

Improved Exponential Estimator for Estimating the Population Mean in the Presence of Non-Response

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Abstract

This paper defines an improvement for estimating the population mean of a study variable using auxiliary information and known values of certain population parameter(s), when there is a non-response in a study as well as on auxiliary variables. Under a simple random sampling without a replacement (SRSWOR) scheme, the mean square error (MSE) of all proposed estimators are obtained and compared with each other. Numerical illustration is also given.

Keywords: Exponential estimator, Study variable, auxiliary variable, mean square error (MSE).

1. Introduction

A non-response in a sample survey creates problems for estimation that cannot be eliminated by simply increasing the sample size. The presence of a non-response distorts parameter estimation by increasing the bias in estimates that results in a larger mean square error. A non-response can be classified as ignorable or non-ignorable depending on if it is correlated with the target variable (Little, 1982; Glynn *et al.*, 1993). The non-response always exists when surveying populations because some individuals hesitate to respond in surveys; and increases notably while studying sensitive issues. The presence of a non-response increases the bias in estimates and ultimately reduces their efficiency.

Incompleteness or non-response in the form of missingness, censoring or grouping is a troubling issue of many data sets. Failure to account for the stochastic nature of incompleteness or non-response can spoil inference about the data. There are several factors that affect the non-response, including type of information collected, official status of the survey in agency, extent of publicity, legal obligations of the respondents, time of visit by the enumerator, and length of the schedule. Hansen and Hurwitz (1946) were the first to deal with the problem of incomplete samples in mail surveys. Hansen and Hurwitz (1946) studied a survey that combined the advantages of mailed questionnaires and personal interviews. The plan first utilizes the economies involved in the use of questionnaires by mailing them to a sample of the population under investigation. A follow-up was then conducted by interviewing a sub-sample of the non-respondents to the mail canvass that substantially eliminated the bias of non-response in the first stage. The optimum allocation of the mail and field samples was obtained by requiring a minimum cost for an assigned precision. The sizes of the two samples depend naturally on the rate of non-response to the mailed questionnaire and the variances of the characteristic under investigation both among the whole population and among non-respondents. These parameters are assumed to be known from previous experience (see El-Badry, 1956). The use of an auxiliary variable x (which is correlated with y) has been emphasized quite extensively in the literature to increase the precision of an estimate of the population parameters of study variable y .

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Various authors including Rao (1986, 1987), Khare and Srivastava (1993, 1995, 1997), Tabasum and Khan (2004, 2006), Singh and Kumar (2008, 2009a, b, c, 2010a, b), Kumar and Bhoulal (2011), Kumar *et al.* (2011) and Kumar (2012) improved the estimating procedure for the population mean in the presence of a non-response using an auxiliary character.

In this paper, under SRSWOR, improved exponential ratio type estimators are proposed to estimate the population mean using some known value of population parameter(s).

2. The Proposed Strategy

Consider a finite population of size N and a random sample of size n drawn without a replacement. In surveys of human populations, frequently n_1 units respond to the items under examination, but the remaining $(n - n_1)$ units do not provide a response. The initial survey may be conducted through mail, telephone calls, or be computer-aided (see Rao, 1986). When a non-response occurs in the initial attempt, Hansen & Hurwitz (1946) suggested a double sampling scheme to estimate population means:

- (a) A simple random sample of size n is selected and the questionnaire is mailed to the sampled units.
- (b) A subsample of size $r = n_2/k$, ($k \geq 1$) units is selected from the n_2 non-responding units where k is the inverse sampling rate at the second phase sample of size n .

