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ON RELATIONS FOR QUOTIENT MOMENTS OF THE GENERALIZED PARETO DISTRIBUTION BASED ON RECORD VALUES AND A CHARACTERIZATION

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Abstract. Generalized Pareto distributions play an important role in reliability, extreme value theory, and other branches of applied probability and statistics. This family of distribution includes exponential distribution, Pareto distribution, and Power distribution. In this paper we establish some recurrences relations satisfied by the quotient moments of the upper record values from the generalized Pareto distribution. Further a characterization of this distribution based on recurrence relations of quotient moments of record values is presented.

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1. Introduction

Record values are found in many situations of daily life as well as in many statistical applications. Often we are interested in observing new records and in recording them: for example, Olympic records or world records in sport. Record values are used in reliability theory. Moreover, these statistics are closely connected with the occurrences times of some corresponding non homogeneous Poisson process used in shock models. The statistical study of record values started with Chandler [8], he formulated the theory of record values as a model for successive extremes in a sequence of independently and identically random variables. Feller [25] gave some examples of record values with respect to gambling problems. Resnick [19] discussed the asymptotic theory of records. Theory of record values and its distributional properties have been extensively studied in the literature, for example, see, Ahsanullah [9], Arnold *et al.* [1,2], Nevzorov [23] and Kamps [21] for reviews on various developments in the area of records.

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We shall now consider the situations in which the record values (e.g. successive largest insurance claims in non-life insurance, highest water-levels or highest temperatures) themselves are viewed as "outliers" and hence the second or third largest values are of special interest. Insurance claims in some non life insurance can be used as one of the examples. Observing successive k largest values in a sequence, Dziubdziela and Kopocinski [24] proposed the following model of k record values, where k is some positive integer.

Let $\{X_n, n \geq 1\}$ be a sequence of identically independently distributed $(i.i.d)$ random variables with probability density function (pdf) $f(x)$ and distribution function (df) $F(x)$. The $j-$th order statistics of a sample $(X_1, X_2, \ldots, X_n)$ is denoted by $X_{j:n}$. For a fix $k \geq 1$ we define the sequence $\{U_n^{(k)}, n \geq 1\}$ of k upper record times of $\{X_n, n \geq 1\}$ as follows

$$U_1^{(k)} = 1,$$
$$U_{n+1}^{(k)} = min\{j > U_n^{(k)} : X_j : j + k + 1 > X_{U_n^{(k)}:U_n^{(k)}+k-1}\}.$$

The sequence $\{Y_n^{(k)}, n \geq 1\}$ with $Y_n^{(k)} = X_{U_n^{(k)}:U_n^{(k)}+k-1}, n = 1, 2, \ldots$ are called the sequences of k upper record values of $\{X_n, n \geq 1\}$.

For $k = 1$ and $n = 1, 2, \ldots$ we write $U_1^{(1)} = U_n$. Then $\{U_n, n \geq 1\}$ is the sequence of record times of $\{X_n, n \geq 1\}$. The sequence $\{Y_n^{(k)}, n \geq 1\}$, where $Y_n^{(k)} = X_{U_n^{(k)}}$ is called the sequence of k upper record values of $\{X_n, n \geq 1\}$. For convenience, we shall also take $Y_0^{(k)} = 0$. Note that $k = 1$ we have $Y_n^{(1)} = X_{U_n}, n \geq 1$, which are record value of $\{X_n, n \geq 1\}$. Moreover $Y_1^{(k)} = min\{X_1, X_2, \ldots, X_k = X_{1:k}\}$.

Let $\{X_n^{(k)}, n \geq 1\}$ be the sequence of k upper record values the joint *pdf* of $X_m^{(k)}$ and $X_n^{(k)}$, $1 \leq m < n$, $n > 2$ is given by

$$\begin{aligned} f_{X_m^{(k)}, X_n^{(k)}}(x, y) = &\frac{k^n}{(m-1)!(n-m-1)!}[-\ln(\bar{F}(x))]^{m-1} \\ &\times [-\ln \bar{F}(y) + \ln \bar{F}(x)]^{n-m-1}[\bar{F}(y)]^{k-1}\frac{f(x)}{\bar{F}(x)}f(y), \; x < y. \end{aligned} \tag{1.1}$$

where $\bar{F}(x) = 1 - F(x)$.

