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Deep Web and MapReduce

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Abstract
This invited paper introduces results on Web science and technology obtained during work with the Korea Advanced Institute of Science and Technology. In the first part, we discuss algorithms for exploring the *deep Web,* which refers to the collection of Web pages that cannot be reached by conventional Web crawlers. In the second part, we discuss sorting algorithms on the *MapReduce* system, which has become a dominant paradigm for massive parallel computing.

Category: Smart and intelligent computing

Keywords: Web; Big data; MapReduce; Parallel computing; Algorithm; Theory

I. INTRODUCTION

From Sep 2011 to Jun 2013, I was a visiting professor under the *World Class University* program of the Korean government, at the Korea Advanced Institute of Science and Technology (KAIST). My affiliation at KAIST was with the Division of Web Science and Technology (WebST), a newly established division with a primary mission to explore the relatively new but exciting area of *Web science.* In this invited paper, some of my research results contributing to that mission will be discussed. During my appointment with KAIST, I had retained my professorship at the Chinese University of Hong Kong, and worked at both institutions alternately during the development of these results, which therefore should be accredited to both institutions.

A. Deep Web

Existing search engines can reach only a small portion of the Internet. They crawl HTML pages interconnected with hyperlinks, which constitute one is known as the *surface Web.* An increasing number of organizations are bringing their data online, by allowing public users to query their back-end databases through context-dependent Web interfaces.

Data acquisition is performed by *interacting* with the interface at runtime, as opposed to following hyperlinks. As a result, the back-end databases cannot be effectively crawled by a search engine with current technology, and are usually referred to as *hidden databases*.

Consider *Yahoo! Autos* (autos.yahoo.com), a popular Website for the online trading of automobiles. A potential buyer specifies filtering criteria through a form, as illustrated in Fig. 1. The query is submitted to the system, which runs it against the back-end database, and returns the result to the user. What makes it non-trivial for a search engine to crawl the database is that setting all search criteria to ANY does not accomplish the task. The reason is that a system typically limits the number k of tuples returned (which is roughly 1000 for *Yahoo! Autos*), and that repeating the same query may not retrieve new tuples, with the same k tuples always being returned.

The ability of crawling a hidden database comes with

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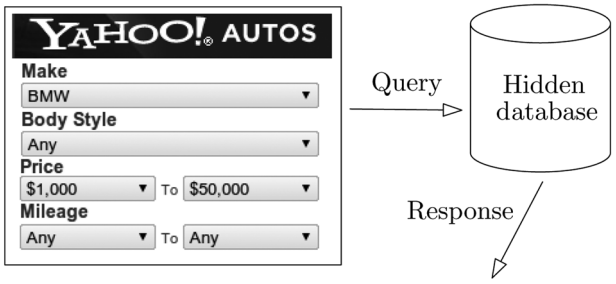

Make	Body style	Price ($)	Mileage (mile)
BMW	sedan	17,500	68,647
BMW	sedan	17,500	76,072
BMW	coupe	3,299	158,573
BMW	convertible	50,000	5,231
	...		

Fig. 1. Form-based querying of a hidden database.

the appeal of enabling virtually any form of processing on the database's content. The challenge, however, is clear: how to obtain *all* the tuples, given that the system limits the number of return tuples for each query. A naive solution is to issue a query for every single location in the data space (for example, in Fig. 1, the data space is the Cartesian product—while one may leverage knowledge of attribute dependencies (e.g., the fact that BMW does not sell trucks in the United States) to prune the data space into a subset of the Cartesian product, the subset is often still too large to enumerate—of the domains of MAKE, BODY STYLE, PRICE, and MILEAGE). However, the number of queries needed can clearly be prohibitive. This gives rise to an interesting problem, as defined in the next subsection, where the objective is to *minimize* the number of queries.

