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STABILITY OF A GENERAL QUADRATIC FUNCTIONAL EQUATION IN NON-ARCHIMEDEAN NORMED SPACES

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Abstract. In this paper, we investigate the stability for the functional equation

$f(x+y+z)+f(x-y)+f(x-z)-f(x-y-z)-f(x+y)-f(x+z)=0$

in non-Archimedean normed spaces.

1. Introduction

A classical question in the theory of functional equations is "when is it true that a mapping, which approximately satisfies a functional equation, must be somehow close to a solution of the equation?". This problem, called *a stability problem of the functional equation,* was formulated by S. M. Ulam [7] in 1940. In the next year, D. H. Hyers [2] gave a partial solution of Ulam problem for the case of an approximate additive mapping. Subsequently, his result was generalized by T. Aoki [1] for an additive mapping and by Th. M. Rassias [6] for a linear mapping with unbounded Cauchy differences.

We introduce some terminologies and notations used in the theory of non-Archimedean spaces (see [3]).

Definition 1.1. A field $\mathbb{K}$ equipped with a function (valuation) $|\cdot|$ from $\mathbb{K}$ into $[0,\infty)$ is called a *non-Archimedean field* if the function $|\cdot| : \mathbb{K} \to [0,\infty)$ satisfies the following conditions:

(i) $|r| = 0$ if and only if $r = 0$;
(ii) $|rs| = |r||s|$;

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(iii) $|r+s| \le \max\{|r|,|s|\}$ for all $r,s \in \mathbb{K}$.

Clearly, $|1| = |-1|$ and $|n| \le 1$ for all $n \in \mathbb{N}$.

DEFINITION 1.2. Let X be a vector space over a scalar field $\mathbb{K}$ with a non-Archimedean nontrivial valuation $|\cdot|$. A function $\|\cdot\| : X \to \mathbb{R}$ is *a non-Archimedean norm* if it satisfies the following conditions:

(i) $\|x\| = 0$ if and only if $x = 0$;
(ii) $\|rx\| = |r|\|x\|$ $(r \in \mathbb{K}, x \in X)$;
(iii) the strong triangle inequality, namely,

$$\|x+y\| \le \max\{\|x\|, \|y\|\}$$

for all $x, y \in X$ and $r \in \mathbb{K}$. The pair $(X, \|\cdot\|)$ is called *a non-Archimedean space* if $\|\cdot\| : X \to \mathbb{R}$ is a non-Archimedean norm on X.

Due to the fact that

$$\|x_n - x_m\| \le \max\{\|x_{j+1} - x_j\| : m \le j \le n-1\} (n > m),$$

a sequence $\{x_n\}$ is Cauchy if and only if $\{x_{n+1} - x_n\}$ converges to zero in a non-Archimedean space. By *a complete non-Archimedean space,* we mean one in which every Cauchy sequence is convergent.

Recently, M. S. Moslehian and Th. M. Rassias [5] discussed the Hyers-Ulam stability of the Cauchy functional equation

$$f(x+y) = f(x) + f(y) \tag{1.1}$$

and the quadratic functional equation

$$f(x+y) + f(x-y) - 2f(x) - 2f(y) = 0 \tag{1.2}$$

in non-Archimedean normed spaces.

Now we consider the *general quadratic functional equation*

$$f(x+y+z)+f(x-y)+f(x-z)-f(x-y-z)-f(x+y)-f(x+z) = 0, \tag{1.3}$$

which solution is called *a general quadratic mapping.* Recently, Kim [4] and Jun et al [3] obtained a stability of the functional equation (1.3) by taking and composing an additive mapping A and a quadratic mapping Q to prove the existence of a general quadratic mapping F which is close to the given function f. In their processing, A is approximate to the odd part $\frac{f(x)-f(-x)}{2}$ of f and Q is close to the even part $\frac{f(x)+f(-x)}{2} - f(0)$ of f, respectively.

In this paper, we get a general stability result of the general quadratic functional equation (1.3) in non-Archimedean normed spaces.

2. Stability of the general quadratic functional equation

Throughout this section, we assume that X is a real linear space and Y is a complete non-Archimedean space with $|2| < 1$.

