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FUZZY INTUITIONISTIC ALMOST (r, s)-CONTINUOUS MAPPINGS

Eun Pyo Lee* and Seung On Lee**

Abstract. We introduce the concepts of fuzzy (r, s)-regular open sets and fuzzy almost (r, s)-continuous mappings on the intuitionistic fuzzy topological spaces in Šostak's sense. Also we investigate the equivalent conditions of the fuzzy almost (r, s)-continuity.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [13]. Chang [3] was the first to introduce the concept of a fuzzy topology on a set X by axiomatizing a collection T of fuzzy subsets of X, where he referred to each member of T as an open set. In his definition of fuzzy topology, fuzziness in the concept of openness of a fuzzy subset was absent. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra, and Samanta [4], and by Ramadan [11]. Azad [2] introduce the concept of fuzzy regular open sets and fuzzy almost continuous mappings in Chang's fuzzy topology.

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Çoker and his colleagues [5, 7, 8] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and M. Demirci [6] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

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Correspondence should be addressed to Seung On Lee, solee@chungbuk.ac.kr.
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In this paper, we introduce the concepts of fuzzy (r,s)-regular open sets and fuzzy almost (r,s)-continuous mappings on the intuitionistic fuzzy topological spaces in Šostak's sense. Also we investigate the equivalent conditions of the fuzzy almost (r,s)-continuity.

2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member μ of I^X is called a fuzzy set of X. For any $\mu \in I^X$, μ^c denotes the complement $1-\mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1}-\mu)$.

Definition 2.1. [1] Let A and B be intuitionistic fuzzy sets on X. Then

(1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.
(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
(3) $A^c = (\gamma_A, \mu_A)$.
(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
(6) $0_\sim = (\tilde{0}, \tilde{1})$ and $1_\sim = (\tilde{1}, \tilde{0})$.

Let f be a mapping from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y. Then:

(1) The image of A under f, denoted by $f(A)$, is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$, is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

A *smooth fuzzy topology* on X is a map $T : I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0}) = T(\tilde{1}) = 1$.
(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$.
(3) $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$.

The pair (X, T) is called a *smooth fuzzy topological space.*

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $0_\sim, 1_\sim \in T$.
(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
(3) If $A_i \in T$ for all i, then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space.*

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.2. [12] Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense* (SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a map $\mathcal{T} : I(X) \to I \otimes I(\mathcal{T}_1, \mathcal{T}_2 : I(X) \to I)$ which satisfies the following properties:

(1) $\mathcal{T}_1(0_\sim) = \mathcal{T}_1(1_\sim) = 1$ and $\mathcal{T}_2(0_\sim) = \mathcal{T}_2(1_\sim) = 0$.
(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.
(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense* (SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

Definition 2.3. [9] Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy (r, s)-open* if $\mathcal{T}_1(A) \geq r$ and $\mathcal{T}_2(A) \leq s$,
(2) *fuzzy (r, s)-closed* if $\mathcal{T}_1(A^c) \geq r$ and $\mathcal{T}_2(A^c) \leq s$.

Definition 2.4. [9] Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy (r, s)-closure* is defined by

$$\mathrm{cl}(A, r, s) = \bigcap \{B \in I(X) \mid A \subseteq B, B \text{ is fuzzy } (r, s)\text{-closed}\}$$

and the *fuzzy (r, s)-interior* is defined by

$$\mathrm{int}(A, r, s) = \bigcup \{B \in I(X) \mid A \supseteq B, B \text{ is fuzzy } (r, s)\text{-open}\}.$$

Lemma 2.5. [9] For an intuitionistic fuzzy set A in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$, we have:

(1) $\text{cl}(A, r, s)^c = \text{int}(A^c, r, s)$.
(2) $\text{int}(A, r, s)^c = \text{cl}(A^c, r, s)$.

