J. Appl. Math. & Informatics Vol. **31***(2013), No. 1 - 2, pp. 211 - 220*
Website: http://www.kcam.biz

FUZZY STRONG IDEALS OF BH-ALGEBRAS WITH DEGREES IN THE INTERVAL $(0, 1]$

EUN MI KIM AND SUN SHIN AHN*

Abstract. In defining a fuzzy strong ideal in BH-algebras, several degrees are provided, and then related properties are investigated.

AMS Mathematics Subject Classification : 06F35, 03G25.
Key words and phrases : (strong) enlarged ideal, fuzzy (strong) ideal with degree.

1. Introduction

Y. Imai and K. Iséki introduced two classes of abstract algebras: BCK-algebras and BCI-algebras ([3,4]). It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. BCK-algebras have some connections with other areas: D. Mundici [7] proved MV-algebras are categorically equivalent to bounded commutative algebra, and J. Meng [8] proved that implicative commutative semigroups are equivalent to a class of BCK-algebras. Y. B. Jun, E. H. Roh, and H. S. Kim [5] introduced the notion of a BH-algebra, which is a generalization of BCK/BCI-algebras. They defined the notions of ideal, maximal ideal and translation ideal and investigated some properties. E. H. Roh and S. Y. Kim [11] estimated the number of BH^*-subalgebras of order i in a transitive BH^*-algebras by using Hao's method. In [2], S. S. Ahn and J. H. Lee introduced the notion of strong ideals in BH-algebra and investigate some properties of it. They also defined the notion of a rough sets in BH-algebras. Using a strong ideal in BH-algebras, they obtained some relations between strong ideals and upper(lower) rough strong ideals in BH-algebras. S. S. Ahn and E. M. Kim [1,6] introduced the notion of (fuzzy) n-fold strong ideal in BH-algebra and investigated some related properties of it.

In this paper, we define the notions of an enlarged (strong) ideal of a BH-algebra X related to a non-empty subset I of X and a fuzzy (strong) ideal of X with some degree and investigate related properties of them.

Received August 26, 2012. Accepted November 5, 2012. *Corresponding author.
© 2013 Korean SIGCAM and KSCAM.

2. Preliminaries

By a *BH-algebra* ([5]), we mean an algebra $(X; *, 0)$ of type (2,0) satisfying the following conditions:

(I) $x * x = 0$,
(II) $x * 0 = x$,
(III) $x * y = 0$ and $y * x = 0$ imply $x = y$, for all $x, y \in X$.

For brevity, we also call X a BH-algebra. In X we can define a binary operation " $\leq$ " by $x \leq y$ if and only if $x * y = 0$. Then $\leq$ is reflexive and antisymmetric. A non-empty subset S of a BH-algebra X is called a *subalgebra* of X if, for any $x, y \in S$, $x * y \in S$, i.e., S is a closed under binary operation.

Definition 2.1 ([5])**.** A non-empty subset A of a BH-algebra X is called an *ideal* of X if it satisfies:

(I1) $0 \in A$,
(I2) $x * y \in A$ and $y \in A$ imply $x \in A$, $\forall x, y \in X$.

An ideal A of a BH-algebra X is said to be a *translation ideal* of X if it satisfies:

(I3) $x * y \in I$, $y * x \in I$ imply $(x * z) * (y * z), (z * x) * (z * y) \in I$ for any $x, y, z \in X$.

Obviously, $\{0\}$ and X are translation ideals of X

Definition 2.2 ([11])**.** A BH-algebra X is called a *BH^*-algebra* if it satisfies the identity $(x * y) * x = 0$ for all $x, y \in X$.

Lemma 2.3. *Let X be a BH^*-algebra. Then the following identity holds:*

$$0 * x = 0, \quad \forall x \in X.$$

Proof. If follows from (II) that $0 * x = (0 * x) * 0 = 0$ for all $x \in X$. Hence $0 * x = 0$. □

Definition 2.4. A BH-algebra $(X; *, 0)$ is said to be *transitive*([11]) if $x * y = 0$ and $y * z = 0$ imply $x * z = 0$.

Lemma 2.5. *An ideal of a BH-algebra X has the following property:*

$$(\forall x \in X)(\forall y \in I)(x \leq y \Rightarrow x \in I).$$

We now review some fuzzy logic concepts. A fuzzy set in a set X is a function $\mu : X \to [0, 1]$. For a fuzzy set μ in X and $t \in [0, 1]$, define $U(\mu; t)$ to be the set $U(\mu; t) = \{x \in X | \mu(x) \geq t\}$, which is called a *level subset* of μ.