For a given mapping $f : X \to Y$, we use the abbreviation

$$\begin{aligned} Df(x,y,z) := {} & f(x+y+z) + f(x-y) + f(x-z) \\ & -f(x-y-z) - f(x+y) - f(x+z) \end{aligned}$$

for all $x, y, z \in X$. Now, we will prove the stability of the general quadratic functional equation (1.3).

Theorem 2.1. *Let $\varphi : X^3 \to [0,\infty)$ be a function such that*

$$\lim_{n\to\infty} \frac{\varphi(2^n x, 2^n y, 2^n z)}{|4|^n} = 0 \ (x, y, z \in X). \tag{2.1}$$

Suppose that $f : X \to Y$ is a mapping satisfying

$$\|Df(x,y,z)\| \le \varphi(x,y,z) \quad (x,y,z \in X). \tag{2.2}$$

Then there exists a unique general quadratic mapping $T : X \to Y$ such that

$$\|f(x) - T(x)\| \le \lim_{n\to\infty} \max\{\psi_j(x) : 0 \le j < n\} \quad (x \in X), \tag{2.3}$$

where $\psi_j : X \to [0,\infty)$ is defined by

$$\begin{aligned} \psi_j(x) \ := \ \max\Big\{ & \frac{\varphi(2^{j-1}x, 2^{j-1}x, 2^j x)}{|2|\cdot|4|^{j+1}}, \frac{\varphi(2^{j-1}x, 2^{j-1}x, 2^{j-1}x)}{|2|\cdot|4|^{j+1}}, \\ & \frac{\varphi(-2^{j-1}x, -2^{j-1}x, -2^j x)}{|2|\cdot|4|^{j+1}}, \\ & \frac{\varphi(-2^{j-1}x, -2^{j-1}x, -2^{j-1}x)}{|2|\cdot|4|^{j+1}}, \frac{\varphi(2^{j+1}x, 2^j x, 2^j x)}{|2|^{j+2}}, \\ & \frac{\varphi(2^j x, 2^{j+1}x, 2^j x)}{|2|^{j+2}}, \frac{\varphi(2^j x, 2^j x, 2^j x)}{|2|^{j+2}} \Big\} \end{aligned}$$

for all $j \ge 0$. In particular, T is given by

$$T(x) = \lim_{n\to\infty} \frac{f(2^n x) + f(-2^n x) - 2f(0)}{2\cdot 4^n} + \frac{f(2^n x) - f(-2^n x)}{2^{n+1}} + f(0)$$

for all $x \in X$.

Proof. Let $J_n f : X \to Y$ be a function defined by

$$J_n f(x) = \frac{f(2^n x) + f(-2^n x) - 2f(0)}{2\cdot 4^n} + \frac{f(2^n x) - f(-2^n x)}{2^{n+1}} + f(0)$$

for all $x \in X$ and $n \in \mathbb{N}$. Notice that $J_0 f(x) = f(x)$ and

$$\begin{aligned}
(2.4)\quad & \|J_j f(x) - J_{j+1} f(x)\| \\
&= \Big\| -\frac{Df(2^{j-1}x, 2^{j-1}x, 2^j x)}{2\cdot 4^{j+1}} - \frac{Df(2^{j-1}x, 2^{j-1}x, 2^{j-1}x)}{2\cdot 4^{j+1}} \\
&\quad -\frac{Df(-2^{j-1}x, -2^{j-1}x, -2^j x)}{2\cdot 4^{j+1}} \\
&\quad -\frac{Df(-2^{j-1}x, -2^{j-1}x, -2^{j-1}x)}{2\cdot 4^{j+1}} + \frac{Df(2^{j+1}x, 2^j x, 2^j x)}{2^{j+2}} \\
&\quad -\frac{Df(2^j x, 2^{j+1}x, 2^j x)}{2^{j+2}} + \frac{Df(2^j x, 2^j x, 2^j x)}{2^{j+2}} \Big\| \\
&\le \max\Big\{ \frac{\|Df(2^{j-1}x, 2^{j-1}x, 2^j x)\|}{|2|\cdot|4|^{j+1}}, \frac{\|Df(2^{j-1}x, 2^{j-1}x, 2^{j-1}x)\|}{|2|\cdot|4|^{j+1}}, \\
&\quad \frac{\|Df(-2^{j-1}x, -2^{j-1}x, -2^j x\|)}{|2|\cdot|4|^{j+1}}, \\
&\quad \frac{\|Df(-2^{j-1}x, -2^{j-1}x, -2^{j-1}x)\|}{|2|\cdot|4|^{j+1}}, \frac{\|Df(2^{j+1}x, 2^j x, 2^j x)\|}{|2|^{j+2}}, \\
&\quad \frac{\|Df(2^j x, 2^{j+1}x, 2^j x)\|}{|2|^{j+2}}, \frac{\|Df(2^j x, 2^j x, 2^j x)\|}{|2|^{j+2}} \Big\} \\
&\le \psi_j(x)
\end{aligned}$$

