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ON SUBTRACTIVE EXTENSION OF SUBSEMIMODULES OF SEMIMODULES

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Abstract. Let R be a commutative semiring with $1_R \neq 0_R$. Characterization of subsemimodules, prime subsemimodules and primary subsemimodules which are subtractive extensions of Q-subsemimodules in R-semimodules are investigated.

1. Introduction

Let R be a commutative semiring with $1_R \neq 0_R$. A commutative monoid $(M, +)$ with a scalar multiplication $R \times M \rightarrow M$, defined by $(r, x) \mapsto rx$ is called left R-semimodule if it satisfies the following conditions for all $r, r^{'} \in R$ and $x, y \in M$:

1) $(rr^{'})x = r(r^{'}x)$;
2) $r(x + y) = rx + ry$;
3) $(r + r^{'})x = rx + r^{'}x$;
4) $1_R x = x$;
5) $r0_M = 0_M = 0_R x$.

Clearly every ring is a semiring and hence every left module over a ring R is a left semimodule over a semiring R. Throughout by an R-semimodule we mean a left R-semimodule. Denote the sets of all non-negative, and positive integers respectively by $\mathbb{Z}_0^+$, and $\mathbb{N}$. The set $\mathbb{Z}_0^+$ is a semiring under usual addition and multiplication of non-negative integers but it is not a ring. If $(M, +)$ is an idempotent commutative monoid, then M is $(\mathbb{Z}_0^+ ,+ , \cdot)$-semimodule with scalar multiplication defined by $rm = 0$ if $r = 0$ and $rm = m$ if $r > 0$ for all $r \in \mathbb{Z}_0^+$ and $m \in M$ [6 , P.151]. A non-empty subset N of an R-semimodule M is called

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subsemimodule of M if N is closed under addition and closed under scalar multiplication. A subsemimodule N of an R-semimodule M is called subtractive subsemimodule (= k-subsemimodule) if x, $x+y \in N$, $y \in M$, then $y \in N$. A proper subsemimodule N of an R-semimodule M is said to be prime (primary) if $rm \in N$, $r \in R, m \in M$, then either $rM \subseteq N$ or $m \in N$ ($r^n M \subseteq N$ for some $n \in \mathbb{N}$ or $m \in N$). A proper subsemimodule N of an R-semimodule M is said to be weakly prime (weakly primary) if $0 \neq rm \in N$, $r \in R, m \in M$, then either $rM \subseteq N$ or $m \in N$ ($r^n M \subseteq N$ for some $n \in \mathbb{N}$ or $m \in N$). Consider $M = (\mathbb{Z}_0^+, +)$ an R-semimodule where $R = (\mathbb{Z}_0^+, +, \cdot)$. For $m \in M$, we denote the subsemimodule $\{am \in M : a \in R\}$ of M by $m\mathbb{Z}_0^+$.

Example 1.1. Consider a semiring $R = (\mathbb{Z}_0^+, +, \cdot)$. Then

1) $9\mathbb{Z}_0^+$ is a primary subsemimodule of an R-semimodule $(\mathbb{Z}_0^+, +)$ but it is not a prime subsemimodule.
2) $4\mathbb{Z}_0^+$ is a weakly primary subsemimodule of an R-semimodule $(\mathbb{Z}_0^+, +)$ but it is not a weakly prime subsemimodule.
3) $6\mathbb{Z}_0^+$ is neither a prime, primary, weakly prime nor a weakly primary subsemimodule of an R-semimodule $(\mathbb{Z}_0^+, +)$.
4) $\{0\}$ is a weakly prime (weakly primary) subsemimodule of an R-semimodule $(\mathbb{Z}_6, +_6)$ but it is not a prime (primary)subsemimodule.

A subsemimodule N of an R-semimodule M is called a Q-subsemimodule (= partitioning subsemimodule) if there exists a subset Q of M such that

1) $M = \cup\{q + N : q \in Q\}$.
2) if q_1, $q_2 \in Q$, then $(q_1 + N) \cap (q_2 + N) \neq \emptyset \Leftrightarrow q_1 = q_2$.