1) Problem Definitions

Consider that a *data space* $\mathbb{D}$ with d attributes $A_1,..., A_d$, each of which has a discrete domain. The domain of A_i denote by $dom(A_i)$ for each $i \in [1, d]$. Then, $\mathbb{D}$ is the Cartesian product of $dom(A_1)$, …, $dom(A_d)$. We refer to each element of the Cartesian product as a *point* in $\mathbb{D}$, representing one possible combination of values of all dimensions.

A_i is a *numeric* attribute if there is a total ordering on $dom(A_i)$. Otherwise, it is a *categorical* attribute. Our discussion distinguishes three types of $\mathbb{D}$:

- **Numeric:** all d attributes of $\mathbb{D}$ are numeric.
- **Categorical:** all d attributes are categorical. In this case, we use U_i to represent the size of $dom(A_i)$, which represents how many distinct values there are in $dom(A_i)$.
- **Mixed:** the first $cat \in [1, d-1]$ attributes $A_1,…, A_{cat}$ are categorical, whereas the other $d - cat$ attributes are numeric. Similar to before, let $U_i = |dom(A_i)|$ for each $i \in [1, cat]$.

To facilitate presentation, we consider the domain of a numeric A_i to be the set of all integers, whereas that of a categorical A_i to be the set of integers from 1 to U_i. Keep in mind, however, that the ordering of these values is irrelevant to a categorical A_i.

Let D be the hidden database of a server with each element of D being a point in $\mathbb{D}$. To avoid ambiguity, we will always refer to elements of D as *tuples*. D is a *bag* (i.e., a multi-set), and it may contain identical tuples.

The server supports *queries* on D. As shown in Fig. 1, each query specifies a predicate on each attribute. Specifically, if A_i is numeric, the predicate is a range condition in the form of:

$$A_i \in [x, y]$$

where $[x, y]$ is an interval in $dom(A_i)$. For a categorical A_i, the predicate is:

$$A_i = x$$

where x is either a value in $dom(A_i)$ or a wildcard ★. In particular, a predicate A_i = ★ means that A_i can be an arbitrary value in $dom(A_i)$, as shown with capturing BODY STYLE = ANY in Fig. 1. If a hidden database server only allows single-value predicates on a numeric attribute (i.e., no range-condition support), then we can simply consider the attribute as categorical.

Given a query q, the bag of tuples in D qualifying all the predicates of q is denoted by $q(D)$. The server does not necessarily return the entire $q(D)$—it does so *only* when $q(D)$ is small. Formally, the response of the server is:

- if $|q(D)| \le k$: the entire $q(D)$ is returned. In this case, we say that q is *resolved*.
- Otherwise: only k tuples—in practice, these are usually k tuples that have the highest priorities (e.g., according to a ranking function) among all the tuples qualifying the query—in $q(D)$ are returned, together with a signal indicating that $q(D)$ still has other tuples. In this case, we say that q *overflows*.

The value of k is a system parameter (e.g., k = 1,000 for *Yahoo! Autos*, as mentioned earlier). It is important to note that in the event that a query q overflows, repeatedly issuing the same q may always result in the same response from the server, and does not help to obtain the other tuples in $q(D)$.

PROBLEM 1 (HIDDEN DATABASE CRAWLING)**.** Retrieve the entire D while minimizing the number of queries.

Recall that D is a bag, and it may have duplicate tuples. We require that no point in the data space $\mathbb{D}$ has *more than k* tuples in D. Otherwise, Problem 1 has no solution at all. To see this, consider the existence of $k + 1$ tuples t_1, ..., t_{k+1} in D, all of which are equivalent to a point $p \in \mathbb{D}$. Then, whenever p satisfies a query, the server can *always* choose to leave t_{k+1} out of its response, making it impossible for any algorithm to extract the entire D. In *Yahoo!*

Autos, this requirement essentially states that there cannot be $k = 1{,}000$ vehicles in the database with exactly the same values for *all* attributes—an assumption that is fairly realistic.

As mentioned in Problem 1, the *cost* of an algorithm is the number of queries issued. This metric is motivated by the fact that most systems have control over how many queries can be submitted by the same IP address within a period of time. Therefore, a crawler must minimize the number of queries to complete a task, in addition to minimizing the burden to the server.

