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Noninformative Priors for the Ratio of the Scale Parameters in the Inverted Exponential Distributions

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Abstract

In this paper, we develop the noninformative priors for the ratio of the scale parameters in the inverted exponential distributions. The first and second order matching priors, the reference prior and Jeffreys prior are developed. It turns out that the second order matching prior matches the alternative coverage probabilities, is a cumulative distribution function matching prior and is a highest posterior density matching prior. In addition, the reference prior and Jeffreys' prior are the second order matching prior. We show that the proposed reference prior matches the target coverage probabilities in a frequentist sense through a simulation study as well as provide an example based on real data is given.

Keywords: Inverted exponential distribution, matching prior, reference prior, scale parameter.

1. Introduction

Exponential distribution is the most exploited distribution for lifetime data analysis. However, its suitability is restricted to a constant hazard rate, which is difficult to justify in many practical problems. This leads to the development of alternative models for lifetime data. A number of distributions such as Weibull and gamma have been extensively used to analyze lifetime data such as situations where the hazard rate is monotonically increasing or decreasing; however, the non-monotonicity of the hazard rate has also been observed in many situations. For example, the hazard rate initially increases with time and reaches a peak after some finite period of time and then declines slowly in some studies of mortality associated with particular diseases. Thus, the need to analyze such data whose hazard rate is non-monotonic was realized and suitable models were proposed. Killer and Kamath (1982), Lin *et al.* (1989) and Dey (2007) advocated the use of inverted exponential distribution as an appropriate model for this situation. Recently Singh *et al.* (2012) proposed Bayes estimators for the parameter and reliability function of inverted exponential distribution under the general entropy loss function for complete, type I and type II censored samples. Abouammoh and Alshingiti (2009) introduced a generalized version of inverted exponential distribution, and discussed the statistical and reliability properties of the distribution.

The problem of comparison of two scale parameters in the inverted exponential distributions has not been considered yet. The ratio of the scale parameters in two inverted exponential distributions has an important meaning. If this ratio is equal to one, then the hazard rate function at time t of these two distributions is identical; therefore, we focus on developing noninformative priors for Bayesian inference of the ratio of two scale parameters.

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There are two different notions of noninformative priors. One is a probability matching prior introduced by Welch and Peers (1963) which matches the coverage probability of Bayesian credible intervals with the corresponding frequentist coverage probability. Interest in such priors has been revived with the work of Stein (1985) and Tibshirani (1989). Among others, we may cite the work of Mukerjee and Dey (1993), DiCiccio and Stern (1994), Datta and Ghosh (1995, 1996), Mukerjee and Ghosh (1997). The other is the reference prior introduced by Bernardo (1979) which maximizes the Kullback-Leibler divergence between the prior and the posterior. Ghosh and Mukerjee (1992), and Berger and Bernardo (1989, 1992) give a general algorithm to derive a reference prior by splitting the parameters into several groups according to their order of inferential importance. Quite often reference priors satisfy the matching criterion described earlier. This approach is very successful in various practical problems (Kang, 2011; Kang *et al.*, 2012).

The outline of the remaining sections is as follows. In Section 2, we develop first order and second order probability matching priors. We reveal that the second order matching prior is a highest posterior density (HPD) matching prior, is a cumulative distribution function (CDF) matching prior and matches the alternative coverage probabilities up to the second order. In addition, we derive the reference prior and Jeffreys prior. It turns out that the reference prior and Jeffreys prior are the second order matching prior. Section 3 devotes to show that the propriety of the posterior distribution for the general prior including the reference prior. In Section 4, we compute the frequentist coverage probabilities under the proposed prior though simulation study and an example is given.

2. The Noninformative Priors

Suppose that X and Y are independently distributed random variables according to the inverted exponential $\mathcal{IE}(\lambda_1)$ with the scale parameter λ_1, and the inverted exponential distribution $\mathcal{IE}(\lambda_2)$ with the scale parameter λ_2. The probability density functions of the inverted exponential distributions with scale parameter λ is given by

$$f(x|\lambda) = \frac{1}{\lambda x^2} \exp\left\{-\frac{1}{x\lambda}\right\}, \quad x > 0,\ \lambda > 0. \tag{2.1}$$

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $\mathcal{IE}(\lambda_1)$ and $Y_1, Y_2, \ldots, Y_m$ be a random sample of size m from $\mathcal{IE}(\lambda_2)$, respectively. We write $\mathbf{X} = (X_1, \ldots, X_n)$ and $\mathbf{Y} = (Y_1, \ldots, Y_m)$. Then we want to develop the noninformative priors of the ratio of scale parameters.