Note that Hansen & Hurwitz (1946) considered mail surveys at the first attempt and personal interviews at the second attempt. In the Hansen & Hurwitz method, the population of size N is should be composed of two strata of 'respondents' and 'non-respondents' having sizes N_1 and $N_2 (= N - N_1)$. Thus we label the data as y_1, \dots, y_{N_1} for the response group, and $y_{N_1+1}, \dots, y_{N_1+N_2}$ for the non-response group. Let $\bar{Y} = \sum_{i=1}^N y_i/N$ and $S_y^2 = \sum_{i=1}^N (y_i - \bar{Y})^2/(N - 1)$ denote the population mean and variance. Let $\bar{Y}_1 = \sum_{i=1}^{N_1} y_i/N_1$ and $S_{y(1)}^2 = \sum_{i=1}^{N_1} (y_i - \bar{Y}_1)^2/(N_1 - 1)$ denote the mean and variance of the response group. Similarly, let $\bar{Y}_2 = \sum_{i=N_1+1}^{N_1+N_2} y_i/N_2$ and $S_{y(2)}^2 = \sum_{i=N_1+1}^{N_1+N_2} (y_i - \bar{Y}_2)^2/(N_2 - 1)$ denote the mean and variance of the non-response group. The population mean can be written as $\bar{Y} = W_1 \bar{Y}_1 + W_2 \bar{Y}_2$, where $W_1 = N_1/N$ and $W_2 = N_2/N$. The sample mean $\bar{y}_1 = \sum_{i=1}^{n_1} y_i/n_1$ is unbiased for \bar{Y}_1 , but has a bias equal to $W_2(\bar{Y}_1 - \bar{Y}_2)$ in estimating the population mean \bar{Y} .

The sample mean $\bar{y}_{2r} = \sum_{i=1}^r y_i/r$ is unbiased for the mean \bar{y}_2 of the n_2 units. An unbiased estimator for the population mean \bar{Y} is

$$\bar{y}^* = w_1 \bar{y}_1 + w_2 \bar{y}_{2r}, \quad (2.1)$$

where $w_1 = n_1/n$ and $w_2 = n_2/n$.

The variance of \bar{y}^* is given by

$$\text{Var}(\bar{y}^*) = \left(\frac{1-f}{n} \right) S_y^2 + \frac{W_2(k-1)}{n} S_{y(2)}^2, \quad (2.2)$$

where $f = n/N$.

Let x_i ($i = 1, 2, \dots, N$) denote an auxiliary variate correlated with the study variable y_i ($i = 1, 2, \dots, N$). The population mean of the auxiliary variable x is $\bar{X} = \sum_{i=1}^N x_i/N$. Let \bar{X}_1 and \bar{X}_2 denote the means of the response and non-response groups. Let \bar{x} denote the mean of all the n units. Let \bar{x}_1 and \bar{x}_2 denote the means of the n_1 responding units and the n_2 non-responding units. Further let $\bar{x}_{2r} = \sum_{i=1}^r x_i/r$ denote the mean of the subsampled units. The population variances of x and y are

denoted by S_x^2 and S_y^2 and the population covariance by S_{xy} . The population correlation coefficient is $\rho = S_{xy}/S_x S_y$. The unbiased estimator of the population mean \bar{X} of the auxiliary variable x is

$$\bar{x}^* = w_1 \bar{x}_1 + w_2 \bar{x}_{2r}. \quad (2.3)$$

The variance of \bar{x}^* is given by

$$\text{Var}(\bar{x}^*) = \left(\frac{1-f}{n} \right) S_x^2 + \frac{W_2(k-1)}{n} S_{x(2)}^2, \quad (2.4)$$

where $S_{x(2)}^2 = \sum_{i=N_1+1}^{N_1+N_2} (x_i - \bar{x}_2)/(N_2 - 1)$.