for all $x \in X$ and $j \ge 0$. It follows from (2.1) and (2.4) that the sequence $\{J_n f(x)\}$ is Cauchy. Since Y is complete, we conclude that $\{J_n f(x)\}$ is convergent. Set

$$T(x) := \lim_{n\to\infty} J_n f(x).$$

Using induction one can show that

$$(2.5)\qquad \|J_n f(x) - f(x)\| \le \max\Big\{\psi_j(x) : 0 \le j < n\}$$

for all $n \in N$ and all $x \in X$. By taking n to approach infinity in (2.5) and using (2.1), one obtains (2.3). Replacing x, y, and z by $2^n x$, $2^n y$, and $2^n z$, respectively, in (2.2) we get

$$\begin{aligned}
\|DJ_n f(x,y,z)\| \;=\; & \Big\| \frac{Df(2^n x, 2^n y, 2^n z) - Df(-2^n x, -2^n y, -2^n z)}{2^{n+1}} \\
& \frac{+Df(2^n x, 2^n y, 2^n z) + Df(-2^n x, -2^n y, -2^n z)}{2\cdot 4^n} \Big\|
\end{aligned}$$

$$\leq \max\left\{\frac{\varphi(2^nx, 2^ny, 2^nz)}{|2|^{n+1}}, \frac{\varphi(-2^nx, -2^ny, -2^nz)}{|2|^{n+1}}, \frac{\varphi(2^nx, 2^ny, 2^nz)}{|2|\cdot|4|^n}, \frac{\varphi(-2^nx, -2^ny, -2^nz)}{|2|\cdot|4|^n}\right\}.$$

Taking the limit as $n \to \infty$ and using (2.1) we get $DT(x, y, z) = 0$. If T' is another general quadratic mapping satisfying (2.3), then

$$\begin{aligned} T'(x) &= \sum_{j=0}^{k-1}\left(- \frac{DT'(2^{j-1}x, 2^{j-1}x, 2^jx)}{2\cdot 4^{j+1}} - \frac{DT'(2^{j-1}x, 2^{j-1}x, 2^{j-1}x)}{2\cdot 4^{j+1}} \right. \\ &\quad - \frac{DT'(-2^{j-1}x, -2^{j-1}x, -2^jx)}{2\cdot 4^{j+1}} \\ &\quad - \frac{DT'(-2^{j-1}x, -2^{j-1}x, -2^{j-1}x)}{2\cdot 4^{j+1}} + \frac{DT'(2^{j+1}x, 2^jx, 2^jx)}{2^{j+2}} \\ &\quad \left. - \frac{DT'(2^jx, 2^{j+1}x, 2^jx)}{2^{j+2}} + \frac{DT'(2^jx, 2^jx, 2^jx)}{2^{j+2}} \right) + J_kT'(x) \\ &= J_kT'(x) \end{aligned}$$

for any $k \in N$ and so

$$\begin{aligned} &\|T(x) - T'(x)\| \\ &= \lim_{k\to\infty} \|J_kT(x) - J_kT'(x)\| \\ &\leq \lim_{k\to\infty} \max\{\|J_kT(x) - J_kf(x)\|, \|J_kf(x) - J_kT'(x)\|\} \\ &\leq \lim_{k\to\infty} |2|^{-2k-1} \max\{\|T(2^kx) - f(2^kx)\|, \|T(-2^kx) - f(-2^kx)\|, \\ &\qquad \|f(2^kx) - T'(2^kx)\|, \|f(-2^kx) - T'(-2^kx)\|\} \\ &\leq \lim_{k\to\infty}\lim_{n\to\infty} \max\left\{|2|^{-1}\psi_j(x), |2|^{-1}\psi_j(-x) : k \leq j < n+k\right\} \\ &= 0 \end{aligned}$$

for all $x \in X$. Therefore $T = T'$. This completes the proof of the uniqueness of T. □

