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CATEGORY OF MAPS AND GOTTLIEB SETS FOR MAPS, AND THEIR DUALS

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Abstract. In this paper, we introduce and study the concepts of WC_k^f-spaces with respect to spaces which are generalized concepts of C_k^f-spaces for maps, and introduce the dual concepts of WC_k^f-spaces with respect to spaces and obtain some dual results.

1. Introduction

Throughout this paper, a space means a space of the homotopy type of a locally finite connected CW complex. All maps shall mean continuous functions. It is known that any space X is filtered by the projective spaces of ΩX by a result of Milnor [8] and Stasheff [10];

$$\Sigma\Omega X = P^1(\Omega X) \hookrightarrow P^2(\Omega X) \hookrightarrow \cdots \hookrightarrow P^\infty(\Omega X) \simeq X.$$

For each k, let $e_k^X : P^k(\Omega X) \to P^\infty(\Omega X) \simeq X$ be the natural inclusion. Let $f : A \to X$ be a map. A space X is called [5] a C_k^f-*space* if the inclusion $e_k^X : P^k(\Omega X) \to X$ is f-cyclic. It is known [5] that a space X is a C_k^f-space for a map $f : A \to X$ if and only if $G^f(Z, X) = [Z, X]$ for any space Z with $cat\ Z \leq k$. For any spaces Z, X, we define $mapcat\ (Z, X) \leq k$ if for any map $g : Z \to X$, $cat\ g \leq k$. It is known that if $cat\ Z \leq k$, then $mapcat\ (Z, X) \leq k$, but the converse does not hold(see Example 2.6).

In this paper, we introduce the concepts of WC_k^f-spaces with respect to spaces which are generalizations of C_k^f-spaces for maps [5] and study some properties of WC_k^f-spaces with respect to spaces. We show that for a space Z with $mapcat\ (Z, X) \leq k$, a space X is a WC_k^f-space with

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respect to Z if and only if $G^f(Z,X) = [Z,X]$. Let $f : A \to X$ and $g : B \to Y$ be any maps and Z a space with *mapcat* $(Z,X) \leq k$. Then we show that the product space $X \times Y$ is a $WC_k^{f\times g}$-space with respect to Z if and only if X is a WC_k^f-space with respect to Z and Y is a WC_k^g-space with respect to Z. We also introduce the dual concepts of WC_k^f-spaces with respect to spaces and obtain some dual results.

2. WC_k^f-spaces with respect to spaces

The *LS category* of X [3], denoted *cat* X, is the least integer k such that X is the union of $k+1$ open sets U_i, each contractible in X. We now recall the following Ganea's theorems [3].

THEOREM 2.1. ([3],[4]) *The category cat* $X \leq k$ *if and only if* $e_k^X : P^k(\Omega X) \to X$ *has a right homotopy inverse.*

The definition of LS category extends from spaces to continuous maps as follows. Let $g : X \to Y$ be a map. The *LS category of* g [3], denoted *cat* g is the least integer k such that X is the union of $k+1$ open sets U_i for which the restriction of g to each U_i is homotopic to a constant map $U_i \to *$. Note that *cat* $X =$ *cat* 1_X.

THEOREM 2.2. [3] *Let* $g : Z \to X$ *be a map. Then the category cat* $g \leq k$ *if and only if there is a map* $\bar{g} : Z \to P^k(\Omega X)$ *such that* $e_k^X \circ \bar{g} \sim g : Z \to X$, *where* $e_k^X : P^k(\Omega X) \to X$ *is the natural inclusion.*

DEFINITION 2.3. *Let* Z, X *be any two spaces. The mapcategory of mapping space from* Z *to* X *is less than equal to* k, *mapcat* $(Z,X) \leq k$, *means that for any map* $g : Z \to X$, *cat* $g \leq k$.

It is clear that *mapcat* $(Z,X) \leq k$ if and only if $(e_k^X)_\# : [Z, P^k(\Omega X)] \to [Z,X]$ is an epimorphism.