2.1. The Probability Matching Priors

Suppose that θ_1 is the parameter of interest and $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_t)^T$ is the unknown parameter. For a prior π, let $\theta_1^{1-\alpha}(\pi, \mathbf{X}, \mathbf{Y})$ denote the $(1-\alpha)^{th}$ posterior quantile of θ_1. We are interested in finding priors π for which

$$P\left[\theta_1 \le \theta_1^{1-\alpha}(\pi, \mathbf{X}, \mathbf{Y})|\boldsymbol{\theta}\right] = 1 - \alpha + o\left(n^{-r}\right). \tag{2.2}$$

for some $r > 0$, as n goes to infinity. Priors π satisfying (2.2) are called matching priors. If $r = 1/2$, then π is referred to as a first order matching prior, while if $r = 1$, π is referred to as a second order matching prior.

In order to find such matching priors π, let

$$\theta_1 = \frac{\lambda_2}{\lambda_1} \quad \text{and} \quad \theta_2 = \lambda_1^n \lambda_2^m.$$

With this parametrization, the likelihood function of parameters (θ_1, θ_2) is given by

$$L(\theta_1, \theta_2) \propto \theta_2^{-1} \exp\left\{-\theta_1^{\frac{m}{n+m}} \theta_2^{-\frac{1}{n+m}} \sum_{i=1}^{n} \frac{1}{x_i} - \theta_1^{-\frac{n}{n+m}} \theta_2^{-\frac{1}{n+m}} \sum_{i=1}^{m} \frac{1}{y_i}\right\}. \tag{2.3}$$

Based on (2.3), the Fisher information matrix is given by

$$\mathbf{I}(\theta_1, \theta_2) = \begin{pmatrix} \frac{nm}{n+m}\theta_1^{-2} & 0 \\ 0 & \frac{1}{n+m}\theta_2^{-2} \end{pmatrix}. \tag{2.4}$$

From the above Fisher information matrix $\mathbf{I}$, θ_1 is orthogonal to θ_2 in the sense of Cox and Reid (1987). Following Tibshirani (1989), the class of first order probability matching prior is characterized by

$$\pi_m^{(1)}(\theta_1, \theta_2) \propto \theta_1^{-1} d(\theta_2), \tag{2.5}$$

where $d(\theta_2) > 0$ is an arbitrary function differentiable in its argument.

The class of prior given in (2.5) can be narrowed down to the second order probability matching priors as given in Mukerjee and Ghosh (1997). A second order probability matching prior is of the form (2.5), and d must satisfy an additional differential equation (2.10) of Mukerjee and Ghosh (1997), that is

$$\frac{1}{6} d(\theta_2) \frac{\partial}{\partial \theta_1}\left\{I_{11}^{-\frac{3}{2}} L_{1,1,1}\right\} + \frac{\partial}{\partial \theta_2}\left\{I_{11}^{-\frac{1}{2}} L_{112} I^{22} d(\theta_2)\right\} = 0, \tag{2.6}$$

where

$$L_{1,1,1} = E\left[\left(\frac{\partial \log L}{\partial \theta_1}\right)^3\right] = \frac{2nm(n-m)}{(n+m)^2}\theta_1^{-3}, \qquad L_{112} = E\left[\frac{\partial^3 \log L}{\partial \theta_1^2 \partial \theta_2}\right] = \frac{2n^2}{(n+m)^3}\theta_1^{-2}\theta_2^{-1},$$

$$I_{11} = \frac{nm}{n+m}\theta_1^{-2}, \qquad I^{22} = (n+m)\theta_2^2.$$

Then (2.6) simplifies to

$$\frac{\partial}{\partial \theta_2}\left\{\frac{2n^2(nm)^{\frac{1}{2}}}{(n+m)^{\frac{3}{2}}}\theta_1^{-1}\theta_2 d(\theta_2)\right\} = 0. \tag{2.7}$$

Hence, a solution of (2.7) is of the form $d(\theta_2) = \theta_2^{-1}$. Thus, the resulting second order probability matching prior is

$$\pi_m^{(2)}(\theta_1, \theta_2) \propto \theta_1^{-1}\theta_2^{-1}. \tag{2.8}$$