Let $(X, \mathcal{T})$ be an intuitionistic fuzzy topological space in Šostak's sense. Then it is easy to see that for each $(r, s) \in I \otimes I$, the family $\mathcal{T}_{(r,s)}$ defined by

$$\mathcal{T}_{(r,s)} = \{A \in I(X) \mid \mathcal{T}_1(A) \geq r \ \text{ and } \ \mathcal{T}_2(A) \leq s\}$$

is an intuitionistic fuzzy topology on X.

Let (X, T) be an intuitionistic fuzzy topological space and $(r, s) \in I \otimes I$. Then the map $T^{(r,s)} : I(X) \to I \otimes I$ defined by

$$T^{(r.s)}(A) = \begin{cases} (1,0) & \text{if } \ A = 0_\sim, 1_\sim \\ (r,s) & \text{if } \ A \in T - \{0_\sim, 1_\sim\} \\ (0,1) & \text{otherwise} \end{cases}$$

becomes an intuitionistic fuzzy topology in Šostak's sense on X.

Definition 2.6. [9] Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy (r, s)-semiopen* if there is a fuzzy (r, s)-open set B in X such that $B \subseteq A \subseteq \text{cl}(B, r, s)$,
(2) *fuzzy (r, s)-semiclosed* if there is a fuzzy (r, s)-closed set B in X such that $\text{int}(B, r, s) \subseteq A \subseteq B$.

Definition 2.7. [10] Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy (r, s)-semiclosure* is defined by

$$\text{scl}(A, r, s) = \bigcap\{B \in I(X) \mid A \subseteq B, B \text{ is fuzzy } (r, s)\text{-semiclosed}\}$$

and the *fuzzy (r, s)-semiinterior* is defined by

$$\text{sint}(A, r, s) = \bigcup\{B \in I(X) \mid A \supseteq B, B \text{ is fuzzy } (r, s)\text{-semiopen}\}.$$

Lemma 2.8. [10] For an intuitionistic fuzzy set A in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$,

(1) $\text{sint}(A, r, s)^c = \text{scl}(A^c, r, s)$.
(2) $\text{scl}(A, r, s)^c = \text{sint}(A^c, r, s)$.

Definition 2.9. [10] Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is said to be

(1) a *fuzzy (r, s)-continuous* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-open set of X for each fuzzy (r, s)-open set B of Y,

(2) a *fuzzy (r, s)-semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-semiopen set of X for each fuzzy (r, s)-open set B of Y,
(3) a *fuzzy (r, s)-irresolute* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-semiopen set of X for each fuzzy (r, s)-semiopen set B of Y.

3. Fuzzy almost (r, s)-continuous mappings

we define the notions of fuzzy (r, s)-regular open sets and fuzzy (r, s)-regular closed sets, and investigate some of their properties.

DEFINITION 3.1. Let A be an intuitionistic fuzzy set on SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy (r, s)-regular open* if $\text{int}(\text{cl}(A, r, s), r, s) = A$,
(2) *fuzzy (r, s)-regular closed* if $\text{cl}(\text{int}(A, r, s), r, s) = A$.

THEOREM 3.2. Let A be an intuitionistic fuzzy set of a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is fuzzy (r, s)-regular open if and only if A^c is fuzzy (r, s)-regular closed.

Proof. It follows from Lemma 2.5. □

REMARK 3.3. Clearly, every fuzzy (r, s)-regular open ((r, s)-regular closed) set is fuzzy (r, s)-open ((r, s)-closed). That the converse need not be true is shown by the following example.

EXAMPLE 3.4. Let $X = \{x, y\}$ and let B be an intuitionistic fuzzy set of X defined as

$$B(x) = (0.3, 0.6), \quad B(y) = (0.5, 0.1).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = B, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Clearly $(\mathcal{T}_1, \mathcal{T}_2)$ is a SoIFT on X. Then the intuitionistic fuzzy set B is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-open set which is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-regular open set. Also, B^c is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-closed set which is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-regular closed set.

THEOREM 3.5. (1) The intersection of two fuzzy (r, s)-regular open sets is a fuzzy (r, s)-regular open set.

(2) The union of two fuzzy (r, s)-regular closed sets is a fuzzy (r, s)-regular closed set.