The following propositions say that a relationship between category of a space and mapcategory of a mapping space.

PROPOSITION 2.4. *cat* $Z \leq k$ *if and only if for any space* X, *mapcat* $(Z, X) \leq k$.

Proof. Suppose that *cat* $Z \leq k$. Then there is a map $s_k^Z : Z \to P^k(\Omega Z)$ such that $e_k^Z \circ s_k^Z \sim 1$. Let X be a space and $g : Z \to X$ a map. We see $e_k^X \circ P^k(\Omega g) \sim g \circ e_k^Z$ by the naturality of the construction of

$P^k(\Omega Z)$ as is shown in the following homotopy commutative diagram;

$$\begin{array}{ccc} P^k(\Omega Z) & \xrightarrow{P^k(\Omega g)} & P^k(\Omega X) \\ {\scriptstyle e_k^Z}\downarrow & & {\scriptstyle e_k^X}\downarrow \\ Z & \xrightarrow{\quad g \quad} & X. \end{array}$$

Thus we have a map $\bar{g} = P^k(\Omega g) \circ s_k^Z : Z \to P^k(\Omega X)$ such that $e_k^X \circ \bar{g} \sim g$. Thus we know *mapcat* $(Z, Y) \leq k$. On the other hand, suppose that for any space X, the mapcategory *mapcat* $(Z, X) \leq k$. Taking $X = Z$ and $g = 1_Z$, then we know that *cat* $Z \leq k$. □

PROPOSITION 2.5. *cat* $X \leq k$ *if and only if for any space* Z, *mapcat* $(Z, X) \leq k$.

Proof. Suppose that *cat* $X \leq k$. Then there is a map $s_k^X : X \to P^k(\Omega X)$ such that $e_k^X \circ s_k^X \sim 1$. Let Z be a space and $g : Z \to X$ a map. Then we have $e_k^X \circ (s_k^X \circ g) \sim 1_X \circ g \sim g$ and *mapcat* $(Z, X) \leq k$. On the other hand, suppose that for any space Z, the mapcategory *mapcat* $(Z, X) \leq k$. Taking $Z = X$ and $g = 1_X$, then we know that *cat* $X \leq k$. □

In general, if *cat* $Z \leq k$, then *mapcat* $(Z, X) \leq k$ for a space X, but the converse does not hold by the following example.

EXAMPLE 2.6. It is well known fact that *cat* $\mathbb{C}P^n = n$. Thus if we take $X = \mathbb{C}P^k$ and $Z = \mathbb{C}P^{k+1}$, then we know, from Proposition 2.5, that *mapcat* $(Z, X) \leq k$, but *cat* $Z = k + 1$.

Let $f : A \to X$ be a map. A based map $g : B \to X$ is called *f-cyclic* [12] if there is a map $\phi : A \times B \to X$ such that the diagram

$$\begin{array}{ccc} A \times B & \xrightarrow{\quad \phi \quad} & X \\ {\scriptstyle j}\uparrow & & {\scriptstyle \nabla}\uparrow \\ A \vee B & \xrightarrow{(f \vee g)} & X \vee X \end{array}$$

is homotopy commute, where $j : A \vee B \to A \times B$ is the inclusion and $\nabla : X \vee X \to X$ is the folding map. Clearly, g is f-cyclic iff f is g-cyclic. In the case, $f = 1_X : X \to X$, $g : B \to X$ is called cyclic [15]. We denote the set of all homotopy classes of f-cyclic maps from B to X by $G^f(B, X) \subset [B, X]$ which is called the *Gottlieb set for a map* $f : A \to X$. If $f = 1_X : X \to X$, then we recover the *Gottlieb set* $G(B, X) = G^{1_X}(B, X)$ defined by Varadarajan [11]. In general,

$G(B,X) \subset G^f(B,X) \subset [B,X]$ for any spaces A, B, X and any map $f : A \to X$.