Definition 2.6 ([11])**.** A fuzzy set μ in a BH-algebra X is called a *fuzzy BH-ideal* (here call it a *fuzzy ideal*) of X if

(FI1) $\mu(0) \geq \mu(x), \forall x \in X$,
(FI2) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}, \forall x, y \in X$.

A fuzzy set μ in a BH-algebra X is called a *fuzzy translation BH-ideal* of X if it satisfies (FI1), (FI2) and

(FI3) $\min\{\mu((x*z)*(y*z)), \mu((z*x)*(z*y))\} \geq \min\{\mu(x*y), \mu(y*x)\}, \forall x, y, z \in X$.

3. Fuzzy ideals in BH-algebras with degrees in $(0, 1]$

In what follows let λ and κ be members of $(0, 1]$, and let n and k denote a natural number and a real number, respectively, such that $k < n$ unless otherwise specified.

Definition 3.1. Let I be a non-empty subset of a BH-algebra X which is not necessary an ideal X. We say that a subset J of X is an *enlarged ideal of X related to I* if it satisfies:

(1) I is a subset of J,
(2) $0 \in J$,
(3) $(\forall x \in X)(\forall y \in I)(x * y \in I \Rightarrow x \in J)$.

Obviously, every ideal is an enlarged ideal of X related to itself. Note that there exists an enlarged ideal of X related to any non-empty subset I of a BH-algebra X.

Example 3.2. (1) Let $X := \{0, 1, 2, 3\}$ be a BH-algebra which is not a BCK/BCI-algebra X with the following table

$*$	0	1	2	3
0	0	1	0	0
1	1	0	0	0
2	2	2	0	3
3	3	3	1	0

Note that $\{0, 2\}$ is not an ideal of X since $1 * 2 = 0 \in \{0, 2\}$ and $1 \notin \{0, 2\}$. Then $\{0, 1, 2\}$ is an enlarged ideal of X related to $\{0, 2\}$. But $\{0, 1, 2\}$ is not an ideal of X since $3 * 2 = 1 \in \{0, 1, 2\}$ and $3 \notin \{0, 1, 2\}$.

(2) Let $X := \{0, 1, 2, 3\}$ be a BH-algebra ([5]) which is not a BCK/BCI-algebra X with the following table

$*$	0	1	2	3
0	0	3	0	2
1	1	0	0	0
2	2	2	0	3
3	3	3	1	0

Note that $\{0, 2\}$ is not an ideal of X since $1 * 2 = 0 \in \{0, 2\}$ and $1 \notin \{0, 2\}$. Then $\{0, 1, 2\}$ is an enlarged ideal of X related to $\{0, 2\}$. But $\{0, 1, 2\}$ is not an ideal of X since $3 * 2 = 1 \in \{0, 1, 2\}$ and $3 \notin \{0, 1, 2\}$.

Definition 3.3. A fuzzy subset μ of a BH-algebra X is called a *fuzzy ideal* of X with degree (λ, κ) if it satisfies:

(1) $(\forall x \in X)(\mu(0) \geq \lambda\mu(x))$,

(2) $(\forall x, y \in X)(\mu(x) \geq \kappa \min\{\mu(x * y), \mu(y)\})$.

Note that if $\lambda \neq \kappa$, then a fuzzy ideal with degree (λ, κ) may not a fuzzy ideal with degree (κ, λ), and vice versa.

Example 3.4. Let $X := \{0, 1, 2, 3\}$ be a BH-algebra ([5]) which is not a BCK/BCI-algebra X with the following table

*	0	1	2	3
0	0	1	0	0
1	1	0	0	0
2	2	2	0	3
3	3	3	3	0

Define a fuzzy subset of ν of X by

$$\nu = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.6 & 0.5 & 0.9 & 0.3 \end{pmatrix}$$

Then ν is a fuzzy ideal of X with degree $(0.6, 0.7)$ but it is not a fuzzy ideal of X with degree $(0.7, 0.6)$ since $\nu(0) = 0.6 \ngeq 0.63 = 0.7 \times \nu(2)$.