The conventional ratio estimator for the population mean \bar{Y} of the study variable y is given by

$$T_{R1} = \frac{\bar{y}^*}{\bar{x}^*} \bar{X}$$

in addition, the alternate ratio estimator uses the \bar{X} and the complete information on x and incomplete information on y character for n sample units, proposed by Rao (1986) is given as

$$T_{R2} = \frac{\bar{y}^*}{\bar{x}} \bar{X}.$$

Similarly, the conventional and alternate product estimators for population mean in presence of non-response are defined by

$$T_{P1} = \bar{y}^* \frac{\bar{x}^*}{\bar{X}} \quad \text{and} \quad T_{P2} = \bar{y}^* \frac{\bar{x}}{\bar{X}}, \quad \text{respectively.}$$

An exponential ratio type estimator for estimating the population mean \bar{Y} of the study variable y proposed by Kumar and Bhogal (2011) is

$$T_{R3} = \bar{y}^* \exp \left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*} \right). \quad (2.5)$$

Several authors have used prior value of certain population parameter(s) to find more precise estimates. Sisodia and Dwivedi (1981), Sen (1978) and Upadhyaya and Singh (1984) used the known coefficient of variation (CV) of the auxiliary variable to estimate the population mean of a study variable in the ratio method of estimation. The use of a prior value of the coefficient of kurtosis to estimate the population variance of study variable y was first made by Singh *et al.* (1973). Singh and Tailor (2003) proposed a modified ratio estimator using the known value of a correlation coefficient. Kadilar and Cingi (2006a) and Khoshenevisan *et al.* (2007) suggested modified ratio estimators using different pairs of known value of population parameter(s).

Following Kadilar and Cingi (2006a) and Khoshenevisan *et al.* (2007), a modified exponential estimator to estimate the population mean \bar{Y} of the study variable y when non-response occurs in a study as well as on an auxiliary variable as

$$t^* = \bar{y}^* \exp \left\{ \frac{\left((a\bar{X} + b) - (a\bar{x}^* + b) \right)}{\left((a\bar{X} + b) + (a\bar{x}^* + b) \right)} \right\}, \quad (2.6)$$

where $a(\neq 0)$, b are either real numbers or the functions of the known parameters of the auxiliary variable x such as Coefficient of Variation (C_x), Coefficient of kurtosis ($\beta_2(x)$) and correlation coefficient (ρ_{yx}).

To obtain the bias and Mean square error (MSE) of t^* , we write

$$\bar{y}^* = \bar{Y} + \varepsilon_0, \quad \bar{x}^* = \bar{X} + \varepsilon_1,$$

such that

$$E(\varepsilon_0) = E(\varepsilon_1) = 0,$$

and

$$\begin{aligned} E(\varepsilon_0^2) &= \text{Var}(\bar{y}^*) = \frac{1}{\bar{Y}} \left\{ \left(\frac{1-f}{n} \right) S_y^2 + \frac{W_2(k-1)}{n} S_{y(2)}^2 \right\}, \\ E(\varepsilon_1^2) &= \text{Var}(\bar{x}^*) = \frac{1}{\bar{X}} \left\{ \left(\frac{1-f}{n} \right) S_x^2 + \frac{W_2(k-1)}{n} S_{x(2)}^2 \right\}, \\ E(\varepsilon_0\varepsilon_1) &= \text{Cov}(\bar{y}^*, \bar{x}^*) = \frac{1}{\bar{Y}\bar{X}} \left\{ \left(\frac{1-f}{n} \right) S_{xy} + \frac{W_2(k-1)}{n} S_{xy(2)} \right\}. \end{aligned}$$

Express t^* in terms of the ε 's as

$$\begin{aligned} t^* &= \bar{Y}(1 + \varepsilon_0) \exp \left\{ \frac{-a\bar{X}\varepsilon_1}{2(a\bar{X} + b) + a\bar{X}\varepsilon_1} \right\} = \bar{Y}(1 + \varepsilon_0) \exp \left\{ \frac{-\phi\varepsilon_1}{1 + \phi\varepsilon_1} \right\} \\ &= \bar{Y}(1 + \varepsilon_0) \exp \left\{ -\phi\varepsilon_1(1 + \phi\varepsilon_1)^{-1} \right\}, \end{aligned} \quad (2.7)$$

where $\phi = a\bar{X}/\{2(a\bar{X} + b)\}$.