Let N be a Q-subsemimodule of an R-semimodule M. Then $M/N_{(Q)} = \{q + N : q \in Q\}$ forms an R-semimodule under the following addition "$\oplus$"and scalar multiplication "$\odot$", $(q_1 + N) \oplus (q_2 + N) = q_3 + N$ where $q_3 \in Q$ is unique such that $q_1 + q_2 + N \subseteq q_3 + N$, and $r \odot (q_1 + N) = q_4 + N$ where $q_4 \in Q$ is unique such that $rq_1 + N \subseteq q_4 + N$. This R-semimodule $M/N_{(Q)}$ is called the quotient semimodule of M by N and denoted by $(M/N_{(Q)}, \oplus, \odot)$ or just $M/N_{(Q)}$. If N is Q-subsemimodule of an R-semimodule M, then there exists a unique $q_0 \in Q$ such that $q_0 + N = N$ [3, Lemma 2.3]. This $q_0 + N$ is the zero element of $M/N_{(Q)}$.

Chaudhari and Bonde [3], have introduced Q-subsemimodule and obtained a relation between subtractive subsemimodules and Q-subsemimodules. Some works on Q-subsemimodules, quotient semimodules, prime subsemimodules and maximal k-subsemimodules may be found in

[2, 8 and 9]. Theory of Q-subsemimodules, prime subsemimodules, primary subsemimodules, direct sum of subsemimodules is recently studied by Chaudhari and Bonde [4 and 5].

In this paper, we introduce the notion of subtractive extension of a subsemimodule of an R-semimodule M and prove its existence. Some characterizations of subtractive extension of Q-subsemimodule in R-semimodules are also obtained.

The following lemma will be used to prove our results.

LEMMA 1.2. [3, *Lemma*1.3] *Let N be subsemimodule of an R-semimodule M and $x, y \in M$ such that $x + N \subseteq y + N$. Then $x + z + N \subseteq y + z + N$ and $rx + N \subseteq ry + N$ for all $z \in M, r \in R$.*

2. Subtractive extension of a subsemimodule

In this section, we introduce the notion of subtractive extension of a subsemimodule of an R-semimodule M and prove its existence.

DEFINITION 2.1. Let N be a subsemimodule of an R-semimodule M. A subsemimodule A of M with $N \subseteq A$ is said to be subtractive extension of N if $x \in N, x + y \in A, y \in M$, then $y \in A$.

Every subtractive subsemimodule of an R-semimodule M containing a subsemimodule N is a subtractive extension of N. But the converse is not true.

EXAMPLE 2.2. Let $R = (\mathbb{Z}_0^+, +, \cdot)$ be a semiring.

1) Consider $M = (\mathbb{Z}_0^+ \cup \{\infty\}, max)$ an R-semimodule. Let $N = \{0, 1, 2, 3, 4, 5\}$ be a subsemimodule of M. Now $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, \infty\}$ is a subsemimodule of M with $N \subseteq A$. Clearly, if $x \in N, x + y \in A$ and $y \in M$, then $y \in A$. Hence A is a subtractive extension of N. But A is not a subtractive subsemimodule of M because $\infty \in A, \infty + 9 \in A$ and $9 \notin A$.
2) Consider $M = (\mathbb{Z}_0^+, +)$ an R-semimodule. For $m \in M$, we have $m\mathbb{Z}_0^+$ is a Q-subsemimodule of M where $Q = \{0, 1, 2, 3, ..., (m-1)\}$. By [2, Theorem 3.2], $m\mathbb{Z}_0^+$ is a subtractive subsemimodule of M. Hence $m\mathbb{Z}_0^+$ is a subtractive extension of every subsemimodule N of M where $N \subseteq m\mathbb{Z}_0^+$.

THEOREM 2.3. *Let N be a Q-subsemimodule of an R-semimodule M and A be a subsemimodule of M with $N \subseteq A$. Denote $\widetilde{A} = \{x \in M :$ there exists $q + N \in M/N_{(Q)}$ such that $x \in q + N$ and $(q +$*

$N) \cap A \neq \emptyset\}$. Then $\widetilde{A}$ is the smallest subtractive extension of N containing A.