Proof. (1) Let A and B be any two fuzzy (r,s)-regular open sets in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$. Then A and B are fuzzy (r,s)-open sets and hence $\mathcal{T}_1(A\cap B)\geq \mathcal{T}_1(A)\wedge\mathcal{T}_1(B)\geq r$ and $\mathcal{T}_2(A\cap B)\leq \mathcal{T}_2(A)\vee\mathcal{T}_2(B)\leq s$. Thus $A\cap B$ is a fuzzy (r,s)-open set. Since $A\cap B\subseteq \mathrm{cl}(A\cap B,r,s)$,

$$\mathrm{int}(\mathrm{cl}(A\cap B,r,s),r,s)\supseteq \mathrm{int}(A\cap B,r,s)=A\cap B.$$

Now, $A\cap B\subseteq A$ and $A\cap B\subseteq B$ imply

$$\mathrm{int}(\mathrm{cl}(A\cap B,r,s),r,s)\subseteq \mathrm{int}(\mathrm{cl}(A,r,s),r,s)=A$$

and

$$\mathrm{int}(\mathrm{cl}(A\cap B,r,s),r,s)\subseteq \mathrm{int}(\mathrm{cl}(B,r,s),r,s)=B.$$

Thus $\mathrm{int}(\mathrm{cl}(A\cap B,r,s),r,s)\subseteq A\cap B$. Therefore

$$\mathrm{int}(\mathrm{cl}(A\cap B,r,s),r,s)=A\cap B$$

and hence $A\cap B$ is a fuzzy (r,s)-regular open set.

(2) Let A and B be any two fuzzy (r,s)-regular closed sets in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$. Then A and B are fuzzy (r,s)-closed sets and hence A^c and B^c are fuzzy (r,s)-open sets. So, $\mathcal{T}_1((A\cup B)^c)=\mathcal{T}_1(A^c\cap B^c)\geq \mathcal{T}_1(A^c)\wedge\mathcal{T}_1(B^c)\geq r$ and $\mathcal{T}_2((A\cup B)^c=\mathcal{T}_2(A^c\cap B^c)\leq \mathcal{T}_2(A^c)\vee\mathcal{T}_2(B^c)\leq s$ and hence $(A\cup B)^c$ is a fuzzy (r,s)-open set. Thus $A\cup B$ is a fuzzy (r,s)-closed set. Since $\mathrm{int}(A\cup B,r,s)\subseteq A\cup B$,

$$\mathrm{cl}(\mathrm{int}(A\cup B,r,s),r,s)\supseteq \mathrm{cl}(A\cup B,r,s)=A\cup B.$$

Now, $A\cup B\supseteq A$ and $A\cup B\supseteq B$ imply

$$\mathrm{cl}(\mathrm{int}(A\cup B,r,s),r,s)\supseteq \mathrm{cl}(\mathrm{int}(A,r,s),r,s)=A$$

and

$$\mathrm{cl}(\mathrm{int}(A\cup B,r,s),r,s)\supseteq \mathrm{cl}(\mathrm{int}(B,r,s),r,s)=B.$$

Thus $\mathrm{cl}(\mathrm{int}(A\cup B,r,s),r,s)\subseteq A\cup B$. Therefore

$$\mathrm{cl}(\mathrm{int}(A\cup B,r,s),r,s)=A\cup B$$

and hence $A\cup B$ is a fuzzy (r,s)-regular closed set. □

THEOREM 3.6. (1) The fuzzy (r,s)-closure of a fuzzy (r,s)-open set is a fuzzy (r,s)-regular closed set.

(2) The fuzzy (r,s)-interior of a fuzzy (r,s)-closed set is a fuzzy (r,s)-regular open set.