Recurrence relations are interesting in their own right. They are useful in reducing the number of operations necessary to obtain a general form for the function under consideration. Furthermore, they are used in characterizing the distributions, which in important area, permitting the identification of population distribution from the properties of the sample. Recurrence relations and identities have attained importance reduces the amount of direct computation and hence reduces the time and labour. They express the higher order moments in terms of lower order moments and hence make the evaluation of higher order moments easy and provide some simple checks to test the accuracy of computation of moments of order statistics.

Recurrence relations for single and product moments of k record values from Weibull, Pareto, generalized Pareto, Burr, exponential and Gumble distribution are derived by Pawalas and Szynal [16, 17, 18]. Sultan [7], Saran and Singh [6], Kumar [3], Kumar and Khan [4] are established recurrence relations for moments of k record values from modified Weibull, linear exponential, exponentiated log-logistic and generalized beta II distributions respectively. Balakrishnan and Ahsanullah [13,14] have proved recurrence relations for single and product moments of record values from generalized Pareto, Lomax and exponential distributions respectively. And similar results for this paper have been done by Lee and Chang [10, 11, 12] and Chang [20] for exponential distribution, Pareto distribution, power function distribution and Weibull distribution respectively. Kamps [22] investigated the importance of recurrence relations of order statistics in characterization.

In this paper, we established some explicit expressions and recurrence relations satisfied by the quotient moments of the upper record values from the generalized Pareto distribution. A characterization of this distribution based on recurrences relations of quotient moments of record values.

A random variable X is said to have generalized Pareto distribution if its *pdf* is of the form

$$f(x) = \frac{\beta(1+\alpha)}{(\alpha x+\beta)^2}\Big(\frac{\beta}{\alpha x+\beta}\Big)^{1/\alpha}, \ x>0, \ \alpha,\beta>0 \tag{1.2}$$

and the corresponding *df* is

$$\bar{F}(x) = \Big(\frac{\beta}{\alpha x+\beta}\Big)^{(1/\alpha)+1}, \ x>0, \ \alpha,\beta>0 \tag{1.3}$$

Where $\alpha > -1, \ \beta > 0$, then f is said to be member of generalized Pareto distribution. It should be noted that for $\alpha > 0$ and $-1 < \alpha < 0$ this model is, respectively, a Pareto distribution and a Power distribution. Moreover the survival function (1.3) tends to the exponential survival function as α tends to zero. This model is a flexible one due to its properties, i.e. it has a linear mean residual life function its coefficient of variation of the residual life is constant and its hazard rate is the reciprocal of linear function.

For more details and some applications of this distribution one may refer to Johnson et al. [15].

2. Relations for the quotient moment

First of all, we may note that for the generalized Pareto distribution in (1.2)

$$\bar{F}(x) = \frac{(\alpha x+\beta)}{(1+\alpha)}f(x) \tag{2.1}$$

The relation in (2.1) will be exploited in this paper to derive recurrence relations for the quotient moments of record values from the generalized Pareto distribution.

We shall first establish the explicit expression for the quotient moments of k record values $E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(m)}^{(k)}\right)^{s+1}}\right)$.

Theorem 2.1. *For generalized Pareto distribution as given in (1.3) and* $1 \leq m \leq n-2,\ k = 1,2,\ldots,\ s = 1,2,\ldots$

$$E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n)}^{(k)}\right)^{s+1}}\right) = [(1+\alpha)k]^n \left(\frac{\alpha}{\beta}\right)^{s-r+1} \sum_{p=0}^{\infty}\sum_{q=0}^{r}(-1)^q \binom{r}{q} \times \frac{(s+1)_p}{p![(1+\alpha)k+\alpha(p+s+1)]^{n-m}[(1+\alpha)k+\alpha(p+q+s-r+1)]^m}. \tag{2.2}$$