It is shown [14] that $G(S^5, S^5 \times S^5) \cong 2\mathbb{Z} \oplus 2\mathbb{Z} \neq G^{i_1}(S^5, S^5 \times S^5) \cong 2\mathbb{Z} \oplus \mathbb{Z} \neq [S^5, S^5 \times S^5] \cong \mathbb{Z} \oplus \mathbb{Z}$. It is introduced [5] that a space X is called a C_k^f*-space* if the inclusion $e_k^X : P^k(\Omega X) \to X$ is f-cyclic.

Lemma 2.7. *Let $f : A \to X$ be a map. Then $g : B \to X$ is f-cyclic if and only if $(g)_\#([Z,B]) \subset G^f(Z,X)$ for any space Z.*

Proof. Suppose that $g : B \to X$ is f-cyclic. Let Z be a space and $\theta : Z \to B$ a map. Since $g : B \to X$ is f-cyclic, there is a map $G : A \times B \to X$ such that $Gj \sim \nabla(f \vee g)$, where $j : A \vee B \to A \times B$ is the inclusion and $\nabla : X \vee X \to X$ is the folding map. Then $\phi = G(1 \times \theta) : A \times Z \to X$ satisfies $\phi j \sim \nabla(f \vee g\theta)$. Thus we have $g_\#([Z,B]) \subset G^f(Z,X)$ for any space Z. On the other hand, taking $Z = B$ and $1_B : B \to B \in [B,B]$. Since $g \sim g_\#(1_B) \in G^f(B,X)$, $g : B \to X$ is f-cyclic. □

Theorem 2.8. [5] *A space X is a C_k^f-space for a map $f : A \to X$ if and only if $G^f(Z,X) = [Z,X]$ for any space Z with cat $Z \leq k$.*

Example 2.9.

(1) The torus T^k is a space with *mapcat*$(T^k, X) \leq k$. It is well known fact that *cat* $T^k = k$. Thus we know, from Proposition 2.4, that *mapcat*$(T^k, X) \leq k$, that is, $(e_k^X)_\# : [T^k, P^k(\Omega X)] \to [T^k, X]$ is an epimorphism.

(2) If a space Z satisfy *cat* $Z \leq k$, then Z is also a space with *mapcat*$(Z, X) \leq k$ from Proposition 2.4.

Definition 2.10. *Let $f : A \to X$ be a map and Z a space with mapcat $(Z,X) \leq k$. Then a space X is called a WC_k^f-space with respect to a space Z if $(e_k^X)_\#([Z, P^k(\Omega X]) \subset G^f(Z,X)$, where $e_k^X : P^k(\Omega X) \to X$ is the natural inclusion.*

Theorem 2.11. *Let $f : A \to X$ be a map and Z space with mapcat $(Z, X) \leq k$. A space X is a WC_k^f-space with respect to Z if and only if $G^f(Z,X) = [Z,X]$.*

Proof. Suppose that X is a WC_k^f-space with respect to Z. Since *mapcat* $(Z,X) \leq k$, $(e_k^X)_\# : [Z, P^k(\Omega X)] \to [Z,X]$ is an epimorphism. Since X is a WC_k^f-space with respect to Z, $[Z,X] = (e_k^X)_\#([Z, P^k(\Omega X)])$ $\subset G^f(Z,X)$ and $G^f(Z,X) = [Z,X]$.

Conversely, assume that $G^f(Z,X) = [Z,X]$. Thus we know $(e_k^X)_\#([Z, P^k(\Omega X)]) = [Z,X] \subset G^f(Z,X)$ and X is a WC_k^f-space with respect to Z. □

We have the following corollary from Theorem 2.8 and Example 2.9.