Proof. (1) Let A be a fuzzy (r,s)-open set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$. Then clearly $\mathrm{int}(\mathrm{cl}(A,r,s),r,s)\subseteq \mathrm{cl}(A,r,s)$ implies that

$$\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A,r,s),r,s),r,s)\subseteq \mathrm{cl}(\mathrm{cl}(A,r,s),r,s)=\mathrm{cl}(A,r,s).$$

Since A is a fuzzy (r,s)-open set, $A = \text{int}(A,r,s)$. Also since $A \subseteq \text{cl}(A,r,s)$, $A = \text{int}(A,r,s) \subseteq \text{int}(\text{cl}(A,r,s),r,s)$. Thus $\text{cl}(A,r,s) \subseteq \text{cl}(\text{int}(\text{cl}(A,r,s),r,s),r,s)$. Therefore

$$\text{cl}(A,r,s) = \text{cl}(\text{int}(\text{cl}(A,r,s),r,s),r,s)$$

and hence $\text{cl}(A,r,s)$ is a fuzzy (r,s)-regular closed set.

(2) Let A be a fuzzy (r,s)-closed set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$. Then clearly $\text{cl}(\text{int}(A,r,s),r,s) \supseteq \text{int}(A,r,s)$ implies that

$$\text{int}(\text{cl}(\text{int}(A,r,s),r,s),r,s) \supseteq \text{int}(\text{int}(A,r,s),r,s) = \text{int}(A,r,s).$$

Since A is a fuzzy (r,s)-closed set, $A = \text{cl}(A,r,s)$. Also since $A \supseteq \text{int}(A,r,s)$, $A = \text{cl}(A,r,s) \supseteq \text{cl}(\text{int}(A,r,s),r,s)$. Thus $\text{int}(A,r,s) \subseteq \text{int}(\text{cl}(\text{int}(A,r,s),r,s),r,s)$. Therefore

$$\text{int}(A,r,s) = \text{int}(\text{cl}(\text{int}(A,r,s),r,s),r,s)$$

and hence $\text{int}(A,r,s)$ is a fuzzy (r,s)-regular open set. □

We are going to introduce the notions of fuzzy almost (r,s)-continuous mappings and investigate some of their properties.

Definition 3.7. Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is called

(1) a *fuzzy almost* (r,s)*-continuous* mapping if $f^{-1}(A)$ is a fuzzy (r,s)-open set of X for each fuzzy (r,s)-regular open set A of Y,
(2) a *fuzzy almost* (r,s)*-open* mapping if $f(B)$ is a fuzzy (r,s)-open set of Y for each fuzzy (r,s)-regular open set B of X,
(3) a *fuzzy almost* (r,s)*-closed* mapping if $f(B)$ is a fuzzy (r,s)-closed set of Y for each fuzzy (r,s)-regular closed set B of X.

Remark 3.8. Clearly a fuzzy (r,s)-continuous mapping is a fuzzy almost (r,s)-continuous mapping. That the converse need not be true is shown by the following example.

Example 3.9. Let $X = \{x,y\}$ and let B be an intuitionistic fuzzy set of X defined as

$$B(x) = (0.2, 0.7), \quad B(y) = (0.6, 0.1).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A),\mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2},\frac{1}{3}) & \text{if } A = B, \\ (0,1) & \text{otherwise,} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = B, \\ (0,1) & \text{otherwise.} \end{cases}$$

Clearly $(\mathcal{T}_1, \mathcal{T}_2)$ and $(\mathcal{U}_1, \mathcal{U}_2)$ are SoIFTs on X. Then the identity map $1_X : (X, \mathcal{T}_1, \mathcal{T}_2) \to (X, \mathcal{U}_1, \mathcal{U}_2)$ is a fuzzy almost $(\frac{1}{2}, \frac{1}{3})$-continuous mapping which is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-continuous mapping.

Theorem 3.10. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is a fuzzy almost (r, s)-continuous mapping.
(2) $f^{-1}(A) \subseteq \text{int}(f^{-1}(\text{int}(\text{cl}(A, r, s), r, s)), r, s)$ for each fuzzy (r, s)-open set A of Y.
(3) $\text{cl}(f^{-1}(\text{cl}(\text{int}(A, r, s), r, s)), r, s) \subseteq f^{-1}(A)$ for each fuzzy (r, s)-closed set A of Y.