Example 3.5. Consider a BH-algebra $X = \{0, 1, 2, 3\}$ as in Example 3.4. Define a fuzzy subset of μ of X by

$$\mu = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.7 & 0.6 & 0.8 & 0.3 \end{pmatrix}$$

Then μ is a fuzzy ideal of X with degree $(0.6, 0.5)$ but it is not a fuzzy ideal of X with degree $(0.9, 0.5)$ since $\mu(0) = 0.7 \ngeq 0.72 = 0.9 \times \mu(2)$.

Obviously, every fuzzy ideal is a fuzzy ideal with degree (λ, κ), but the converse may not be true. In fact, the fuzzy ideal μ with degree $(0.6, 0.5)$ in Example 3.5 is not fuzzy ideal of X since $\mu(0) = 0.7 \ngeq 0.8 = \mu(2)$. Note that a fuzzy ideal with degree (λ, κ) is a fuzzy ideal if and only if $(\lambda, \kappa) = (1, 1)$. If $\lambda_1 \geq \lambda_2$ and $\kappa_1 \geq \kappa_2$, then every fuzzy ideal with degree (λ_1, κ_1) is a fuzzy ideal with (λ_2, κ_2), but the converse is not true as shown by Example 3.5.

Proposition 3.6. *Every fuzzy ideal of a BH-algebra X with degree (λ, κ) satisfies the following assertions:*

(1) $(\forall x, y \in X)(x \leq y \Rightarrow \mu(x) \geq \lambda\kappa\mu(y))$.
(2) *if X is a BH^*-algebra X, then*

$$\mu(x * y) \geq \lambda\kappa\mu(x),\ \forall x, y \in X.$$

Proof. (1) Let $x, y \in X$ be such that $x \leq y$. Then $x * y = 0$. Hence

$$\begin{aligned} \mu(x) &\geq \kappa \min\{\mu(x * y), \mu(y)\} \\ &= \kappa \min\{\mu(0), \mu(y)\} \\ &\geq \kappa \min\{\lambda\mu(y), \mu(y)\} \\ &= \lambda\kappa\mu(y). \end{aligned}$$

(2) By Definition 3.3(1), we have

$$\begin{aligned}\mu(x*y) \geq& \kappa\min\{\mu((x*y)*x), \mu(x)\}\\ =&\kappa\min\{\mu(0), \mu(y)\}\\ \geq&\kappa\min\{\lambda\mu(x), \mu(x)\}\\ =&\lambda\kappa\mu(x).\end{aligned}$$

for any $x, y \in X$. □

Corollary 3.7. *Let μ be a fuzzy ideal of a BH-algebra with degree (λ, κ). If $\lambda = \kappa$, then the following assertions hold:*

(1) $(\forall x, y \in X)(x \leq y \Rightarrow \mu(x) \geq \lambda^2\mu(y))$.
(2) *if X is a BH^*-algebra X, then*

$$\mu(x*y) \geq \lambda^2\mu(x),\ \forall x, y \in X.$$

Note that a fuzzy subset μ of a BH-algebra X is a fuzzy ideal of X if and only if

$$(\forall t \in [0.1])(U(\mu; t) \in \mathcal{I}(X) \cup \{\emptyset\}),$$

where $\mathcal{I}(X)$ is the set of all ideals of X. But, we know that for a fuzzy subset μ of a BH-algebra X there exist $\lambda, \kappa \in (0, 1)$ and $t \in [0, 1]$ such that

(1) μ is a fuzzy ideal of X with degree (λ, κ),
(2) $U(\mu; t) \notin \mathcal{I}(X) \cup \{\emptyset\}$.

Example 3.8. Consider the fuzzy ideal μ of X with degree $(0.6, 0.5)$ in Example 3.5. If $t \in (0.6, 0.7]$, then $U(\mu; t) = \{0, 2\}$ is not an ideal of X since $1 * 2 = 0 \in \{0, 2\}$ but $1 \notin \{0, 2\}$.