Corollary 2.12. *X is a C_k^f-space if and only if for each space Z with cat $Z \leq k$, X is a WC_k^f-space with respect to Z.*

Theorem 2.13. *Let $f : A \to X$ and $g : B \to Y$ be any maps and Z a space with mapcat $(Z, X) \leq k$. Then the product space $X \times Y$ is a $WC_k^{f\times g}$-space with respect to Z if and only if X is a WC_k^f-space with respect to Z and Y is a WC_k^g-space with respect to Z.*

Proof. Suppose $X \times Y$ is a $WC_k^{f\times g}$-space with respect to Z. It is known [5] that $G^{f\times g}(Z, X \times Y) \cong G^f(Z, X) \times G^g(Z, Y)$ for any space Z. Since Z is a space with *mapcat* $(Z, X) \leq k$, we have, from Theorem 2.11, that $G^f(Z, X) \times G^g(Z, Y) \cong G^{f\times g}(Z, X \times Y) = [Z, X \times Y] = [Z, X] \times [Z, Y]$ and hence $G^f(Z, X) = [Z, X]$ and $G^g(Z, Y) = [Z, Y]$. Thus X is a WC_k^f-space with respect to Z and Y is a WC_k^g-space with respect to Z.

Conversely, suppose that X is a WC_k^f-space with respect to Z and Y is a WC_k^g-space with respect to Z. Then $G^f(Z, X) = [Z, X]$ and $G^g(Z, Y) = [Z, Y]$ by Theorem 2.11. It follows that $G^{f\times g}(Z, X \times Y) \cong G^f(Z, X) \times G^g(Z, Y) = [Z, X] \times [Z, Y] = [Z, X \times Y]$. Thus $X \times Y$ is a $WC_k^{f\times g}$-space with respect to Z. □

3. DWC_k^p-spaces with respect to spaces

In [3], Ganea introduced the concept of cocategory of a space as follows; Let X be a any space. Define a sequence of cofibrations

$$\mathcal{C}_k : X \xrightarrow{e'_k} F_k \xrightarrow{s'_k} B_k \ (k \geq 0)$$

as follows, let $\mathcal{C}_0 : X \xrightarrow{e'_0} cX \xrightarrow{s'_0} \Sigma X$ be the standard cofibration. Assuming $\mathcal{C}_k$ to be defined, let F'_{k+1} be the fibre of s'_k and $e''_{k+1} : X \to F'_{k+1}$ lift e'_k. Define F_{k+1} as the reduced mapping cylinder of e''_{k+1}, let e'_{k+1} : $X \to F_{k+1}$ is the obvious inclusion map, and let $B_{k+1} = F_{k+1}/e'_{k+1}(X)$ and $s'_{k+1} : F_{k+1} \to F_{k+1}/e_{k+1}(X)$ the quotient map.

Definition 3.1. [3] *The cocategory of X, cocat X, is the least integer $k \geq 0$ for which there is a map $r : F_k \to X$ such that $r \circ e'_k \sim 1$. If there is no such integer, cocat $X = \infty$.*

The following remark can easily obtained from the above definition.

REMARK 3.2. *cocat* $X \leq k$ *if and only if* $e'_k : X \to F_k$ *has a left homotopy inverse.*

For a map $p : X \to A$, a based map $g : X \to B$ is *p-cocyclic* [9] if there is a map $\theta : X \to A \vee B$ such that $j\theta \sim (p \times g)\Delta$, where $j : A \vee B \to A \times B$ is the inclusion and $\Delta : X \to X \times X$ is the diagonal map. *The dual Gottlieb set for a map* $p : X \to A$, $DG^p(X, B)$, is the set of all homotopy classes of p-cocyclic maps from X to B. In the case $p = 1_X : X \to X$, we call a 1-cocyclic map is just a cocyclic map, and denoted by, $DG(X, B)$, which is the set of all homotopy classes of cocyclic maps from X to B.

In general, $DG(X, B) \subset DG^p(X, B) \subset [X, B]$ for any map $p : X \to A$ and any space B. However, there is an example in [13] such that $DG(X, B) \neq DG^p(X, B) \neq [X, B]$.