Proof. From (1.1), we have

$$E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n)}^{(k)}\right)^{s+1}}\right) = \frac{k^n}{(m-1)!(n-m-1)!} \times \int_0^{\infty} x^r[-\ln(\bar{F}(x))]^{m-1}\frac{f(x)}{[\bar{F}(x)]}G(x)dx, \tag{2.3}$$

where

$$G(x) = \int_x^{\infty} y^{-(s+1)}[-\ln(\bar{F}(y)) + \ln(\bar{F}(x))]^{n-m-1}[\bar{F}(x)]^{k-1}f(y)dy. \tag{2.4}$$

By setting $w = -ln(\bar{F}(y)) + ln(\bar{F}(x))$ in (2.4), we obtain

$$\begin{aligned}
G(x) &= \left(\frac{\alpha}{\beta}\right)^{(s+1)} \int_0^{\infty}[1-(\bar{F}(x)e^{-w})^{\alpha/(1+\alpha)}]^{-(s+1)} \times \left(\bar{F}(x)e^{-w}\right)^{\frac{\alpha(s+1)}{(1+\alpha)}+k} w^{n-m-1}dw \\
&= \left(\frac{\alpha}{\beta}\right)^{(s+1)} \sum_{p=0}^{\infty} \frac{(s+1)_{(p)}}{p!}[\bar{F}(x)]^{\frac{\alpha(p+s+1)+(1+\alpha)k}{1+\alpha}} \times \int_0^{\infty} w^{n-m-1}e^{-\frac{w[\alpha(p+s+1)+(1+\alpha)k]}{1+\alpha}}dw \\
&= \left(\frac{\alpha}{\beta}\right)^{(s+1)} \sum_{p=0}^{\infty} \frac{(s+1)_{(p)}}{p!} \frac{(1+\alpha)^{n-m}\Gamma(n-m)[\bar{F}(x)]^{\frac{\alpha(p+s+1)+(1+\alpha)k}{1+\alpha}}}{[(1+\alpha)k+\alpha(p+s+1)]^{n-m}}.
\end{aligned}$$

On substituting the above expression of $G(x)$ in (2.3), we get

$$E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n)}^{(k)}\right)^{s+1}}\right) = \frac{k^n(1+\alpha)^{n-m}}{(m-1)!}\left(\frac{\alpha}{\beta}\right)^{(s+1)}\sum_{p=0}^{\infty}\frac{(s+1)_{(p)}}{p![(1+\alpha)k+\alpha(p+s+1)]^{n-m}} \times \int_0^{\infty} x^r[-\ln(\bar{F}(x))]^{m-1}[\bar{F}(x)]^{\frac{\alpha(p+s+1)+(1+\alpha)k}{1+\alpha}-1}f(x)dx. \tag{2.5}$$

Again by setting $z = -ln(\bar{F}(x))$ in (2.5) and simplifying the resulting expression, we establish the result given in (2.2). □

Corollary 2.1. *For* $m \geq 1$, $r = 0, 1, 2, \ldots$ *and* $s = 1, 2, \ldots,$

$$E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(m+1)}^{(k)}\right)^{s+1}}\right) = [(1+\alpha)k]^{m+1}\left(\frac{\alpha}{\beta}\right)^{s-r+1}\sum_{p=0}^{\infty}\sum_{q=0}^{r}(-1)^q\binom{r}{q} \times \frac{(s+1)_p}{p![(1+\alpha)k+\alpha(p+s+1)][(1+\alpha)k+\alpha(p+q+s-r+1)]^m}. \tag{2.6}$$

Proof. Upon substituting $n = m + 1$ in (2.2) and simplifying, then we get the result given in (2.6). □

Remark 2.1. Setting $k = 1$ in (2.2) we deduce the explicit expression for the quotient moments of record values from the generalized Pareto distribution.