Proof. (1) ⇒ (2) Let f be a fuzzy almost (r, s)-continuous mapping and A be a fuzzy (r, s)-opens set of Y. Since A is fuzzy (r, s)-open and $A \subseteq \text{cl}(A, r, s)$,

$$A = \text{int}(A, r, s) \subseteq \text{int}(\text{cl}(A, r, s), r, s).$$

By Theorem 3.6 (2), $\text{int}(\text{cl}(A, r, s), r, s)$ is a fuzzy (r, s)-regular open set of Y. Since f is fuzzy almost (r, s)-continuous, $f^{-1}(\text{int}(\text{cl}(A, r, s), r, s))$ is a fuzzy (r, s)-open set of X. Hence

$$\begin{aligned} f^{-1}(A) &\subseteq f^{-1}(\text{int}(\text{cl}(A, r, s), r, s)) \\ &= \text{int}(f^{-1}(\text{int}(\text{cl}(A, r, s), r, s)), r, s). \end{aligned}$$

(2) ⇒ (3) Let A be a fuzzy (r, s)-closed set of Y. Then A^c is a fuzzy (r, s)-open set of Y. By (2),

$$f^{-1}(A^c) \subseteq \text{int}(f^{-1}(\text{int}(\text{cl}(A^c, r, s), r, s)), r, s).$$

Hence

$$\begin{aligned} f^{-1}(A) &= (f^{-1}(A^c))^c \\ &\supseteq (\text{int}(f^{-1}(\text{int}(\text{cl}(A^c, r, s), r, s)), r, s)^c \\ &= \text{cl}(f^{-1}(\text{cl}(\text{int}(A, r, s), r, s)), r, s). \end{aligned}$$

(3) ⇒ (1) Let A be a fuzzy (r, s)-regular open set of Y. Then $A = \text{int}(\text{cl}(A, r, s), r, s)$. Since A^c is a fuzzy (r, s)-regular closed set of Y, A^c is a fuzzy (r, s)-closed set of Y. By (3),

$$\text{cl}(f^{-1}(\text{cl}(\text{int}(A^c, r, s), r, s)), r, s) \subseteq f^{-1}(A^c).$$

Hence

$$\begin{aligned} f^{-1}(A) &= (f^{-1}(A^c))^c \\ &\subseteq (\mathrm{cl}(f^{-1}(\mathrm{cl}(\mathrm{int}(A^c, r, s), r, s)), r, s))^c \\ &= \mathrm{int}(f^{-1}(\mathrm{int}(\mathrm{cl}(A, r, s), r, s)), r, s) \\ &= \mathrm{int}(f^{-1}(A), r, s) \\ &\subseteq f^{-1}(A). \end{aligned}$$

Thus $f^{-1}(A) = \mathrm{int}(f^{-1}(A), r, s)$ and hence $f^{-1}(A)$ is a fuzzy (r, s)-open set of X. Therefore f is a fuzzy almost (r, s)-continuous mapping. □

THEOREM 3.11. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \times I$. Then f is a fuzzy almost (r, s)-open mapping if and only if $(\mathrm{int}(B, r, s)) \subseteq \mathrm{int}(f(B), r, s)$ for each fuzzy (r, s)-semiclosed set B of X.

Proof. Let f be a fuzzy almost (r, s)-open mapping and B be a fuzzy (r, s)-semiclosed set of X. Then

$$\mathrm{int}(B, r, s) \subseteq \mathrm{int}(\mathrm{cl}(B, r, s), r, s) \subseteq B.$$

Note that $\mathrm{cl}(B, r, s)$ is a fuzzy (r, s)-closed set of X. By Theorem 3.6 (2), $\mathrm{int}(\mathrm{cl}(B, r, s), r, s)$ is a fuzzy (r, s)-regular open set of X. Since f is a fuzzy almost (r, s)-open mapping, $f(\mathrm{int}(\mathrm{cl}(B, r, s), r, s))$ is a fuzzy (r, s)-open set of Y. Thus we have

$$\begin{aligned} f(\mathrm{int}(B, r, s) &\subseteq f(\mathrm{int}(\mathrm{cl}(B, r, s), r, s)) \\ &= \mathrm{int}(f(\mathrm{int}(\mathrm{cl}(B, r, s), r, s)), r, s) \\ &\subseteq \mathrm{int}(f(B), r, s). \end{aligned}$$