Assume that $|\varepsilon_1/\bar{X}| < 1$, so that the right hand side of (2.7) are expandable in terms of a power series. Expanding the right hand side of (2.7) and neglecting the terms in ε 's with a power greater than two, we have

$$t^* = \bar{Y}(1 + \varepsilon_0 - \phi\varepsilon_1 + \phi^2\varepsilon_1^2 - \phi\varepsilon_0\varepsilon_1). \quad (2.8)$$

Subtracting \bar{Y} from both sides of (2.8), and taking expectations of both sides, we get the bias of t^* up to the first order of approximation, as

$$B(t^*) = \frac{1}{\bar{X}} \left\{ \left(\frac{1-f}{n} \right) R\phi(\phi - K_{yx})S_x^2 + \frac{W_2(k-1)}{n} R\phi(\phi - K_{yx(2)})S_{x(2)}^2 \right\}, \quad (2.9)$$

where $K_{yx} = S_{yx}/S_x^2$ and $K_{yx(2)} = S_{yx(2)}/S_{x(2)}^2$.

Subtracting \bar{Y} from both sides of (2.8), squaring both sides and retaining terms in ε up to the second power, we have

$$(t^* - \bar{Y})^2 = \bar{Y}^2(\varepsilon_0 - \phi\varepsilon_1)^2. \quad (2.10)$$

Taking expectations of both sides of (2.10), we get the MSE of t^* to the first degree of approximation as

$$\text{MSE}(t^*) = \left(\frac{1-f}{n} \right) \{ S_y^2 + R^2\phi^2 S_x^2 - 2R\phi S_{yx} \} + \frac{W_2(k-1)}{n} \{ S_{y(2)}^2 + R^2\phi^2 S_{x(2)}^2 - 2R\phi S_{yx(2)} \}. \quad (2.11)$$

3. Some Members of the Suggested Estimator t^*

The following are the members of the suggested estimator t^* which can be simply obtained by substituting the suitable choice of constants a and b .

Estimator(s)	Values of	
	a	b
$t_0^* = \bar{y}^*$	0	0
$t_1^* = \bar{y}^* \exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*}\right)$	1	1
$t_2^* = \bar{y}^* \exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^* + 2\beta_2(x)}\right)$	1	$\beta_2(x)$
$t_3^* = \bar{y}^* \exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^* + 2C_x}\right)$	1	C_x
$t_4^* = \bar{y}^* \exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^* + 2\rho_{yx}}\right)$	1	ρ_{yx}
$t_5^* = \bar{y}^* \exp\left(\frac{\beta_2(x)(\bar{X} - \bar{x}^*)}{\beta_2(x)(\bar{X} + \bar{x}^*) + 2C_x}\right)$	$\beta_2(x)$	C_x
$t_6^* = \bar{y}^* \exp\left(\frac{C_x(\bar{X} - \bar{x}^*)}{C_x(\bar{X} + \bar{x}^*) + 2\beta_2(x)}\right)$	C_x	$\beta_2(x)$
$t_7^* = \bar{y}^* \exp\left(\frac{C_x(\bar{X} - \bar{x}^*)}{C_x(\bar{X} + \bar{x}^*) + 2\rho_{yx}}\right)$	C_x	ρ_{yx}
$t_8^* = \bar{y}^* \exp\left(\frac{\rho_{yx}(\bar{X} - \bar{x}^*)}{\rho_{yx}(\bar{X} + \bar{x}^*) + 2C_x}\right)$	ρ_{yx}	C_x
$t_9^* = \bar{y}^* \exp\left(\frac{\beta_2(x)(\bar{X} - \bar{x}^*)}{\beta_2(x)(\bar{X} + \bar{x}^*) + 2\rho_{yx}}\right)$	$\beta_2(x)$	ρ_{yx}
$t_{10}^* = \bar{y}^* \exp\left(\frac{\rho_{yx}(\bar{X} - \bar{x}^*)}{\rho_{yx}(\bar{X} + \bar{x}^*) + 2\beta_2(x)}\right)$	ρ_{yx}	$\beta_2(x)$