Theorem 2.2. *For generalized Pareto distribution as given in (1.3) and* $1 \leq m \leq n-2$, $k = 1, 2, \ldots,$ $s = 0, 1, 2, \ldots$

$$E\left(\frac{\left(X_{U(m)}^{(k)}\right)^{r+1}}{\left(X_{U(n)}^{(k)}\right)^{s}}\right) = [(1+\alpha)k]^{n}\left(\frac{\alpha}{\beta}\right)^{s-r+1}\sum_{p=0}^{\infty}\sum_{q=0}^{r+1}(-1)^q\binom{r+1}{q} \times \frac{(s)_p}{p![(1+\alpha)k+\alpha(p+s)]^{n-m}[(1+\alpha)k+\alpha(p+q+s-r-1)]^m}. \tag{2.7}$$

Proof. Proof can be established on line of Theorem 2.1. □

Corollary 2.2. *For* $m \geq 1$, *and* $r, s = 0, 1, 2, \ldots,$

$$E\left(\frac{\left(X_{U(m)}^{(k)}\right)^{r+1}}{\left(X_{U(m+1)}^{(k)}\right)^{s}}\right) = [(1+\alpha)k]^{m+1}\left(\frac{\alpha}{\beta}\right)^{s-r+1}\sum_{p=0}^{\infty}\sum_{q=0}^{r+1}(-1)^q\binom{r+1}{q} \times \frac{(s)_p}{p![(1+\alpha)k+\alpha(p+s)][(1+\alpha)k+\alpha(p+q+s-r-1)]^m}. \tag{2.8}$$

Proof. Upon substituting $n = m + 1$ in (2.7) and simplifying, then we get the result given in (2.8). □

Remark 2.2. Setting $k = 1$ in (2.7) we deduce the explicit expression for the quotient moments of record values from the generalized Pareto distribution.

Making use of (2.1), we can derive recurrence relations for the quotient moments of k upper record values

Theorem 2.3. *For $1 \leq m \leq n-2$, $r = 0,1,2,\ldots$ and $s = 1,2,\ldots$,*

$$\left(1+\frac{\alpha(s+1)}{(1+\alpha)k}\right) E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n)}^{(k)}\right)^{s+1}}\right) = E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n-1)}^{(k)}\right)^{s+1}}\right) - \frac{\beta(s+1)}{(1+\alpha)k} E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n)}^{(k)}\right)^{s+2}}\right) \tag{2.9}$$

Proof. From equation (1.1) for $1 \leq m \leq n-1$, $r = 0,1,2,\ldots$ and $s = 1,2,\ldots$,

$$E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n)}^{(k)}\right)^{s+1}}\right) = \frac{k^n}{(m-1)!(n-m-1)!} \times \int_0^\infty x^r[-\ln(\bar{F}(x))]^{m-1}\frac{f(x)}{[\bar{F}(x)]} I(x)dx, \tag{2.10}$$

where

$$I(x) = \int_x^\infty y^{-(s+1)}\left[-\ln(\bar{F}(y))+\ln(\bar{F}(x))\right]^{n-m-1}\left[\bar{F}(x)\right]^{k-1} f(y)dy.$$

Integrating $I(x)$ by parts treating $[\bar{F}(y)]^{k-1}f(y)$ for integration and the rest of the integrand for differentiation, and substituting the resulting expression in (2.10), we get

$$E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n)}^{(k)}\right)^{s+1}}\right) - E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n-1)}^{(k)}\right)^{s+1}}\right) = -\frac{(s+1)k^n}{k(m-1)!(n-m-1)!}$$
$$\int_0^\infty\int_x^\infty \frac{x^r}{y^{s+2}} \times [-\ln(\bar{F}(x))]^{m-1}[-\ln(\bar{F}(y))+\ln(\bar{F}(x))]^{n-m-1}[\bar{F}(y)]^k\frac{f(x)}{\bar{F}(x)}dydx$$

the constant of integration vanishes since the integral in $I(x)$ is a definite integral. On using the relation (2.1), we obtain

