this algorithm uses the advantages of the

ACE algorithm and PEG algorithm. The parameter standard  $L_{d_i}$  shows as Fig. 2.

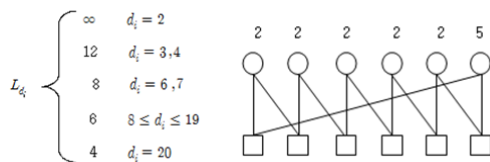


Fig 2. Parameter standard limiting cycles

The above standard is equivalent to the (6, 2), or (4, 5) standard of the ACE. In the limiting cycles the cycle that length is 12 can only appear when the variables nodes are connected, and the degree is 5. That appearance in the worst case, the rest of the node in the cycle degree is 2, at this time the ACE value of this cycle is 3, also greater than 2, through thus method, it can conform to the 6, 2 standard. Likewise, the cycle that length is 8 can only appear when the variables nodes the degree is 8, so when in the worst case, all the cycle which the length is 8, the ACE value is 6, also satisfies (4, 5) standard.

From the PEG algorithm process, progressively establishing edges is drawn between symbol and check nodes in an edge-by-edge manner, as show in Fig. 3. The shortest cycle passing through edge-by-edge is guaranteed to be no shorter than  $2(l+2)$ . The degree of variable nodes corresponding to the minimum depth standard is used instead of cycle length standard.  $N_{v_j}^l$  is as the check nodes set that the length to less than  $2l$ . It means that the graph is from the point of  $v_j$ , including the set of check nodes with depth  $l$ .  $\overline{N_{v_j}^l}$  is the rest of the nodes check nodes set  $c_i$ , removing the  $N_{v_j}^l$ . When edge is added, if choosing from the set  $N_{v_j}^l$ , it will appear a cycle that the length is less than  $(2l+2)$ , but if choosing from the set  $\overline{N_{v_j}^l}$ , it will not appear a cycle that the

length is less than  $(2l+2)$ . So a node is chosen from the set  $\overline{N_{v_j}^l}$ . Different from the PEG algorithm, the degree  $d_j$  of each variable node a minimum depth number  $l_{d_j}$  are given. If the variable node has degree  $d_j$ , when it extended to the depth  $l_{d_j}+1$ , it will not extended any more, and choose a node from the set  $\overline{N_{v_j}^l}$ , This limits the length of the cycle which must be greater than  $(2l_{d_i}+2)$ . Then it can limit the length of the cycle.

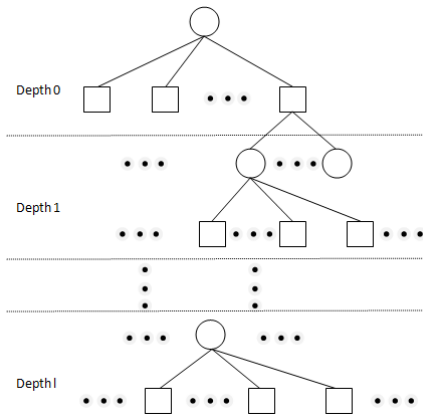


Fig. 3 Tree constructing the shortest cycles of the algorithm(Variable nodes indicated by circle, check nodes indicated by block)

At this time, the further transformation of the above standards is needed, and the length of the cycles standard to the depth layer standard must be changed. Corresponding depth layer standard is given in Table 1.

Table 1. The variable node degree and minimum depth standard

The number of minimum depth	The degree of the variable node
$\infty$	$d_i = 2$
5	$d_i = 3, 4$
3	$d_i = 5, 6, 7$
2	$8 \leq d_i \leq 19$
1	$d_i \geq 20$

#### IV. Experiment and Results

With the proposed algorithm which is new algorithm with high performance, the simulations on the its encoding and decoding were done together with the Random, PEG, and ACE algorithm. Code was (1008, 504) LDPC code. and the degree distributions of the variable node and check node,  $\lambda(x)$  and  $\rho(x)$ [8], are as follows.

$$\lambda(x) = 0.25105x + 0.30938x^2 + 0.00104x^3 + 0.43853x^9$$

$$\text{and } \rho(x) = 0.63676x^6 + 0.36324x^7$$

where  $x$  is LDPC code. And the simulations was under both Erasure channel and Gauss channel. The performance of LDPC code constructed by the proposed algorithm are indicated in Fig. 4, in this Fig. 4, the performance of PEG algorithm, ACE algorithm and proposed algorithm are much better than use a random graph of LDPC code, especially in low delete probability(BEC). The decoding performance with proposed algorithm have similar performance with ACE algorithm and PEG algorithm(we can see in the Fig. 4 when the probability of Erasure is 0.36). But in a high delete probability, such as the probability of Erasure is 0.42, the decoding performance of this proposed algorithm is more outstanding. The outstanding performance can be seen also under the Gauss channel in the Fig. 5. with a low signal-to-noise ratio (BIAWGN) in the channel(from the Fig. 5 when the SNR from 1 to 2). It was verified that the proposed algorithm has better than other algorithms in performance, because of small cycles in iterative decoding. Graph of the code constructed by the proposed algorithm had good connectivity. After the SNR is 2, the influence of small cycle on decoding performance is reduced. and the performance is not outstanding.

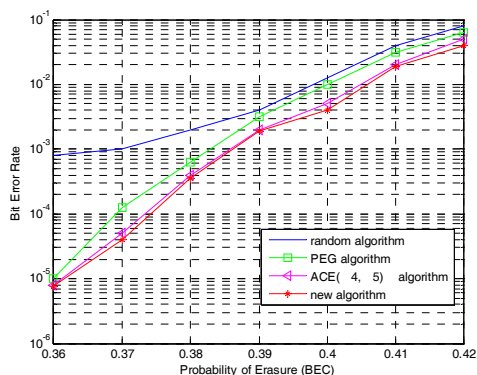


Fig. 4 Performance comparison of iterative decoding of LDPC codes under the erasure channel

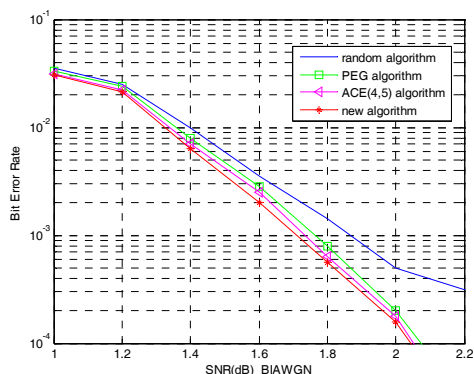


Fig. 5 Performance comparison of iterative decoding of LDPC codes under the gauss channel

## V. Conclusions

An algorithm having new edge growth with depth constraints for constructing a good Tanner graph of LDPC(Low density parity check) codes is proposed. In this algorithm, the stopping set is reduced in the Tanner graph, and the connectivity of the Tanner graph is increased. Time complexity of this algorithm is  $O(nd)$  ( $n$  is the number of node and  $d$  is the degree). The Time complexity is same with PEG algorithm, and lower than ACE algorithm. But the performance is more outstanding than PEG algorithm(in the Fig. 4 and Fig. 5), The

simulation result shows that the improved performance of LDPC codes constructed by this new algorithm.

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### 저자 소개



#### 서희종(Hee-Jong Suh)

1975년 한국항공대학교 항공통신공  
학과 졸업(공학사)

1996년 중앙대학교 대학원 전자공  
학과 졸업(공학박사)

2006년 전남대학교 전기전자통신컴퓨터공학부 교수

※ 관심분야 : 그래프이론, 컴퓨터통신망구조, 네트워크