In addition to the above estimators, a large number of estimators can be generated from the proposed estimator t^* at (2.6) by substituting the different values of a and b . The expressions of MSE of the above said estimators are as follows:

$$\text{MSE}(t_0^*) = \left(\frac{1-f}{n}\right) S_y^2 + \frac{W_2(k-1)}{n} S_{y(2)}^2, \tag{3.1}$$

$$\text{MSE}(t_1^*) = \left(\frac{1-f}{n}\right) \{S_y^2 + \phi_1^2 R^2 S_x^2 - 2\phi_1 R S_{yx}\} + \frac{W_2(k-1)}{n} \{S_{y(2)}^2 + \phi_1^2 R^2 S_{x(2)}^2 - 2\phi_1 R S_{yx(2)}\}. \tag{3.2}$$

$$\text{MSE}(t_2^*) = \left(\frac{1-f}{n}\right) \{S_y^2 + \phi_2^2 R^2 S_x^2 - 2\phi_2 R S_{yx}\} + \frac{W_2(k-1)}{n} \{S_{y(2)}^2 + \phi_2^2 R^2 S_{x(2)}^2 - 2\phi_2 R S_{yx(2)}\}, \tag{3.3}$$

$$\text{MSE}(t_3^*) = \left(\frac{1-f}{n}\right) \{S_y^2 + \phi_3^2 R^2 S_x^2 - 2\phi_3 R S_{yx}\} + \frac{W_2(k-1)}{n} \{S_{y(2)}^2 + \phi_3^2 R^2 S_{x(2)}^2 - 2\phi_3 R S_{yx(2)}\}, \tag{3.4}$$

$$\text{MSE}(t_4^*) = \left(\frac{1-f}{n}\right) \{S_y^2 + \phi_4^2 R^2 S_x^2 - 2\phi_4 R S_{yx}\} + \frac{W_2(k-1)}{n} \{S_{y(2)}^2 + \phi_4^2 R^2 S_{x(2)}^2 - 2\phi_4 R S_{yx(2)}\}, \tag{3.5}$$

$$\text{MSE}(t_5^*) = \left(\frac{1-f}{n}\right) \{S_y^2 + \phi_5^2 R^2 S_x^2 - 2\phi_5 R S_{yx}\} + \frac{W_2(k-1)}{n} \{S_{y(2)}^2 + \phi_5^2 R^2 S_{x(2)}^2 - 2\phi_5 R S_{yx(2)}\}, \tag{3.6}$$

$$\text{MSE}(t_6^*) = \left(\frac{1-f}{n}\right)\{S_y^2 + \phi_6^2 R^2 S_x^2 - 2\phi_6 RS_{yx}\} + \frac{W_2(k-1)}{n}\{S_{y(2)}^2 + \phi_6^2 R^2 S_{x(2)}^2 - 2\phi_6 RS_{yx(2)}\}, \quad (3.7)$$

$$\text{MSE}(t_7^*) = \left(\frac{1-f}{n}\right)\{S_y^2 + \phi_7^2 R^2 S_x^2 - 2\phi_7 RS_{yx}\} + \frac{W_2(k-1)}{n}\{S_{y(2)}^2 + \phi_7^2 R^2 S_{x(2)}^2 - 2\phi_7 RS_{yx(2)}\}, \quad (3.8)$$

$$\text{MSE}(t_8^*) = \left(\frac{1-f}{n}\right)\{S_y^2 + \phi_8^2 R^2 S_x^2 - 2\phi_8 RS_{yx}\} + \frac{W_2(k-1)}{n}\{S_{y(2)}^2 + \phi_8^2 R^2 S_{x(2)}^2 - 2\phi_8 RS_{yx(2)}\}, \quad (3.9)$$

$$\text{MSE}(t_9^*) = \left(\frac{1-f}{n}\right)\{S_y^2 + \phi_9^2 R^2 S_x^2 - 2\phi_9 RS_{yx}\} + \frac{W_2(k-1)}{n}\{S_{y(2)}^2 + \phi_9^2 R^2 S_{x(2)}^2 - 2\phi_9 RS_{yx(2)}\}, \quad (3.10)$$