There are alternative ways through which matching can be accomplished. Datta *et al.* (2000) provided a theorem which establishes the equivalence of second order matching priors and HPD matching priors (DiCiccio and Stern, 1994; Ghosh and Mukerjee, 1995) within the class of first order matching priors. The equivalence condition is that $I_{11}^{-3/2} L_{111}$ dose not depend on θ_1. Since

$$L_{111} = E\left[\frac{\partial^3 \log L}{\partial \theta_1^3}\right] = \frac{2nm(2n+m)}{(n+m)^2}\theta_1^{-3},$$

$I_{11}^{-3/2}L_{111}$ does not depend on θ_1. Therefore, the second order probability matching prior (2.8) is a HPD matching prior.

Mukerjee and Ghosh (1997) gave conditions that the second order matching prior satisfies a CDF matching. Since $\pi_m^{(2)}(\theta_1, \theta_2) \propto \theta_1^{-1}\theta_2^{-1}$, $I^{11} = \{(n+m)/nm\}\theta_1^2$ and $L_{122} = E[\partial^3 \log L/\partial\theta_1\partial\theta_2^2] = 0$, their conditions can be checked by the following equations.

$$\Delta_3 = \frac{\partial^2}{\partial\theta_1^2}\left\{I^{11}\pi_m^{(2)}(\theta)\right\} - 2\frac{\partial}{\partial\theta_1}\left\{I^{11}\frac{\partial}{\partial\theta_1}\pi_m^{(2)}(\theta)\right\} - \frac{\partial}{\partial\theta_2}\left\{L_{112}I^{11}I^{22}\pi_m^{(2)}(\theta)\right\} - \frac{\partial}{\partial\theta_1}\left\{L_{122}I^{11}I^{22}\pi_m^{(2)}(\theta)\right\} = 0,$$

$$\Delta_4 = \frac{\partial}{\partial\theta_1}\left\{L_{111}(I^{11})^2\pi_m^{(2)}(\theta)\right\} = 0.$$

Therefore, the second order matching prior is a CDF matching prior.

Now, we want to know whether the second order matching prior is the alternative coverage probabilities matching prior of Mukerjee and Reid (1999) up to second order or not. Since

$$L_{11,1} = E\left[\frac{\partial^2 \log L}{\partial\theta_1^2}\frac{\partial \log L}{\partial\theta_1}\right] = -\frac{2n^2m}{(n+m)^2}\theta_1^{-3},$$

$$L_{11,2} = E\left[\frac{\partial^2 \log L}{\partial\theta_1^2}\frac{\partial \log L}{\partial\theta_2}\right] = -\frac{nm}{(n+m)^2}\theta_1^{-2}\theta_2^{-1}$$

and $d(\theta_2) = \theta_2^{-1}$. Then

$$\frac{\partial}{\partial\theta_2}\left\{L_{112}I^{22}I_{11}^{-\frac{1}{2}}d(\theta_2)\right\} = 0, \qquad \frac{\partial}{\partial\theta_2}\left\{L_{11,2}I^{22}I_{11}^{-\frac{1}{2}}d(\theta_2)\right\} = 0,$$

$$\frac{\partial}{\partial\theta_1}\left\{I_{11}^{-\frac{3}{2}}L_{111}\right\} = 0, \qquad \frac{\partial}{\partial\theta_1}\left\{I_{11}^{-\frac{3}{2}}L_{11,1}\right\} = 0,$$

therefore the second order matching prior (2.8) matches the alternative coverage probabilities (Mukerjee and Reid, 1999).

2.2. The Reference Priors

Reference priors introduced by Bernardo (1979), and extended further by Berger and Bernardo (1992) have become very popular over the years for the development of noninformative priors. In this section, we derive the reference priors for different groups of ordering of (θ_1, θ_2).

Due to the orthogonality of the parameters, the reference priors can be found easily by Datta and Ghosh (1995). If θ_1 is the parameter of interest, then the reference prior for group of ordering of $\{(\theta_1, \theta_2)\}$ is

$$\pi_1(\theta_1, \theta_2) \propto \theta_1^{-1}\theta_2^{-1}.$$

For group of ordering of $\{\theta_1, \theta_2\}$, the reference prior is

$$\pi_2(\theta_1, \theta_2) \propto \theta_1^{-1}\theta_2^{-1}.$$

From the above results, we know that Jeffreys prior π_1 and the reference prior π_2 are the second order matching prior and are the same.