COROLLARY 2.2. *Let X and Y be non-Archimedean normed spaces over $\mathbb{K}$ with $|2| < 1$. If Y is complete and for some $2 < r$, $f : X \to Y$ satisfies the condition*

$$\|Df(x, y, z)\| \leq \theta(\|x\|^r + \|y\|^r + \|z\|^r)$$

for all $x, y, z \in X$. Then there exists a unique general quadratic mapping $T : X \to Y$ such that

$$\|f(x) - T(x)\| \leq 3|2|^{-3-r}\theta\|x\|^r. \tag{2.6}$$

Proof. Let $\varphi(x, y, z) = \theta(\|x\|^r + \|y\|^r + \|z\|^r)$. Since $|2| < 1$ and $r - 2 > 0$,

$$\lim_{n\to\infty} |4|^{-n}\varphi(2^n x, 2^n y, 2^n z) = \lim_{n\to\infty} |2|^{n(r-2)}\varphi(x, y, z) = 0$$

for all $x, y, z \in Y$. Therefore the conditions of Theorem 2.1 are satisfied. It is easy to see that $\psi_0(x) = 3|2|^{-3-r}\theta\|x\|^r$. By Theorem 2.1 there is a unique general quadratic mapping $T : X \to Y$ such that (2.6) holds. □

THEOREM 2.3. *Let $\varphi : X^3 \to [0, \infty)$ be a function such that*

$$\lim_{n\to\infty} |2|^n\varphi(2^{-n}x, 2^{-n}y, 2^{-n}z) = 0 \ (x, y, z \in X). \tag{2.7}$$

Suppose that $f : X \to Y$ is a mapping satisfying

$$\|Df(x, y, z)\| \leq \varphi(x, y, z) \quad (x, y, z \in X). \tag{2.8}$$

Then there exists a unique general quadratic mapping $T : X \to Y$ such that

$$\|f(x) - T(x)\| \leq \lim_{n\to\infty} \max\{\psi_j(x) : 0 \leq j < n\} \quad (x \in X), \tag{2.9}$$

where $\psi_j : X \to [0, \infty)$ is defined by

$$\begin{aligned}
&\psi_j(x) \\
&:= \max\Big\{|2|^{2j-1}\varphi\left(\frac{x}{2^{j+2}}, \frac{x}{2^{j+2}}, \frac{x}{2^{j+1}}\right), |2|^{2j-1}\varphi\left(\frac{x}{2^{j+2}}, \frac{x}{2^{j+2}}, \frac{x}{2^{j+2}}\right), \\
&\qquad |2|^{2j-1}\varphi\left(\frac{-x}{2^{j+2}}, \frac{-x}{2^{j+2}}, \frac{-x}{2^{j+1}}\right), |2|^{2j-1}\varphi\left(\frac{-x}{2^{j+2}}, \frac{-x}{2^{j+2}}, \frac{-x}{2^{j+2}}\right), \\
&\qquad |2|^{j-1}\varphi\left(\frac{x}{2^{j}}, \frac{x}{2^{j+1}}, \frac{x}{2^{j+1}}\right), |2|^{j-1}\varphi\left(\frac{x}{2^{j+1}}, \frac{x}{2^{j}}, \frac{x}{2^{j+1}}\right), \\
&\qquad |2|^{j-1}\varphi\left(\frac{x}{2^{j+1}}, \frac{x}{2^{j+1}}, \frac{x}{2^{j+1}}\right)\Big\}
\end{aligned}$$

for all $j \geq 0$. In particular, T is given by

$$\begin{aligned}
T(x) \quad = \quad & \lim_{n\to\infty} \frac{4^n}{2}\left(f(2^{-n}x) + f(-2^{-n}x) - 2f(0)\right) \\
& +2^{n-1}\left(f\left(\frac{x}{2^n}\right) - f\left(-\frac{x}{2^n}\right)\right) + f(0)
\end{aligned}$$

for all $x \in X$.