We will use n to denote the number of tuples in D. It is clear that the number of queries needed to extract the entire D is at least n/k. Of course, this ideal cost may not always be possible. Hence, there are two central technical questions that need to be answered. The first, on the upper bound side, relates to how to solve Problem 1 by performing only a small number of queries even in the *worst* case. The second, on the lower bound side, concerns how many queries are *compulsory* for solving the problem in the worst case.

2) Our Results

We have concluded a systematic study of *hidden database crawling* as defined in Problem 1. At a high level, our first contribution is a set of algorithms that are both provably fast in the worst case, and efficient on practical data. Our second contribution is a set of lower-bound results establishing the *hardness* of the problem. These results explicitly clarify how the hardness is affected by the underlying factors, and thus reveal valuable insights into the characteristics of the problem. Furthermore, the lower bounds also prove that our algorithms are already optimal asymptotically, and cannot be improved by more than a constant factor.

Our first main result is:

THEOREM 1. *There is an algorithm for solving Problem 1 whose cost is:*

- $O(d \cdot \frac{n}{k})$ *when* $\mathbb{D}$ *is numeric;*
- *at most* U_1 *when* $\mathbb{D}$ *is categorical and* $cat = 1$ *(i.e., there is only one categorical attribute);*
- *at most* $\frac{n}{k} \cdot \sum_{i=1}^{d} min\left\{U_i, \frac{n}{k}\right\} + \sum_{i=1}^{d} U_i$ *when* $\mathbb{D}$ *is categorical and* $cat > 1$*;*
- *at most* $U_1 + O(d \cdot \frac{n}{k})$ *when* $\mathbb{D}$ *is mixed and* $cat = 1$*;*
- *otherwise (i.e.,* $\mathbb{D}$ *is mixed and* $cat > 1$*): at most* $\frac{n}{k} \cdot \sum_{i=1}^{cat} \min\left\{U_i, \frac{n}{k}\right\} + \sum_{i=1}^{cat} U_i + O\left((d - cat)\frac{n}{k}\right)$.

The above can be conveniently understood as follows: our algorithm pays an additive cost of $O(n/k)$ for each numeric attribute A_i, whereas it pays $\frac{n}{k} \cdot \min\{U_i, \frac{n}{k}\} + U_i$ for each categorical A_i. The only exception is when $cat = 1$: in this scenario, we pay merely U_1 for the (only) categorical attribute A_1. The cost of each numeric attribute is *irrelevant* to its domain size.

Our second main result complements the preceding one:

THEOREM 2. *None of the results in Theorem 1 can be improved by more than a constant factor in the worst case.*

Besides establishing the optimality of our upper bounds in Theorem 1, Theorem 2 has its own interesting implications. First, it indicates the unfortunate fact that for all types of $\mathbb{D}$, the best achievable query time in the worst case is much higher than the ideal cost of n/k. Nevertheless, Theorem 1 suggests that we can achieve this cost *asymptotically* when d is a constant and all attributes are numeric. Second, as the number cat of categorical attributes increases from 1 to 2, the discrepancy of the time complexities in Theorem 1 is not an artifact, but rather, it is due to an inherent *leap* in the hardness of the problem (which is true regardless of the number of numeric attributes). That is, while we pay only $O(U_1)$ extra queries for the (sole) categorical attribute when $cat = 1$, as cat grows to 2 and beyond, the cost paid by any algorithm for each categorical A_i has an extra term of $\frac{n}{k} \min\{U_i, \frac{n}{k}\}$. Given that the term is multiplicative, this finding implies (perhaps surprisingly) that, in the worst case, it may be infeasible to crawl a hidden database with a large size n, and at least 2 categorical attributes such that at least one of them has a large domain.

In this paper, we prove Theorems 1 and 2 only for the case where $\mathbb{D}$ is numeric. The rest of the proof is available elsewhere [1].