3. Propriety of the Posterior Distributions

We investigate the propriety of posteriors for a general class of priors which includes the reference prior and the matching prior. We consider the class of priors

$$\pi(\theta_1, \theta_2) \propto \theta_1^{-a}\theta_2^{-b}, \tag{3.1}$$

where $a > 0$ and $b > 0$. The following general theorem can be proved.

Theorem 1. *The posterior distribution of (θ_1, θ_2) under the general prior (3.1) is proper if $bn-a+1 > 0$ and $bm + a - 1 > 0$.*

Proof: Note that the joint posterior for θ_1 and θ_2 given $\mathbf{x}$ and $\mathbf{y}$ is

$$\pi(\theta_1, \theta_2|\mathbf{x}, \mathbf{y}) \propto \theta_1^{-a}\theta_2^{-b-1} \exp\left\{-\theta_1^{\frac{m}{n+m}}\theta_2^{\frac{-1}{n+m}}\sum_{i=1}^{n} x_i^{-1} - \theta_1^{\frac{-n}{n+m}}\theta_2^{\frac{-1}{n+m}}\sum_{i=1}^{m} y_i^{-1}\right\}. \tag{3.2}$$

Then integrating with respect to θ_2 in (3.2), we can get

$$\pi(\theta_1|\mathbf{x}, \mathbf{y}) \propto \theta_1^{-bm-a}\left[\sum_{i=1}^{n} x_i^{-1} + \theta_1^{-1}\sum_{i=1}^{m} y_i^{-1}\right]^{-b(n+m)}. \tag{3.3}$$

Therefore

$$\int_0^\infty \pi(\theta_1|\mathbf{x}, \mathbf{y})d\theta_1 \propto \int_0^\infty \theta_1^{bn-a}\left(\theta_1 + \frac{\sum_{j=1}^{m} y_j^{-1}}{\sum_{i=1}^{n} x_i^{-1}}\right)^{-b(n+m)} d\theta_1. \tag{3.4}$$

Letting $z = \theta_1/(\theta_1 + k)$, where $k = \sum_{j=1}^{n_2} y_j / \sum_{i=1}^{n_1} x_i$, the above integration results in beta function. Thus the integration (3.4) is finite if $bn - a + 1 > 0$ and $bm + a - 1 > 0$. This completes the proof. □

Theorem 2. *Under the general prior (3.1), the marginal posterior density of θ_1 is given by*

$$\pi(\theta_1|\mathbf{x}, \mathbf{y}) = \frac{\Gamma[b(n+m)]}{\Gamma[bn-a+1]\Gamma[bm+a-1]}\left(\sum_{i=1}^{n} x_i^{-1}\right)^{bn-a+1}\left(\sum_{i=1}^{m} y_i^{-1}\right)^{bm+a-1}$$
$$\times\, \theta_1^{-bm-a}\left[\sum_{i=1}^{n} x_i^{-1} + \theta_1^{-1}\sum_{i=1}^{m} y_i^{-1}\right]^{-b(n+m)}. \tag{3.5}$$

The marginal posterior density (3.5) of θ_1 is a beta distribution with parameter $bn-a+1$ and $bm+a-1$. Therefore we have the marginal posterior density of θ_1, and so we can make Bayesian inference for θ_1 using (3.5).

4. Numerical Studies

We evaluate the frequentist coverage probabilities by investigating the credible interval of the marginal posterior density of θ_1 under the reference prior given in Section 3 for several configurations of (λ_1, λ_2) and (n, m). That is to say, the frequentist coverage of a α^{th} posterior quantile should be close to α. Table 4.1 provides numerical values of the frequentist coverage probabilities of 0.05 and 0.95 posterior

Table 1: Frequentist coverage probability of 0.05 and 0.95 posterior quantiles of θ_1