Theorem 3.9. *Let μ be a fuzzy subset of a BH-algebra X. For any $t \in (0, 1]$ with $t \leq \lambda$, if $U(\mu; t)$ is an enlarged ideal of X related to $U(\mu; \frac{1}{\max\{\lambda,\kappa\}})$, then μ is a fuzzy ideal of X with degree (λ, κ).*

Proof. Assume that $\mu(0) < t \leq \lambda\mu(x)$ for some $x \in X$ and $t \in (0, \lambda]$. Then $\mu(x) \geq \frac{t}{\lambda} \geq \frac{t}{\max\{\lambda,\kappa\}}$. Hence $x \in U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$, i.e., $U(\mu; \frac{t}{\max\{\lambda,\kappa\}}) \neq \emptyset$. Since $U(\mu; t)$ is an enlarged ideal of X related to $U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$, $0 \in U(\mu; t)$, i.e., $\mu(0) \geq t$. This is a contradiction, and thus $\mu(0) \geq \lambda\mu(x)$ for all $x \in X$.

Now suppose that there exist $a, b \in X$ such that $\mu(a) < \kappa\min\{\mu(a*b), \mu(b)\}$. If we take $t := \kappa\min\{\mu(a*b), \mu(b)\}$, then $t \in (0, \kappa] \subseteq (0, \max\{\lambda, \kappa\}]$, $a*b \in U(\mu; \frac{t}{\kappa}) \subseteq U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$ and $b \in U(\mu; \frac{t}{\kappa}) \subseteq U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$. It follows from Definition 3.1(3) that $a \in U(\mu; t)$ so that $\mu(a) \geq t$, which is impossible. Therefore

$$\mu(x) \geq \kappa\min\{\mu(x*y), \mu(y)\}$$

for all $x, y \in X$. Hence μ is a fuzzy ideal of X with degree (λ, κ). □

Corollary 3.10. *Let μ be a fuzzy subset of a BH-algebra X. For any $t \in [0, 1]$ with $t \leq \frac{k}{n}$, if $U(\mu; t)$ is an enlarged ideal of X related to $U(\mu; \frac{n}{k}t)$, then μ is a fuzzy ideal of X with degree $(\frac{k}{n}, \frac{k}{n})$.*

Theorem 3.11. *Let $t \in [0,1]$ be such that $U(\mu;t)(\neq \emptyset)$ is not necessary an ideal of a BH-algebra X. If μ is a fuzzy ideal of X with degree (λ,κ), then $U(\mu; t\min\{\lambda,\kappa\})$ is an enlarged ideal of X related to $U(\mu;t)$.*

Proof. Since $t\min\{\lambda,\kappa\} \leq t$, we get $U(\mu;t) \subseteq U(\mu; t\min\{\lambda,\kappa\})$. Since $U(\mu;t)(\neq \emptyset)$, there exists $x \in U(\mu;t)$ and so $\mu(x) \geq t$, Using Definition 3.3(1), we have $\mu(0) \geq \lambda\mu(x) \geq \lambda t \geq t\min\{\lambda,\kappa\}$, which implies that $0 \in U(\mu; t\min\{\lambda,\kappa\})$. Let $x,y \in X$ be such that $x * y \in U(\mu;t)$ and $y \in U(\mu;t)$. Then $\mu(x*y) \geq t$ and $\mu(y) \geq t$. It follows from Definition 3.3(2) that

$$\mu(x) \geq \kappa\min\{\mu(x*y),\mu(y)\} \geq \kappa t \geq t\min\{\lambda,\kappa\}$$

so that $x \in U(\mu; t\min\{\lambda,\kappa\})$. Therefore $U(\mu; t\min\{\lambda,\kappa\})$ is an enlarged ideal of X related to $U(\mu;t)$. □

Theorem 3.12. *Let μ be a fuzzy ideal of a BH-algebra X with degree (λ,κ). If the inequality $x*y \leq z$ holds in X, then*

$$\mu(x) \geq \min\{\kappa\mu(y), \lambda\kappa^2\mu(z)\}$$

for all $x,y,z \in X$.

Proof. Assume that $x*y \leq z$ for all $x,y,z \in X$. Then $(x*y)*z = 0$ and hence

$$\begin{aligned}\mu(x*y) \geq& \kappa\min\{\mu((x*y)*z),\mu(z)\}\\ =&\kappa\min\{\mu(0),\mu(z)\}\\ \geq&\kappa\min\{\lambda\mu(z),\mu(z)\}\\ =&\kappa\lambda\mu(z).\end{aligned}$$

It follows that

$$\begin{aligned}\mu(x) \geq&\kappa\min\{\mu(x*y),\mu(y)\}\\ \geq&\kappa\min\{\lambda\kappa\mu(z),\mu(y)\}\\ =&\min\{\kappa\mu(y),\lambda\kappa^2\mu(z)\}\end{aligned}$$

for all $x,y,z \in X$. □

Corollary 3.13. *Let μ be a fuzzy ideal of a BH-algebra X with degree (λ,κ). If $\lambda = \kappa$ and the inequality $x*y \leq z$ holds in X, then*

$$\mu(x) \geq \min\{\kappa\mu(y), \kappa^3\mu(z)\}$$

for all $x,y,z \in X$.