Proof. We have $q_0 + N = N$ is the zero element of $M/N_{(Q)}$ and $N \subseteq A \Rightarrow q_0 \in (q_0 + N) \cap A \Rightarrow q_0 \in \widetilde{A} \Rightarrow \widetilde{A} \neq \emptyset$. Let $x, y \in \widetilde{A}, r \in R$. Then there exist unique $q_1, q_2 \in Q$ such that $x \in q_1 + N, y \in q_2 + N$ and $(q_1+N) \cap A \neq \emptyset, (q_2+N) \cap A \neq \emptyset$. Now $x+y \in (q_1+N) \oplus (q_2+N) = q+N$ where q is a unique element of Q such that $q_1 + q_2 + N \subseteq q + N$. Since $(q_1 + N) \cap A \neq \emptyset$ and $(q_2 + N) \cap A \neq \emptyset$, $(q + N) \cap A \neq \emptyset$. So $x + y \in \widetilde{A}$. Similarly $rx \in \widetilde{A}$. If $a \in A$, then there exists a unique $q \in Q$ such that $a \in q + N$. So $(q + N) \cap A \neq \emptyset$. Now $a \in \widetilde{A}$. Thus $\widetilde{A}$ is a subsemimodule of M containing A. Let $x \in N, x + y \in \widetilde{A}, y \in M$. So $x + y \in q' + N$ where $q' + N \in M/N_{(Q)}$ and $(q' + N) \cap A \neq \emptyset$. Since N is a Q-subsemimodule of M, there exists a unique $q'' \in Q$ such that $y \in q'' + N$. As $x \in N \Rightarrow x + y \in q'' + N$. So $(q' + N) \cap (q'' + N) \neq \emptyset \Rightarrow q' = q'' \Rightarrow q'' + N = q' + N \Rightarrow (q'' + N) \cap A = (q' + N) \cap A \neq \emptyset$. Hence $y \in \widetilde{A}$. Thus $\widetilde{A}$ is a subtractive extension of N. Now let B be any subtractive extension of N containing A. Now $a \in \widetilde{A} \Rightarrow a \in q''' + N$ and $(q''' + N) \cap A \neq \emptyset \Rightarrow (q''' + N) \cap B \neq \emptyset$. Let $b = q''' + n \in (q''' + N) \cap B$ for some $n \in N$. Since B is a subtractive extension of N, $q''' \in B$. So $a \in q''' + N \subseteq B$. Thus $\widetilde{A}$ is the smallest subtractive extension of N containing A. □

THEOREM 2.4. *Let N be a Q-subsemimodule of an R-semimodule M. If A, B are subsemimodules of M containing N, then*

1) *A is a subtractive extension of $N \Leftrightarrow A = \widetilde{A}$*
2) *$\widetilde{\widetilde{A}} = \widetilde{A}$*
3) *$A \subseteq B \Rightarrow \widetilde{A} \subseteq \widetilde{B}$*
4) *$A = B \Rightarrow \widetilde{A} = \widetilde{B}$.*

Proof. Trivial. □

In the above theorem, reverse implications are not true.

EXAMPLE 2.5. Consider $M = (\mathbb{Z}_0^+, +)$ a semimodule over $R = (\mathbb{Z}_0^+, +, \cdot)$.

1) Let $N = 8\mathbb{Z}_0^+$ be a Q-subsemimodule of M, $A = 4\mathbb{Z}_0^+, B = \{0, 6, 8, 10, ...\}$. Then A, B are subsemimodules of M with $N \subseteq A, B$ and $\widetilde{A} = 4\mathbb{Z}_0^+ \subseteq 2\mathbb{Z}_0^+ = \widetilde{B}$ but A is not a subset of B.