Proof. Let $J_nf : X \to Y$ be a function defined by

$$J_nf(x) = \lim_{n\to\infty} \frac{4^n}{2}\left(f(2^{-n}x) + f(-2^{-n}x) - 2f(0)\right) + 2^{n-1}\left(f\left(\frac{x}{2^n}\right) - f\left(-\frac{x}{2^n}\right)\right) + f(0)$$

for all $x \in X$ and $n \in \mathbb{N}$. Notice that $J_0f(x) = f(x)$ and

$$\begin{aligned}
&\|J_jf(x) - J_{j+1}f(x)\| \\
&= \Big\|\frac{4^j}{2}\Big(Df\left(\frac{x}{2^{j+2}}, \frac{x}{2^{j+2}}, \frac{x}{2^{j+1}}\right) + Df\left(\frac{x}{2^{j+2}}, \frac{x}{2^{j+2}}, \frac{x}{2^{j+2}}\right) \\
&\quad + Df\left(\frac{-x}{2^{j+2}}, \frac{-x}{2^{j+2}}, \frac{-x}{2^{j+1}}\right) + Df\left(\frac{-x}{2^{j+2}}, \frac{-x}{2^{j+2}}, \frac{-x}{2^{j+2}}\right)\Big) \\
&\quad - 2^{j-1}\Big(Df\left(\frac{x}{2^{j}}, \frac{x}{2^{j+1}}, \frac{x}{2^{j+1}}\right) - Df\left(\frac{x}{2^{j+1}}, \frac{x}{2^{j}}, \frac{x}{2^{j+1}}\right) \\
&\quad + Df\left(\frac{x}{2^{j+1}}, \frac{x}{2^{j+1}}, \frac{x}{2^{j+1}}\right)\Big)\Big\| \\
&\le \psi_j(x)
\end{aligned} \tag{2.10}$$

for all $x \in X$ and $j \ge 0$. It follows from (2.7) and (2.10) that the sequence $\{J_nf(x)\}$ is Cauchy. Since Y is complete, we conclude that $\{J_nf(x)\}$ is convergent. Set

$$T(x) := \lim_{n\to\infty} J_nf(x).$$

Using induction one can show that

$$\|J_nf(x) - f(x)\| \le \max\Big\{\psi_j(x) : 0 \le j < n\Big\} \tag{2.11}$$

for all $n \in N$ and all $x \in X$. By taking n to approach infinity in (2.11) and using (2.7) one obtains (2.9). Replacing x, y, and z by $2^{-n}x$, $2^{-n}y$, and $2^{-n}z$, respectively, in (2.8), we get

$$\begin{aligned}
&\|DJ_nf(x,y,z)\| \\
&= \Big\|2^{n-1}Df\left(\frac{x}{2^n}, \frac{y}{2^n}, \frac{z}{2^n}\right) - 2^{n-1}Df\left(\frac{-x}{2^n}, \frac{-y}{2^n}, \frac{-z}{2^n}\right) \\
&\quad + 2^{2n-1}Df\left(\frac{x}{2^n}, \frac{y}{2^n}, \frac{z}{2^n}\right) + 2^{2n-1}Df\left(\frac{-x}{2^n},, \frac{-y}{2^n}, \frac{-z}{2^n}\right)\Big\| \\
&\le \max\Big\{|2|^{n-1}\varphi\left(\frac{x}{2^n}, \frac{y}{2^n}, \frac{z}{2^n}\right), |2|^{n-1}\varphi\left(\frac{-x}{2^n}, \frac{-y}{2^n}, \frac{-z}{2^n}\right)\Big\}.
\end{aligned}$$