Let $g : X \to Z$ be a map. A cocategory of a map is less than equal to k, *cocat* $g \leq k$,[3] if there is a map $\bar{g} : F_k^X \to Z$ such that $\bar{g} \circ e_k'^X \sim g : X \to Z$.

DEFINITION 3.3. *Let* X, Z *be any two spaces. The mapcocategory of mapping space from* X *to* Z *is less than equal to* k*, mapcocat* $(X, Z) \leq k$*, means that for any map* $g : X \to Z$*, cocat* $g \leq k$*.*

It is clear that *mapcocat* $(X, Z) \leq k$ if and only if $(e_k'^X)^\# : [F_k^X, Z] \to [X, Z]$ is an epimorphism.

PROPOSITION 3.4. *cocat* $Z \leq k$ *if and only if for any space* X*, mapcocat* $(X, Z) \leq k$*.*

Proof. Suppose that *cocat* $Z \leq k$. Then there is a map $s_k'^Z : F_k^Z \to Z)$ such that $s_k'^Z \circ e_k'^Z \sim 1$. Let X be a space and $g : X \to Z$ a map. We see $F_k(g) \circ e_k'^X \sim e_k'^Z \circ g$ by the naturality of the construction of F_k^Z as is shown in the following homotopy commutative diagram:

$$\begin{array}{ccc} F_k^X & \xrightarrow{F_k(g)} & F_k^Z \\ {\scriptstyle e_k'^X}\uparrow & & {\scriptstyle e_k'^Z}\uparrow \\ X & \xrightarrow{g} & Z. \end{array}$$

Thus we have a map $\bar{g} = s_k'^Z \circ F_k(g) : F_k^X \to Z$ such that $\bar{g} \circ e_k'^X \sim g$. Thus we know *mapcocat* $(X, Z) \leq k$. On the other hand, suppose that for any space X, the mapcocategory *mapcocat* $(X, Z) \leq k$. Taking $X = Z$ and $g = 1_Z$, then we know that *cocat* $Z \leq k$. □

PROPOSITION 3.5. *cocat* $X \leq k$ *if and only if for any space* Z*, mapcocat* $(X, Z) \leq k$*.*

Proof. Suppose that *cocat* $X \leq k$. Then there is a map $s_k^{\prime X} : F_k^X \to X$ such that $s_k^{\prime X} \circ e_k^{\prime X} \sim 1$. Let Z be a space and $g : X \to Z$ a map. Then we have $(g \circ s_k^{\prime X}) \circ e_k^{\prime X} \sim 1_X \circ g \sim g$ and *mapcat* $(X, Z) \leq k$. On the other hand, suppose that for any space Z, the mapcocategory *mapcocat* $(X, Z) \leq k$. Taking $Z = X$ and $g = 1_X$, then we know that *cocat* $X \leq k$. □

It is introduced [18] that a space X is called DC_k^p-space for a map $p : X \to A$ if $e_k^{\prime X} : X \to F_k^X$ is p-cocyclic.

Lemma 3.6. *Let $p : X \to A$ be a map. Then $g : X \to B$ is p-cocyclic if and only if $(g)^{\#}([B, Z]) \subset DG^p(X, Z)$ for any space Z.*

Proof. Suppose that $g : X \to B$ is p-cocyclic. Let Z be a space and $h : B \to Z$ a map. Since $g : X \to B$ is p-cocyclic, there is a map $\theta : X \to A \vee B$ such that $j\theta \sim (p \times g)\Delta$, where $j : A \vee B \to A \times B$ is the inclusion and $\Delta : X \to X \times X$ is the diagonal map. Then $\phi = (1 \vee h)\theta : X \to A \vee Z$ satisfies $j\phi \sim (p \times hg)\Delta$. Thus we have $g^{\#}([B, Z]) \subset DG^p(X, Z)$ for any space Z. On the other hand, taking $Z = B$ and $1_B : B \to B \in [B, B]$. Since $g \sim g^{\#}(1_B) \in DG^p(X, B)$, $g : X \to B$ is p-cocyclic. □