$$\text{MSE}(t_{10}^*) = \left(\frac{1-f}{n}\right)\{S_y^2 + \phi_{10}^2 R^2 S_x^2 - 2\phi_{10} RS_{yx}\} + \frac{W_2(k-1)}{n}\{S_{y(2)}^2 + \phi_{10}^2 R^2 S_{x(2)}^2 - 2\phi_{10} RS_{yx(2)}\}, \quad (3.11)$$

where

$$\begin{aligned} \phi_1 &= \frac{\bar{X}}{2(\bar{X} + 1)}, & \phi_2 &= \frac{\bar{X}}{2(\bar{X} + \beta_2(x))}, & \phi_3 &= \frac{\bar{X}}{2(\bar{X} + C_x)}, & \phi_4 &= \frac{\bar{X}}{2(\bar{X} + \rho_{yx})}, \\ \phi_5 &= \frac{\beta_2(x)\bar{X}}{2(\beta_2(x)\bar{X} + C_x)}, & \phi_6 &= \frac{C_x\bar{X}}{2(C_x\bar{X} + \beta_2(x))}, & \phi_7 &= \frac{C_x\bar{X}}{2(C_x\bar{X} + \rho_{yx})}, & \phi_8 &= \frac{\rho_{yx}\bar{X}}{2(\rho_{yx}\bar{X} + C_x)}, \\ \phi_9 &= \frac{\beta_2(x)\bar{X}}{2(\beta_2(x)\bar{X} + \rho_{yx})}, & \phi_{10} &= \frac{\rho_{yx}\bar{X}}{2(\rho_{yx}\bar{X} + \beta_2(x))}. \end{aligned}$$

4. Modified Estimators

Following Kadilar and Cingi (2006b), a modified estimator combining estimators t_1^* and t_i^* ; ($i = 2, 3, \dots, 10$) as

$$T_i^* = \alpha t_1^* + (1 - \alpha)t_i^*; \quad i = 2, 3, \dots, 10, \quad (4.1)$$

where α is any chosen constant such that MSE of T_i^* is minimum and t_i^* , ($i = 2, 3, \dots, 10$) are estimators listed in Section 3.

The MSE of T_i^* to the first degree of approximation as

$$\begin{aligned} \text{MSE}(T_i^*) &= \left(\frac{1-f}{n}\right)\left\{S_y^2 + R^2 S_x^2 \left(\frac{\alpha}{2} + \varphi_i - \alpha\varphi_i\right)^2 - 2RS_{yx} \left(\frac{\alpha}{2} + \varphi_i - \alpha\varphi_i\right)\right\} \\ &+ \frac{W_2(k-1)}{n}\left\{S_{y(2)}^2 + R^2 S_{x(2)}^2 \left(\frac{\alpha}{2} + \varphi_i - \alpha\varphi_i\right)^2 - 2RS_{yx(2)} \left(\frac{\alpha}{2} + \varphi_i - \alpha\varphi_i\right)\right\} \end{aligned} \quad (4.2)$$

which is minimal, when

$$\alpha = \frac{2(B - \varphi_i RA)}{(1 - 2\varphi_i)RA} = \alpha_{opt}(\text{say}), \quad (4.3)$$

where

$$A = \left(\frac{1-f}{n}\right)S_x^2 + \frac{W_2(k-1)}{n}S_{x(2)}^2, \quad B = \left(\frac{1-f}{n}\right)S_{yx} + \frac{W_2(k-1)}{n}S_{yx(2)}.$$

Substitute (4.3) in (4.2), the optimum MSE of T_i^* is given as

$$\min \text{MSE}(T_i^*) = \left(\frac{1-f}{n}\right) S_y^2 + \frac{W_2(k-1)}{n} S_{y(2)}^2 - \frac{B^2}{A} = \text{MSE}(T_{opt}^*). \quad (4.4)$$

5. Efficiency Comparison

From (3.1), (3.2) to (3.11) and (4.4), we get

$$\min \text{MSE}(T_i^*) \leq \text{MSE}(t_0^*),$$

only when

$$\frac{B^2}{A} \geq 0 \quad (5.1)$$

If this condition (*i.e.* (5.1)) is satisfied, then the proposed estimator T_i^* at its optimum is more efficient than the usual unbiased estimator $t_0^* = \bar{y}^*$.