$$E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n)}^{(k)}\right)^{s+1}}\right) - E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n-1)}^{(k)}\right)^{s+1}}\right) = -\frac{(s+1)k^n}{k(1+\alpha)(m-1)!(n-m-1)!}$$
$$\times\left\{\alpha\int_0^\infty\int_x^\infty \frac{x^r}{y^{s+1}}[-\ln(\bar{F}(x))]^{m-1}f(x)[-\ln(\bar{F}(y))+\ln(\bar{F}(x))]^{n-m-1}[\bar{F}(y)]^{k-1}\frac{f(y)}{\bar{F}(x)}dydx\right.$$
$$\left.+\beta\int_0^\infty\int_x^\infty \frac{x^r}{y^{s+2}}[-\ln(\bar{F}(x))]^{m-1}f(x)\times[-\ln(\bar{F}(y))+\ln(\bar{F}(x))]^{n-m-1}[\bar{F}(y)]^{k-1}\frac{f(y)}{\bar{F}(x)}dydx\right\}$$

and hence the result given in (2.10). □

Corollary 2.3. *For* $m \geq 1$, $r = 0, 1, 2, \ldots$ *and* $s = 1, 2, \ldots$,

$$\begin{aligned}
&\left(1 + \frac{\alpha(s+1)}{(1+\alpha)k}\right) E\left(\frac{\left(X_{U(m)}^{(k)}\right)^{r}}{\left(X_{U(m+1)}^{(k)}\right)^{s+1}}\right) \\
&= E\left((X_{U(m)}^{(k)})^{r-s-1}\right) - \frac{\beta(s+1)}{(1+\alpha)k} E\left(\frac{\left(X_{U(m)}^{(k)}\right)^{r}}{\left(X_{U(m+1)}^{(k)}\right)^{s+2}}\right).
\end{aligned} \tag{2.11}$$

Proof. Upon substituting $n = m + 1$ in (2.9) and simplifying, then we get the result given in (2.11). □

Remark 2.3. Setting $k = 1$ in (2.9) we deduce the recurrence relation for the quotient moments of upper record values from the generalized Pareto distribution.

Theorem 2.4. *For* $1 \leq m \leq n - 2$ *and* $r, s = 0, 1, 2, \ldots$,

$$\begin{aligned}
&\left(1 + \frac{\alpha s}{(1+\alpha)k}\right) E\left(\frac{\left(X_{U(m)}^{(k)}\right)^{r+1}}{\left(X_{U(n)}^{(k)}\right)^{s}}\right) \\
&= E\left(\frac{\left(X_{U(m)}^{(k)}\right)^{r+1}}{\left(X_{U(n-1)}^{(k)}\right)^{s}}\right) - \frac{\beta s}{(1+\alpha)k} E\left(\frac{\left(X_{U(m)}^{(k)}\right)^{r+1}}{\left(X_{U(n)}^{(k)}\right)^{s+1}}\right)
\end{aligned} \tag{2.12}$$

Proof. Proof follows on the line of Theorem 2.2. □

Corollary 2.4. *For* $m \geq 1$, *and* $r, s = 0, 1, 2, \ldots$,

$$\begin{aligned}
&\left(1 + \frac{\alpha s}{(1+\alpha)k}\right) E\left(\frac{\left(X_{U(m)}^{(k)}\right)^{r+1}}{\left(X_{U(m+1)}^{(k)}\right)^{s}}\right) \\
&= E\left(\left(X_{U(m)}^{(k)}\right)^{r-s+1}\right) - \frac{\beta s}{(1+\alpha)k} E\left(\frac{\left(X_{U(m)}^{(k)}\right)^{r+1}}{\left(X_{U(m+1)}^{(k)}\right)^{s+1}}\right)
\end{aligned} \tag{2.13}$$

Proof. Upon substituting $n = m + 1$ in (2.12) and simplifying, then we get the result given in (2.13). □

Remark 2.4. Setting $k = 1$ in (2.12) we deduce the recurrence relation for the quotient moments of upper record values from the generalized Pareto distribution.