4. Fuzzy strong ideals in BH-algebras with degrees in $(0,1]$

Definition 4.1. Let I be a non-empty subset of a BH-algebra X which is not necessary a strong ideal X. We say that a subset J of X is an *enlarged strong ideal of X related to I* if it satisfies:

(1) I is a subset of J,
(2) $0 \in J$,
(3) $(\forall x,y,z \in X)((x*y)*z \in I$ and $y \in I \Rightarrow x*z \in J)$.

Obviously, every strong ideal is an enlarged ideal of X related to itself. Note that there exists an enlarged strong ideal of X related to any non-empty subset I of a BH-algebra X.

Example 4.2. Let $X := \{0, 1, 2, 3, 4, 5\}$ be a BH-algebra ([2]) which is not a BCK/BCI-algebra X with the following table

$*$	0	1	2	3	4	5
0	0	0	0	0	0	5
1	1	0	0	0	0	1
2	2	2	0	0	0	1
3	3	2	1	0	1	1
4	4	4	4	4	0	1
5	5	5	5	5	5	0

Note that $\{0, 2\}$ is not a strong ideal of X since $(3 * 2) * 4 = 0 \in \{0, 2\}$ and $3 * 4 = 1 \notin \{0, 2\}$. Then $\{0, 1, 2, 3, 4\}$ is an enlarged strong ideal of X related to $\{0, 2\}$.

Theorem 4.3. *Let I be a non-empty subset of a BH-algebra X. Every enlarged strong ideal of X related to I is an enlarged ideal of X related to I.*

Proof. Let J be an enlarged strong ideal of X related to I. Putting $z := 0$ in Definition 4.1(3), we have

$$(\forall x, y \in X)((x * y) * 0 = x * y \in I \text{ and } y \in I \Rightarrow x * 0 = x \in J).$$

Hence J is an enlarged strong ideal of X related to I. □

The converse of Theorem 4.3 is not true in general as seen in the following example.

Example 4.4. Let $X := \{0, 1, 2, 3\}$ be a BH-algebra which is not a BCK/BCI-algebra with the following table:

$*$	0	1	2	3
0	0	3	0	2
1	1	0	0	0
2	2	2	0	3
3	3	3	3	0

Note that $\{0, 2\}$ is not an ideal of X since $1*2 = 0 \in \{0, 2\}$, but $1 \notin \{0, 2\}$. Then $\{0, 1, 2\}$ is an enlarged ideal of X related to $\{0, 2\}$. But it is not an enlarged strong ideal of X since $(2 * 2) * 3 = 2 \in \{0, 2\}$ but $2 * 3 = 3 \notin \{0, 1, 2\}$.

Definition 4.5. A fuzzy set μ in a BH-algebra X is called a *fuzzy strong ideal* of X if (FI1) and

(FI4) $\mu(x * z) \geq \min\{\mu((x * y) * z), \mu(y)\}, \forall x, y \in X$.

Example 4.6. Let $X := \{0, 1, 2, 3\}$ be a BH-algebra with the following table:

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	2	0	0
3	3	2	2	0

Note that $\{0, 2\}$ is not an ideal of X since $1*2 = 0 \in \{0, 2\}$, but $1 \notin \{0, 2\}$. Then $\{0, 1, 2\}$ is an enlarged ideal of X related to $\{0, 2\}$. But it is not an enlarged strong ideal of X since $(3 * 2) * 0 = 2 \in \{0, 2\}$ but $3 * 0 = 3 \notin \{0, 1, 2\}$.

Definition 4.7. A fuzzy subset μ of a BH-algebra X is called a *fuzzy strong ideal* of X with degree (λ, κ) if it satisfies:

(1) $(\forall x \in X)(\mu(0) \geq \lambda\mu(x))$,
(2) $(\forall x, y, z \in X)(\mu(x * z) \geq \kappa\min\{\mu((x * y) * z), \mu(y)\})$.