B. MapReduce

We are in an era of information explosion, where industry, academia, and governments are accumulating data at an unprecedentedly high speed. This brings forward the urgent need for fast computation over colossal datasets whose sizes can reach the order of terabytes or higher. In recent years, the database community has responded to this challenge by building massive parallel computing platforms which use hundreds or even thousands of commodity machines. The most notable platform thus far is *MapReduce,* which has attracted a significant amount of attention in research.

Since its invention [2], MapReduce has gone through years of improvement into a mature paradigm. At a high level, a MapReduce system involves a number of share-nothing machines which communicate only by sending messages over the network. A MapReduce algorithm instructs these machines to perform a computational task collaboratively. Initially, the input dataset is distributed

across the machines, typically in a non-replicate manner, with each object on one machine. The algorithm executes in *rounds* (sometimes also called *jobs* in the literature), each having three phases: *map, shuffle,* and *reduce.* The first two enable the machines to exchange data. In the map phase, each machine prepares the information to be delivered to other machines, while the shuffle phase takes care of the actual data transfer. No network communication occurs in the reduce phase, where each machine performs calculation with its local storage. The current round finishes after the reduce phase. If the computational task has not completed, another round starts.

As with traditional parallel computing, a MapReduce system aims to achieve a high degree of load balancing, as well as the minimization of space, CPU, I/O, and network costs at each individual machine. Although these principles have guided the design of MapReduce algorithms, previous practices have mostly been on a best-effort basis, paying relatively less attention to enforcing serious constraints on different performance metrics. Our work aims to remedy the situation by studying algorithms that promise outstanding efficiency in multiple aspects simultaneously.

1) Minimal MapReduce Algorithms

Let S be the set of input objects for the underlying problem. Let n be the *problem cardinality*, which is the number of objects in S, and t be the number of machines used in the system. Define $m = n/t$, where m is the number of objects per machine when S is evenly distributed across the machines. Consider an algorithm for solving a problem on S. We say that the algorithm is *minimal* if it has all of the following properties:

- *Minimum footprint:* at all times, each machine uses only $O(m)$ space of storage.
- *Bounded net-traffic:* in each round, every machine sends and receives at most $O(m)$ words of information over the network.
- *Constant round:* the algorithm must terminate after a constant number of rounds.
- *Optimal computation:* every machine performs only $O(T_{seq}/t)$ amount of computation *in total* (i.e., summing over all rounds), where T_{seq} is the time needed to solve the same problem on a single sequential machine. The algorithm should achieve a speedup of t by using t machines in parallel.

It is fairly intuitive why minimal algorithms are appealing. First, a *minimum footprint* ensures that each machine keeps $O(1/t)$ of the dataset S at any moment. This effectively prevents *partition skew*, where some machines are forced to handle considerably more than m objects, as is a major cause of inefficiency in MapReduce [3].

Second, *bounded net-traffic* guarantees that the shuffle phase of each round transfers at most $O(m \cdot t) = O(n)$ words of network traffic overall. The duration of the phase equals roughly the time for a machine to send and receive $O(m)$ words, because the data transfers between different machines are in parallel. Furthermore, this property is also useful when one wants to make an algorithm *stateless* for the purpose of fault tolerance.

The third property *constant round* is not new, as it has been the goal of many previous MapReduce algorithms. Importantly, this and the previous properties imply that there can be only $O(n)$ words of network traffic during the *entire* algorithm. Finally, *optimal computation* echoes the very original motivation of MapReduce to accomplish a computational task t times faster than leveraging only one machine.

2) Our Results

The core of this work comprises a neat minimal algorithm for:

Sorting. The input is a set S of n objects drawn from an ordered domain. When the algorithm terminates, all the objects must have been distributed across the t machines in a sorted fashion. That is, we can order the machines from 1 to t such that all objects in machine i precede those in machine j for all $1 \le i \le j \le t$.