2) Let $N = 6\mathbb{Z}_0^+$ be a Q-subsemimodule of M, $A = 2\mathbb{Z}_0^+, B = \{0, 6, 8, 10, ...\}$. Then A, B are subsemimodules of M with $A \neq B$ and $N \subseteq A, B$. Here $\widetilde{A} = 2\mathbb{Z}_0^+ = \widetilde{B}$.

Chaudhari and Bonde [3, Lemma 3.4], proved that if A, N are subsemimodules of an R-semimodule M with N a Q-subsemimodule and $N \subseteq A$, then N is a $Q \cap A$-subsemimodule of A. However, we prove the following lemma which gives a characterization of a subtractive extension of a Q-subsemimodule N of an R-semimodule M and will be used in the subsequent theorem.

Lemma 2.6. *Let N be a Q-subsemimodule of an R-semimodule M and A be a subsemimodule of M with $N \subseteq A$. Then A is a subtractive extension of N if and only if N is a $Q \cap A$-subsemimodule of A.*

Proof. Let $q_0 + N$ be the zero element of $M/N_{(Q)}$. Let A be a subtractive extension of N and $a \in A$. Then there exists unique $q \in Q$ such that $a \in q + N$. So $a = q + n$ for some $n \in N$. Since A is a subtractive extension of N, $q \in A$. Hence $q \in Q \cap A$. If $(q_1 + N) \cap (q_2 + N) \neq \emptyset$ for some $q_1, q_2 \in Q \cap A$, then $q_1 = q_2$ because N is a Q-subsemimodule of M. Thus N is a $Q \cap A$-subsemimodule of A. Conversely suppose that N is a $Q \cap A$-subsemimodule of A. Let $x \in N, x + y \in A$ for some $y \in M$. Since N is a $Q \cap A$-subsemimodule of A and N a Q-subsemimodule of M there exist unique $q_1 \in Q \cap A, q_2 \in Q$ such that $x + y \in q_1 + N$ and $y \in q_2 + N$. But then $x + y \in (q_0 + N) \oplus (q_2 + N) = q_2 + N$, since $x \in N = q_0 + N$. So $(q_1 + N) \cap (q_2 + N) \neq \emptyset$. Hence $q_2 = q_1 \in A$. Now $y \in q_2 + N \subseteq A$. □

Theorem 2.7. *Let N be a Q-subsemimodule of an R-semimodule M and A a subsemimodule of M. Then the following statements are equivalent:*

1) *A is a subtractive extension of N*
2) *$A/N_{(Q \cap A)}$ is a subsemimodule of an R-semimodule $M/N_{(Q)}$*
3) *$A/N_{(Q \cap A)} \subseteq M/N_{(Q)}$*

Proof. (1) ⇒ (2) Let A be a subtractive extension of N. By Lemma 2.6, N is a $Q \cap A$-subsemimodule of A. Let $q_1 + N, q_2 + N \in A/N_{(Q \cap A)}$. Therefore $q_1, q_2 \in Q \cap A \subseteq Q$. Hence there exists a unique $q_3 \in Q$ such that $(q_1 + N) \oplus (q_2 + N) = q_3 + N$ where $q_1 + q_2 + N \subseteq q_3 + N$. Now $q_1 + q_2 = q_3 + n$ for some $n \in N$. But $q_3 + n = q_1 + q_2 \in A$. Since A is a subtractive extension of N, $q_3 \in A$. Now $q_3 + N \in A/N_{(Q \cap A)}$. Similarly, if $r \in R, q + N \in A/N_{(Q \cap A)}$, then $r \odot (q + N) \in A/N_{(Q \cap A)}$. Hence $A/N_{(Q \cap A)}$ is a subsemimodule of $M/N_{(Q)}$.