Note that if $\lambda \neq \kappa$, then a fuzzy strong ideal with degree (λ, κ) may not a fuzzy strong ideal with degree (κ, λ), and vice versa.

Example 4.8. Consider a BH-algebra $X = \{0, 1, 2, 3, 4, 5\}$ as in Example 4.2. Define a fuzzy subset μ of X by

$$\mu = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.7 & 0.9 & 0.8 & 0.6 & 0.5 & 0.3 \end{pmatrix}$$

Then μ is a fuzzy strong ideal of X with degree $(0.6, 0.5)$ but it is not a fuzzy strong ideal of X with degree $(0.8, 0.5)$ since $\nu(0) = 0.7 \ngeq 0.72 = 0.8 \times \mu(1)$.

Obviously, every fuzzy strong ideal is a fuzzy strong ideal with degree (λ, κ), but the converse may not be true. In fact, the fuzzy strong ideal μ with degree $(0.6, 0.5)$ in Example 4.8 is not a fuzzy strong ideal of X since $\mu(0) = 0.7 < \mu(1) = 0.9$. Note that a fuzzy strong ideal with degree (λ, κ) is a fuzzy strong ideal if and only if $(\lambda, \kappa) = (1, 1)$. If $\lambda_1 \geq \lambda_2$ and $\kappa_1 \geq \kappa_2$, then every fuzzy strong ideal with degree (λ_1, κ_1) is a fuzzy strong ideal with (λ_2, κ_2), but the converse is not true as shown by Example 4.8.

Theorem 4.9. *Let μ be a fuzzy subset of a BH-algebra X. For any $t \in (0, 1]$ with $t \leq \lambda$, if $U(\mu; t)$ is an enlarged strong ideal of X related to $U(\mu; \frac{1}{\max\{\lambda,\kappa\}})$, then μ is a fuzzy strong ideal of X with degree (λ, κ).*

Proof. Assume that $\mu(0) < t \leq \lambda\mu(x)$ for some $x \in X$ and $t \in (0, \lambda]$. Then $\mu(x) \geq \frac{t}{\lambda} \geq \frac{t}{\max\{\lambda,\kappa\}}$. Hence $x \in U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$, i.e., $U(\mu; \frac{t}{\max\{\lambda,\kappa\}}) \neq \emptyset$. Since $U(\mu; t)$ is an enlarged strong ideal of X related to $U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$, $0 \in U(\mu; t)$, i.e., $\mu(0) \geq t$. This is a contradiction, and thus $\mu(0) \geq \lambda\mu(x)$ for all $x \in X$.

Now suppose that there exist $a, b, c \in X$ such that $\mu(a * c) < \kappa\min\{\mu((a * b) * c)), \mu(b)\}$. If we take $t := \kappa\min\{\mu((a * b) * c), \mu(b)\}$, then $t \in (0, \kappa] \subseteq (0, \max\{\lambda, \kappa\}]$, $(a * b) * c \in U(\mu; \frac{t}{\kappa}) \subseteq U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$ and $b \in U(\mu; \frac{t}{\kappa}) \subseteq$

$U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$. It follows from Definition 4.1(3) that $a * c \in U(\mu; t)$ so that $\mu(a * c) \geq t$, which is impossible. Therefore

$$\mu(x * z) \geq \kappa \min\{\mu((x * y) * z), \mu(y)\}$$

for all $x, y, z \in X$. Hence μ is a fuzzy strong ideal of X with degree (λ, κ). □

Corollary 4.10. *Let μ be a fuzzy subset of a BH-algebra X. For any $t \in [0, 1]$ with $t \leq \frac{k}{n}$, if $U(\mu; t)$ is an enlarged strong ideal of X related to $U(\mu; \frac{n}{k}t)$, then μ is a fuzzy strong ideal of X with degree $(\frac{k}{n}, \frac{k}{n})$.*

Theorem 4.11. *Let $t \in [0, 1]$ be such that $U(\mu; t)(\neq \emptyset)$ is not necessary an ideal of a BH-algebra X. If μ is a fuzzy strong ideal of X with degree (λ, κ), then $U(\mu; t\min\{\lambda, \kappa\})$ is an enlarged strong ideal of X related to $U(\mu; t)$.*