$\min \text{MSE}(T_i^*) \leq \text{MSE}(t_0^*)$, we obtain the following condition

$$(B - R\varphi_i A)^2 \geq 0. \quad (5.2)$$

It is envisaged that all the proposed estimators T_i^* , ($i = 2, 3, \dots, 10$) are more efficient than the suggested estimators t_i^* , ($i = 2, 3, \dots, 10$) in Section 3 because the condition in (5.2) is always satisfied.

6. Numerical Study

Source: Khare and Sinha (2007). The description of the population is given below:

The data on the physical growth of the upper socio-economic group of 95 school children of Varanasi under an ICMR study (1983–1984), Department of Pediatrics, B.H.U., was analyzed. The first 25% (*i.e.* 24 children) units were considered as non-responding units. Here, weight (Kg.) of the children is taken as study variable y and skull circumference (cm) of the children is taken as auxiliary variable x . The parameters were:

$$\begin{aligned} \bar{Y} &= 19.4968; & \bar{X} &= 51.1726; & S_y^2 &= 9.26618; & S_x^2 &= 2.3662; & S_{y(2)}^2 &= 5.5424; \\ S_{x(2)}^2 &= 1.6079; & \rho_{yx} &= 0.328; & \rho_{yx(2)} &= 0.477; & S_{xy} &= 1.5359; & S_{xy(2)} &= 1.4240; \\ N &= 95; & n &= 35; & W_2 &= 0.25. \end{aligned}$$

Here, compute the percent of relative efficiencies (PREs) for different estimators of population mean \bar{Y} with respect to usual unbiased estimator \bar{y}^* for the varying values of k (Table 1).

Table 1 shows that the proposed estimator is more efficient than the usual unbiased estimator \bar{y}^* . The PREs of all estimators increase relative to the value of k . It is also noted that the proposed estimator T_i^* , ($i = 2, 3, \dots, 10$) under optimum conditions perform better than the estimators proposed and listed in Section 3. The choice of the estimators depends mainly upon the availability of information about known values of the parameter(s) ($C_x, \rho_{yx}, \beta_2(x)$).

Table 1: Percent relative efficiencies (PREs) of the proposed estimators with respect to the usual unbiased estimator \bar{y}^* for different values of k .

Estimator(s)	(1/k)			
	(1/2)	(1/3)	(1/4)	(1/5)
$t_0^* = \bar{y}^*$	100.00	100.00	100.00	100.00
t_1^*	106.28	106.75	107.09	107.35
t_3^*	106.39	106.88	107.25	107.53
t_4^*	106.37	106.84	107.71	107.49
t_7^*	105.39	105.81	106.11	106.34
t_8^*	106.39	106.88	107.25	107.51
t_{opt}^*	114.60	116.42	117.88	119.02

7. Conclusion

Exponential ratio type estimators are developed using some known value of the population parameter(s) of the auxiliary variable x such as Coefficient of Variation (C_x), Coefficient of kurtosis ($\beta_2(x)$) and Correlation Coefficient (ρ_{yx}). Also suggested is a modified estimator T_i^* ($i = 2, 3, \dots, 10$). The properties of proposed estimators (*i.e.* biases and Mean Square Errors (MSE)'s) are studied as well as compared to the proposed estimator under optimum conditions with the usual unbiased estimator and the estimators t_i^* ($i = 2, 3, \dots, 10$). Theoretically and numerically, it was shown that the proposed estimator T_i^* , ($i = 2, 3, \dots, 10$) under optimum conditions performs better than the proposed estimators listed in Section 3.

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