(2) $\Rightarrow$ (3) Trivial.
(3) $\Rightarrow$ (1) Let $x \in N, x+y \in A$ where $y \in M$. Then $x \in N = q_0 + N$ where $q_0 + N$ is the zero element of $M/N_{(Q)}$ and hence zero element of $A/N_{(Q\cap A)}$. Now there exist unique $q_1 \in Q \cap A, q_2 \in Q$ such that $x+y \in q_1 + N$ and $y \in q_2 + N$. But then $x + y \in (q_0 + N) \oplus (q_2 + N) = q_2 + N$. So $q_1, q_2 \in Q$ are such that $(q_1 + N) \cap (q_2 + N) \neq \emptyset$. Hence $q_1 = q_2 \in A$. So $y \in q_2 + N \subseteq A$. □

If N is a Q-subsemimodule of an R-semimodule M, then following theorem gives a relationship between subsemimodules of the quotient semimodule $M/N_{(Q)}$ and subsemimodules of M which are subtractive extensions of N.

THEOREM 2.8. *Let N be a Q-subsemimodule of an R-semimodule M. Then a subset L of $M/N_{(Q)}$ is a subsemimodule of $M/N_{(Q)}$ if and only if there exists a subsemimodule A of M such that A is a subtractive extension of N and $A/N_{(Q\cap A)} = L$.*

Proof. Let L be a subsemimodule of an R-semimodule $M/N_{(Q)}$. Denote $A = \{x \in M : there \ exists \ a \ unique \ q \in Q \ such \ that \ x + N \subseteq q + N \in L\}$, clearly $N \subseteq A$. Let $x, y \in A, r \in R$. Then there exist unique $q_1, q_2 \in Q$ such that $x + N \subseteq q_1 + N \in L, y + N \subseteq q_2 + N \in L$. Again there exist unique $q_3, q_4 \in Q$ such that $(q_1 + N) \oplus (q_2 + N) = q_3 + N \in L$ and $r \odot (q_1 + I) = q_4 + N \in L$ where $q_1 + q_2 + N \subseteq q_3 + N$ and $rq_1 + N \subseteq q_4 + N$. By Lemma 1.2, $x+y \in x+y+N \subseteq q_1+q_2+N \subseteq q_3+N$ and $rx \in rx + N \subseteq rq_1 + N \subseteq q_4 + N$. So $x + y, rx \in A$. Hence A is a subsemimodule of M with $N \subseteq A$. Now let $x \in N, x+y \in A, y \in M$. So there exists a unique $q \in Q$ such that $x+y \in q+N \in L$. Since N is a Q-subsemimodule of M, there exists a unique $q' \in Q$ such that $y \in q' + N$. Since $x \in N, x + y \in q' + N$. So $(q + N) \cap (q' + N) \neq \emptyset \Rightarrow q = q'$. Now $y \in q' + N = q + N \in L$. Thus $y \in A$. Hence A is a subtractive extension of N. Clearly $A/N_{(Q\cap A)} \subseteq L$. Now if $q + N \in L$, then $q \in A$. So $L \subseteq A/N_{(Q\cap A)}$. Thus$A/N_{(Q\cap A)} = L$. Conversely, suppose that A is a subtractive extension of N and $A/N_{(Q\cap A)} = L$. Then by Theorem 2.7, L is a subsemimodule of $M/N_{(Q)}$. □

COROLLARY 2.9. *Let N be a Q-subsemimodule of an R-semimodule M and let T, L be subtractive extensions of N. Then $T/N_Q = L/N_Q$ if and only if $T = L$.*

Proof. Let $T/N_Q = L/N_Q$ and $a \in T$. Then $a \in q_1 + N$ for some unique $q_1 \in Q$. So $a = q_1 + c$ for some unique $c \in N$. Since T is a subtractive extension of N, $q_1 \in T$. Thus $q_1 + N \subseteq T/N_Q = L/N_Q$.

It follows that $q_1 \in L$. Now $a = q_1 + c \in L$. Hence $T \subseteq L$. Similarly $L \subseteq T$. Converse is trivial. $\square$

The following theorem gives a relationship between prime subsemimodules of M which are subtractive extensions of a Q-subsemimodule N of M and prime subsemimodules of the quotient semimodule $M/N_{(Q)}$.