Taking the limit as $n \to \infty$ and using (2.7) we get $DT(x,y,z) = 0$. If T' is another general quadratic mapping satisfying (2.9), then

$$\begin{aligned}
&T'(x) - J_kT'(x)\\
&= \sum_{j=0}^{k-1}\Bigg(\frac{4^j}{2}\bigg(DT'\left(\frac{x}{2^{j+2}},\frac{x}{2^{j+2}},\frac{x}{2^{j+1}}\right) + DT'\left(\frac{x}{2^{j+2}},\frac{x}{2^{j+2}},\frac{x}{2^{j+2}}\right)\\
&\quad +DT'\left(\frac{-x}{2^{j+2}},\frac{-x}{2^{j+2}},\frac{-x}{2^{j+1}}\right) + DT'\left(\frac{-x}{2^{j+2}},\frac{-x}{2^{j+2}},\frac{-x}{2^{j+2}}\right)\bigg)\\
&\quad -2^{j-1}\bigg(DT'\left(\frac{x}{2^{j}},\frac{x}{2^{j+1}},\frac{x}{2^{j+1}}\right) - DT'\left(\frac{x}{2^{j+1}},\frac{x}{2^{j}},\frac{x}{2^{j+1}}\right)\\
&\quad +DT'\left(\frac{x}{2^{j+1}},\frac{x}{2^{j+1}},\frac{x}{2^{j+1}}\right)\bigg)\Bigg) = 0
\end{aligned}$$

for any $k \in \mathbb{N}$ and so

$$\begin{aligned}
&\|T(x) - T'(x)\|\\
&= \lim_{k\to\infty} \|J_kT(x) - J_kT'(x)\|\\
&\le \lim_{k\to\infty} \max\{\|J_kT(x) - J_kf(x)\|, \|J_kf(x) - J_kT'(x)\|\}\\
&\le \lim_{k\to\infty} |2|^{k-1}\max\bigg\{\left\|T\left(\frac{x}{2^k}\right) - f\left(\frac{x}{2^k}\right)\right\|, \left\|T\left(-\frac{x}{2^k}\right) - f\left(-\frac{x}{2^k}\right)\right\|,\\
&\qquad\qquad \left\|f\left(\frac{x}{2^k}\right) - T'\left(\frac{x}{2^k}\right)\right\|, \left\|f\left(-\frac{x}{2^k}\right) - T'\left(-\frac{x}{2^k}\right)\right\|\bigg\}\\
&\le \lim_{k\to\infty} |2|^{k-1}\lim_{n\to\infty}\max\{\psi_j\left(\frac{x}{2^k}\right), \psi_j\left(\frac{-x}{2^k}\right) : 0 \le j < n\}\\
&= \lim_{k\to\infty} |2|^{-1}\lim_{n\to\infty}\max\{\psi_j(x), \psi_j(-x) : k \le j < n+k\}\\
&= 0 \qquad (x \in X).
\end{aligned}$$

Therefore $T = T'$. This completes the proof of the uniqueness of T. □

COROLLARY 2.4. *Let X and Y be non-Archimedean normed spaces over $\mathbb{K}$ with $|2| < 1$. If Y is complete and for some $0 \le r < 1$, $f : X \to Y$ satisfies the condition*

$$\|Df(x,y,z)\| \le \theta(\|x\|^r + \|y\|^r + \|z\|^r)$$

for all $x, y, z \in X$. Then there exists a unique general quadratic mapping $T : X \to Y$ such that

$$\|f(x) - T(x)\| \le 3|2|^{-1-2r}\theta\|x\|^r. \tag{2.12}$$

Proof. Let $\varphi(x,y,z) = \theta(\|x\|^r + \|y\|^r + \|z\|^r)$. Since $|2| < 1$ and $1-r>0$,

$$\lim_{n\to\infty} |2|^n \varphi(2^{-n}x, 2^{-n}y, 2^{-n}z) = \lim_{n\to\infty} |2|^{n(1-r)}\varphi(x,y,z) = 0$$

for all $x,y,z \in X$. Therefore the conditions of Theorem 2.3 are satisfied. It is easy to see that $\psi_0(x) = 3|2|^{-1-2r}\theta\|x\|^r$. By Theorem 2.3, there is a unique general quadratic mapping $T : X \to Y$ satisfying (2.12). □

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