Theorem 3.7. [18] *A space X is a DC_k^p-space for a map $p : X \to A$ if and only if $DG^p(X, Z) = [X, Z]$ for any space Z with cocat $Z \leq k$.*

Remark 3.8. *If a space Z satisfy cocat $Z \leq k$, then also Z is a space with mapcocat$(Z, X) \leq k$ from Proposition 3.4.*

Definition 3.9. *Let $p : X \to A$ be a map and Z a space with mapcocat $(X, Z) \leq k$. A space X is called a DWC_k^p−space with respect to a space Z if $(e_k^{\prime X})^{\#}([F_k^X, Z]) \subset DG^p(X, Z)$.*

Theorem 3.10. *Let $p : X \to A$ be a map and Z a space with mapcocat $(X, Z) \leq k$. Then a space X is a DWC_k^p−space with respect to Z if and only if $DG^p(X, Z) = [X, Z]$.*

Proof. Suppose that X is a DWC_k^p-space with respect to Z. Since *mapcocat* $(X, Z) \leq k$, $(e_k^{\prime X})^{\#} : [F_k^X, Z] \to [X, Z]$ is an epimorphism. Since X is a DWC_k^p-space with respect to Z, $[X, Z] = (e_k^{\prime X})^{\#}([F_k^X, Z]) \subset DG^p(X, Z)$ and $DG^p(X, Z) = [X, Z]$.

Conversely, assume that $DG^p(X, Z) = [X, Z]$. Thus we know $(e_k^{\prime X})^{\#}$ $([F_k^X, Z]) = [X, Z] \subset G^p(X, Z)$ and X is a DWC_k^p-space with respect to Z. □

We have the following corollary from Theorem 3.7 and Remark 3.8.

Corollary 3.11. *X is a DC_k^p-space if and only if for each space Z with cocat $Z \leq k$, X is a DWC_k^p-space with respect to Z.*

Let $p : X \to A$ and $q : Y \to A$ be any maps. Then it is known [18] that the relation $DG^{\nabla(p\vee q)}(X \vee Y, B) \equiv DG^p(X, B) \times DG^q(Y, B)$ holds for any space B.

Theorem 3.12. *Let $p : X \to A$ and $q : Y \to A$ be any maps and Z a space with mapcocat $(X, Z) \leq k$. Then the wedge space $X \vee Y$ is a $DWC_k^{\nabla(p\vee q)}$-space with respect to Z if and only if X is a DWC_k^p-space with respect to Z and Y is a DWC_k^q-space with respect to Z.*

Proof. If $X \vee Y$ is a $DWC_k^{\nabla(p\vee q)}$-space with respect to Z, then we know, from Theorem 3.10 and the above fact, that $DG^p(X, Z) \times DG^q(Y, Z) \equiv DG^{\nabla(p\vee q)}(X \vee Y, Z) = [X \vee Y, Z] \equiv [X, Z] \times [Y, Z]$. Then we have $DG^p(X, Z) = [X, Z]$ and $DG^q(Y, Z) = [Y, Z]$. Thus we know that X is a DWC_k^p-space with respect to Z and Y is a DWC_k^q-space with respect to Z. On the other hand, suppose that X is a DWC_k^p-space with respect to Z and Y is a DWC_k^q-space with respect to Z. Then $DG^p(X, Z) = [X, Z]$, $DG^q(Y, Z) = [Y, Z]$. Thus we know $DG^{\nabla(p\vee q)}(X \vee Y, Z) \equiv DG^p(X, Z) \times DG^q(Y, Z) = [X, Z] \times [Y, Z] \equiv [X \vee Y, Z]$. Thus $X \vee Y$ is a $DWC_k^{\nabla(p\vee q)}$-space with respect to Z. □

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