λ_1	λ_2	n	m	π_r	
				0.05	0.95
0.1	0.1	5	5	0.048	0.953
		5	10	0.051	0.950
		10	10	0.050	0.947
		10	20	0.052	0.949
	1.0	5	5	0.051	0.957
		5	10	0.049	0.945
		10	10	0.051	0.947
		10	20	0.055	0.949
	10.0	5	5	0.049	0.951
		5	10	0.052	0.952
		10	10	0.052	0.946
		10	20	0.051	0.954
1.0	0.1	5	5	0.049	0.945
		5	10	0.047	0.949
		10	10	0.054	0.945
		10	20	0.049	0.948
	1.0	5	5	0.052	0.951
		5	10	0.047	0.948
		10	10	0.048	0.948
		10	20	0.053	0.949
	10.0	5	5	0.052	0.950
		5	10	0.052	0.946
		10	10	0.050	0.952
		10	20	0.050	0.951
10.0	0.1	5	5	0.049	0.955
		5	10	0.052	0.950
		10	10	0.048	0.950
		10	20	0.052	0.949
	1.0	5	5	0.050	0.951
		5	10	0.050	0.951
		10	10	0.047	0.947
		10	20	0.053	0.947
	10.0	5	5	0.052	0.954
		5	10	0.053	0.948
		10	10	0.045	0.948
		10	20	0.050	0.952

quantiles for the reference prior. The computation of these numerical values is based on the following algorithm for any fixed true (λ_1, λ_2) and any prespecified and probability value α. Let $\theta_1^\alpha(\pi)$ be the posterior α^{th} quantile of θ_1 under the prior π given $\mathbf{X}$ and $\mathbf{Y}$. That is, $F(\theta_1^\alpha(\pi)|\mathbf{X},\mathbf{Y}) = \alpha$, where $F(\cdot|\mathbf{X},\mathbf{Y})$ is the marginal posterior distribution of θ_1. Then the frequentist coverage probability of this one sided credible interval of θ_1 is

$$P(\alpha; \theta_1, \theta_2) = P(\theta_1 \le \theta_1^\alpha(\pi)|\theta_1, \theta_2). \tag{4.1}$$

The computed $P(\alpha; \theta_1, \theta_2)$ when $\alpha = 0.05(0.95)$ is shown in Table 1. In particular, for fixed (n, m) and (λ_1, λ_2), we take 10,000 independent random samples of $\mathbf{X} = (X_1, \ldots, X_n)$ and $\mathbf{Y} = (Y_1, \ldots, Y_m)$ from the inverted exponential distributions, respectively.

In Table 1, we can observe that the reference prior meets very well the target coverage probabilities even if the small sample sizes. Note that the results of table are not too sensitive to the change of the values of (λ_1, λ_2); therefore, we recommend to use the reference prior.

Example 1. This example is taken from Singh *et al.* (2012). The data was originally reported by Bjerkedal (1960), and it represents the survival time (in days) of guinea pigs injected with different doses of tubercle bacilli. The regimen number is the common logarithm of the number of bacillary units in 0.5ml of challenge solution; that is, regimen 6.6 corresponds to 4.0×10^6 bacillary units per 0.5ml ($\ln(4.0 \times 10^6) = 6.6$). Corresponding to regimen 6.6, 72 observations are listed below. For estimation of ratio of scale parameters, we randomly divided this data into two groups. The data sets are given by

Group 1: 15, 22, 24, 32, 38, 44, 52, 53, 54, 55, 60, 60, 61, 65, 67, 68, 70, 70, 76, 81, 83, 84, 91, 96, 99, 110, 127, 143, 146, 146, 175, 211, 233, 263, 341, 376.

Group 2: 12, 24, 32, 33, 34, 38, 43, 48, 54, 56, 57, 58, 58, 59, 60, 60, 62, 63, 65, 72, 73, 75, 76, 85, 87, 95, 98, 109, 121, 129, 131, 175, 258, 258, 297, 341.

For this data, the maximum likelihood estimate (MLE) of θ_1 is 1.0837 and the corresponding 95% confidence interval of θ_1 is (0.6803, 1.7264). Bayes estimate and the 95% credible interval based on the reference prior are 1.1147 and (0.6803, 1.7264), respectively. The estimates and the intervals for ratio of scale parameters based on the MLE and the reference prior give almost the same results.

5. Concluding Remarks

In the inverted exponential models, we have found the second order matching prior, the reference prior and Jeffreys prior for the ratio of the scale parameters. We revealed the relevance of the second order matching prior and other matching criteria. It turns out that the reference prior and Jeffreys prior are the second order matching prior. As illustrated in our numerical study, the reference prior meets very well the target coverage probabilities, even if the sample size is small. Therefore we recommend the use of the reference prior for Bayesian inference of the ratio of the scale parameters in two independent inverted exponential distributions.

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