Proof. Since $t\min\{\lambda, \kappa\} \leq t$, we get $U(\mu; t) \subseteq U(\mu; t\min\{\lambda, \kappa\})$. Since $U(\mu; t)(\neq \emptyset)$, there exists $x \in U(\mu; t)$ and so $\mu(x) \geq t$, Using Definition 4.7(1), we have $\mu(0) \geq \lambda\mu(x) \geq \lambda t \geq t\min\{\lambda, \kappa\}$, which implies that $0 \in U(\mu; t\min\{\lambda, \kappa\})$. Let $x, y, z \in X$ be such that $(x*y)*z \in U(\mu; t)$ and $y \in U(\mu; t)$. Then $\mu((x*y)*z) \geq t$ and $\mu(y) \geq t$. It follows from Definition 4.7(2) that

$$\mu(x * z) \geq \kappa \min\{\mu((x * y) * z), \mu(y)\} \geq \kappa t \geq t\min\{\lambda, \kappa\}$$

so that $x * z \in U(\mu; t\min\{\lambda, \kappa\})$. Therefore $U(\mu; t\min\{\lambda, \kappa\})$ is an enlarged strong ideal of X related to $U(\mu; t)$. □

Theorem 4.12. *Let μ be a fuzzy strong ideal of a BH-algebra X with degree (λ, κ). If the inequality $x * y \leq z$ holds in X, then*

$$\mu(x * z) \geq \kappa\lambda\mu(y), \ \forall x, y, z \in X.$$

Proof. Assume that $x * y \leq z$ for all $x, y, z \in X$. Then $(x * y) * z = 0$ and hence

$$\begin{aligned} \mu(x * z) &\geq \kappa \min\{\mu((x * y) * z), \mu(y)\} \\ &= \kappa \min\{\mu(0), \mu(y)\} \\ &\geq \kappa \min\{\lambda\mu(y), \mu(y)\} \\ &= \kappa\lambda\mu(y) \end{aligned}$$

for all $x, y, z \in X$. □

Corollary 4.13. *Let μ be a fuzzy strong ideal of a BH-algebra X with degree (λ, κ). If $\lambda = \kappa$ and the inequality $x * y \leq z$ holds in X, then*

$$\mu(x * z) \geq \kappa^2\mu(y), \ \forall x, y, z \in X.$$

References

1. S. S. Ahn and E.M. Kim, *On n-fold strong ideals of BH-algebras*, Honam Math. Journal, **33** (2011), 271-277.
2. S. S. Ahn and J. H. Lee, *Rough strong ideals in BH-algebras*, Honam Math. Journal, **32**(2010), 203-215.
3. K. Iséki, *On BCI-algebras*, Mathematics Seminar Notes, **8**(1980), 125-130.
4. K. Iséki and S. Tanaka , *An introduction to the theory of BCK-algebras*, Math. Japonica, **23**(1978), 1-26.
5. Y. B. Jun, E. H. Roh and H. S. Kim, *On BH-algebras*, Scientae Math. **1**(1998), 347-354.
6. E. M. Kim and S. S. Ahn, *On fuzzy n-fold strong ideals of BH-algebras*, J. Appl. Math. & Informatics, **30** (2012), 665-676.
7. D. Mundici, *MV-algebras are categorically equivalent to bounded commutative BCK-algebras*, Math. Japon., **31**(1986), 889-894.
8. J. Meng, *Implicative commutative semigroups are equivalent to a class of BCK-algebras*, Semigroup Forum, **50**(1995), 89-96.
9. J. Meng and Y. B. Jun, *BCK-algebras*, Kyung Moon Sa, Seoul, 1994.
10. Y. B. Jun, E. H. Roh and Q. Zhang, *On fuzzy translation BH-ideals in BH-algebras*, J. Fuzzy Math., **8**(2008), 361-370.
11. E. H. Roh and S. Y. Kim, *On BH^*-subalgebras of transitive BH^*-algebras*, Far East J. Math. Sci., **1**(1999), 255-263.

Eun Mi Kim is working as a teacher in Sanggye High School and is interested in BH/BCK-algebras and fuzzy Algebras.

Department of Mathematics Education, Dongguk University, Seoul 100-715, Korea.
e-mail: duchil@hanmail.net

Sun Shin Ahn is working as a professor at Dongguk University and is interested in BH/BCK-algebras and fuzzy algebras.

Department of Mathematics Education, Dongguk University, Seoul, 100-715, Korea.
e-mail: sunshine@dongguk.edu