THEOREM 2.10. *Let N be a Q-subsemimodule of an R-semimodule M and P be a subtractive extension of N. Then P is a prime subsemimodule of M if and only if $P/N_{(Q\cap P)}$ is a prime subsemimodule of $M/N_{(Q)}$.*

Proof. Let P be a prime subsemimodule of M. Suppose that $r \in R, q_1 + N \in M/N_{(Q)}$ are such that $r \odot (q_1 + N) = q_2 + N \in P/N_{(Q\cap P)}$ where $q_2 \in Q \cap P$ is unique such that $rq_1 + N \subseteq q_2 + N$. Therefore $rq_1 = q_2 + n \in P$ for some $n \in N \subseteq P$. As P is a prime subsemimodule, either $rM \subseteq P$ or $q_1 \in P$. If $q_1 \in P$, then $q_1 \in Q \cap P$ and hence $q_1 + N \in P/N_{(Q\cap P)}$. Suppose that $rM \subseteq P$. For $q + N \in M/N_{(Q)}$, let $r \odot (q + N) = q_3 + N$ where q_3 is a unique element of Q such that $rq + N \subseteq q_3 + N$. Therefore $rq = q_3 + n' \in P$ for some $n' \in N$, since P is a subtractive extension of $N, q_3 \in P$. So $q_3 \in Q \cap P$. Now $r \odot (q + N) = q_3 + N \in P/N_{(Q\cap P)}$ and hence $r \odot M/N_{(Q)} \subseteq P/N_{(Q\cap P)}$. So $P/N_{(Q\cap P)}$ is a prime subsemimodule of $M/N_{(Q)}$. Conversely suppose that $P/N_{(Q\cap P)}$ is a prime subsemimodule of $M/N_{(Q)}$. Let $rm \in P$ where $r \in R, m \in M$. Now there exists a unique $q_1 \in Q$ such that $m \in q_1 + N$ and $rm \in r \odot (q_1 + N) = q_2 + N$ where q_2 is a unique element of Q such that $rq_1 + N \subseteq q_2 + N$. Now $rm \in P, rm \in q_2 + N$ implies $q_2 \in P$, as P is a subtractive extension of N. Hence $r \odot (q_1 + N) = q_2 + N \in P/N_{(Q\cap P)}$. As $P/N_{(Q\cap P)}$ is a prime subsemimodule, either $r \odot M/N_{(Q)} \subseteq P/N_{(Q\cap P)}$ or $q_1 + N \in P/N_{(Q\cap P)}$. If $q_1 + N \in P/N_{(Q\cap P)}$, then $q_1 \in P$. Hence $m \in q_1 + N \subseteq P$. So assume that $r \odot M/N_{(Q)} \subseteq P/N_{(Q\cap P)}$. For $x \in M$, there exists a unique $q_3 \in Q$ such that $x \in q_3 + N$ and $rx \in r \odot (q_3 + N) = q_4 + N$ where q_4 is a unique element of Q such that $rq_3 + N \subseteq q_4 + N$. Now $q_4 + N = r \odot (q_3 + N) \in P/N_{(Q\cap P)} \Rightarrow q_4 \in P$. Now $rx \in q_4 + N \subseteq P$. So $rM \subseteq P$. Hence P is a prime subsemimodule of M. $\square$

Adopting the proof of Theorem 2.10, we have the following theorems.

THEOREM 2.11. *Let N be a Q-subsemimodule of an R-semimodule M and P a subtractive extension of N. Then P is a primary subsemimodule of M if and only if $P/N_{(Q\cap P)}$ is a primary subsemimodule of $M/N_{(Q)}$.*

THEOREM 2.12. *Let N be a Q-subsemimodule of an R-semimodule M and P be a subtractive extension of N. Then*

1) *If P is a weakly prime (weakly primary) subsemimodule of M, then $P/N_{(Q\cap P)}$ is a weakly prime (weakly primary) subsemimodule of $M/N_{(Q)}$.*
2) *If N and $P/N_{(Q\cap P)}$ are weakly prime (weakly primary) subsemimodules of M and $M/N_{(Q)}$ respectively, then P is a weakly prime (weakly primary) subsemimodule of M.*

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