3. Characterization

Theorem 3.1. *Let $k \geq 1$ is a fix positive integer, r be a non- negative integer and X be an absolutely continuous random variable with pdf $f(y)$ and cdf $F(y)$ on the support (o, ∞), then*

$$\begin{aligned} &\left(1+\frac{\alpha(s+1)}{(1+\alpha)k}\right) E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n)}^{(k)}\right)^{s+1}}\right) \\ =&E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n-1)}^{(k)}\right)^{s+1}}\right) - \frac{\beta(s+1)}{(1+\alpha)k} E\left(\frac{\left(X_{U(m)}^{(k)}\right)^r}{\left(X_{U(n)}^{(k)}\right)^{s+2}}\right) \end{aligned} \tag{3.1}$$

if and only if

$$\bar{F}(y) = \left(\frac{\beta}{\alpha y + \beta}\right)^{(1/\alpha)+1}, \ y > 0, \ \alpha, \beta > 0.$$

Proof. The necessary part follows immediately from equation (2.9). On the other hand if the recurrence relation in equation (3.1) is satisfied, then on using equation (1.1), we have

$$\begin{aligned} &\frac{k^n}{(m-1)!(n-m-1)!} \int_0^\infty \int_x^\infty \frac{x^r}{y^{s+1}} [-\ln(\bar{F}(x))]^{m-1} f(x) \\ &\quad \times [-\ln(\bar{F}(y)) + \ln(\bar{F}(x))]^{n-m-1} [\bar{F}(y)]^{k-1} \frac{f(y)}{\bar{F}(x)} dy dx \\ =&\frac{k^n(n-m-1)}{k(m-1)!(n-m-1)!} \int_0^\infty \int_x^\infty \frac{x^r}{y^{s+1}} [-\ln(\bar{F}(x))]^{m-1} f(x) \\ &\quad \times [-\ln(\bar{F}(y)) + \ln(\bar{F}(x))]^{n-m-2} [\bar{F}(y)]^{k-1} \frac{f(y)}{\bar{F}(x)} dy dx \\ &- \frac{\alpha(s+1)k^n}{k(1+\alpha)(m-1)!(n-m-1)!} \int_0^\infty \int_x^\infty \frac{x^r}{y^{s+1}} [-\ln(\bar{F}(x))]^{m-1} f(x) \\ &\quad \times [-\ln(\bar{F}(y)) + \ln(\bar{F}(x))]^{n-m-1} [\bar{F}(y)]^{k-1} \frac{f(y)}{\bar{F}(x)} dy dx \\ &- \frac{\beta(s+1)k^n}{k(1+\alpha)(m-1)!(n-m-1)!} \int_0^\infty \int_x^\infty \frac{x^r}{y^{s+2}} [-\ln(\bar{F}(x))]^{m-1} f(x) \\ &\quad \times [-\ln(\bar{F}(y)) + \ln(\bar{F}(x))]^{n-m-1} [\bar{F}(y)]^{k-1} \frac{f(y)}{\bar{F}(x)} dy dx. \end{aligned} \tag{3.2}$$

Integrating the first integral on the right hand side of equation (3.2) by parts and simplifying the resulting expression, we find that

$$\frac{(s+1)k^n}{k(m-1)!(n-m-1)!}\int_0^\infty\int_x^\infty \frac{x^r}{y^{s+2}}[-\ln(\bar{F}(x))]^{m-1} \times[-\ln(\bar{F}(y))+\ln(\bar{F}(x))]^{n-m-1}[\bar{F}(y)]^{k-1}\frac{f(x)}{\bar{F}(x)} \times\left\{\bar{F}(y)-\left(\frac{\alpha y}{1+\alpha}+\frac{\beta}{1+\alpha}\right)f(y)\right\}dydx=0. \tag{3.3}$$

Now applying a generalization of the Müntz-Szász Theorem (Hwang and Lin, [15]) to equation (3.3), we get

$$\frac{f(y)}{\bar{F}(y)}=\frac{1+\alpha}{\alpha y+\beta}$$

which proves that

$$\bar{F}(y)=\left(\frac{\beta}{\alpha y+\beta}\right)^{(1/\alpha)+1},\ y>0,\ \alpha,\beta>0.$$

□

4. Conclusion

In this study some exact expressions and recurrence relations for the quotient moments of record values from the generalized Pareto distribution have been established. Further, recurrence relation of the quotient moments of record values has been utilized to obtain a characterization of the generalized Pareto.

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