Conversely, let B be a fuzzy (r, s)-regular open set of X. Then B is fuzzy (r, s)-open and hence $\mathrm{int}(B, r, s) = B$. Since $\mathrm{int}(\mathrm{cl}(B, r, s), r, s) = B$, B is a fuzzy (r, s)-semiclosed set of X. So

$$\begin{aligned} f(B) &= f(\mathrm{int}(B, r, s)) \\ &\subseteq \mathrm{int}(f(B) r, s) \\ &\subseteq f(B). \end{aligned}$$

Thus $f(B) = \mathrm{int}(f(B), r, s)$ and hence $f(B)$ is a fuzzy (r, s)-open set of Y. Therefore f is a almost (r, s)-open mapping. □

THEOREM 3.12. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a fuzzy (r, s)-semicontinuous mapping and a fuzzy almost (r, s)-open mapping. Then f is a fuzzy (r, s)-irresolute mapping.

Proof. Let A be a fuzzy (r,s)-semiclosed set of Y. Then $\mathrm{int}(\mathrm{cl}(A,r,s), r,s) \subseteq A$. Clearly, $\mathrm{cl}(A,r,s)$ is a fuzzy (r,s)-closed set of Y. Since f is a fuzzy (r,s)-semicontinuous mapping, $f^{-1}(\mathrm{cl}(A,r,s))$ is a fuzzy (r,s)-semiclosed set of X. So

$$\begin{aligned} f^{-1}(\mathrm{cl}(A,r,s)) &\supseteq \mathrm{int}(\mathrm{cl}(f^{-1}(\mathrm{cl}(A,r,s)),r,s),r,s) \\ &\supseteq \mathrm{int}(\mathrm{cl}(f^{-1}(A),r,s),r,s). \end{aligned}$$

Thus we have

$$\begin{aligned} \mathrm{int}(\mathrm{cl}(f^{-1}(A),r,s),r,s) &= \mathrm{int}(\mathrm{int}(\mathrm{cl}(f^{-1}(A),r,s),r,s),r,s) \\ &\subseteq \mathrm{int}(f^{-1}(\mathrm{cl}(A,r,s)),r,s). \end{aligned}$$

Note that A is a fuzzy (r,s)-semiclosed set of Y. Since f is a fuzzy almost (r,s)-open mapping and $f^{-1}(\mathrm{cl}(A,r,s))$ is a fuzzy (r,s)-semiclosed set of X,

$$\begin{aligned} f(\mathrm{int}(f^{-1}(\mathrm{cl}(A,r,s)),r,s)) &\subseteq \mathrm{int}(ff^{-1}(\mathrm{cl}(A,r,s)),r,s) \\ &\subseteq \mathrm{int}(\mathrm{cl}(A,r,s),r,s) \\ &\subseteq A. \end{aligned}$$

Hence we have

$$\begin{aligned} \mathrm{int}(\mathrm{cl}(f^{-1}(A),r,s),r,s) &\subseteq f^{-1}f(\mathrm{int}(\mathrm{cl}(f^{-1}(A),r,s),r,s)) \\ &\subseteq f^{-1}f(\mathrm{int}(f^{-1}(\mathrm{cl}(A,r,s)),r,s)) \\ &\subseteq f^{-1}(A). \end{aligned}$$

Thus $f^{-1}(A)$ is a fuzzy (r,s)-semiclosed set of X and hence f is a fuzzy (r,s)-irresolute mapping. □

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*
Department of Mathematics
Seonam University
Namwon 590-711, Republic of Korea
E-mail: eplee55@nate.com

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Department of Mathematics
Chungbuk National University
Cheongju 361-763, Republic of Korea
E-mail: solee@chungbuk.ac.kr