Sorting can be settled in $O(n \log n)$ time on a sequential computer. There has been progress in developing MapReduce algorithms for this important operation. The state of the art is *TeraSort* [4], which won Jim Gray's benchmark contest in 2009. *TeraSort* comes close to being minimal when a crucial parameter is set appropriately. As will be made clear later, the algorithm requires manual tuning of the parameter, an improper choice of which can incur severe performance penalties.

Our work was initialized by an attempt to justify theoretically why *TeraSort* often achieves excellent sorting time with only 2 rounds. In the first round, the algorithm extracts a random sample set S_{samp} of the input S, and then picks $t - 1$ sampled objects as the *boundary objects*. Conceptually, these boundary objects divide S into t segments. In the second round, each of the t machines acquires all the objects in a distinct segment, and sorts them. The size of S_{samp} is the key to efficiency. If S_{samp} is too small, the boundary objects may be insufficiently scattered, which can cause partition skew in the second round. Conversely, an over-sized S_{samp} entails expensive sampling overhead. In the standard implementation of *TeraSort,* the sample size is left as a parameter, although it always seems to admit a good choice that gives outstanding performance [4].

We provide a rigorous explanation of the above phenomenon. Our theoretical analysis clarifies how to set the size of S_{samp} to guarantee the minimality of *TeraSort.* In the meantime, we also remedy a conceptual drawback of *TeraSort.* Strictly speaking, this algorithm does not fit in the MapReduce framework, because it requires that,

besides network messages, the machines should be able to communicate by reading/writing a common distributed file. Once this is disabled, the algorithm requires one more round. We present an elegant fix so that the algorithm still terminates in 2 rounds even by strictly adhering to MapReduce. Our findings with *TeraSort* have immediate practical significance, given the essential role of sorting in a large number of MapReduce programs.

It is worth noting that a minimal algorithm for sorting leads to minimal algorithms for several fundamental database problems, including ranking, *group-by*, *semi-join*, and *skyline* [5].

II. CRAWLING THE DEEP WEB

This section explains how to solve Problem 1 when the data space $\mathbb{D}$ is numeric. In Section II-A, we first define some atomic operators, and present an algorithm that is intuitive, but has no attractive performance bounds. Then, in Sections II-B and II-C, we present another algorithm, which achieves the optimal performance, as proven in Section II-D.

A. Basic Operations and Baseline Algorithm

Recall that in a numeric $\mathbb{D}$, the predicate of a query q on each attribute is a range condition. Thus, q can be regarded as a d-dimensional (axis-parallel) rectangle, such that its result $q(D)$ consists of the tuples of D covered by that rectangle. If the predicate of q on attribute A_i ($i \in [1, d]$) is $A_i \in [x_1, x_2]$, we say that $[x_1, x_2]$ is the *extent* of the rectangle of q along A_i. Henceforth, we may use the symbol q to refer to its rectangle also, when no ambiguity can be caused. Clearly, settling Problem 1 is equivalent to determining the entire $q(D)$ where q is the rectangle covering the whole $\mathbb{D}$.

Split. A fundamental idea to extract all the tuples in $q(D)$ is to refine q into a set S of smaller rectangles, such that each rectangle $q' \in S$ can be resolved (i.e., $q'(D)$ has at most k tuples). Note that this always happens as long as rectangle q' is sufficiently small. In an the extreme case, when q' has degenerated into a point in $\mathbb{D}$, the query q' is definitely resolved (otherwise, there would be at least $k + 1$ tuples of D at this point). Therefore, a basic operation in our algorithms for Problem 1 is *split*.

Given a rectangle q, we may perform two types of splitting, depending on how many rectangles q is divided into:

1) **2-way split:** Let $[x_1, x_2]$ be the extent of q on A_i (for some $i \in [1, d]$). A *2-way split at a value* $x \in [x_1, x_2]$ partitions q into rectangles q_{left} and q_{right}, by dividing the A_i-extent of q at x. Formally, on any attribute other than A_i, q_{left} and q_{right} have the same extents as q. Along A_i, however, the extent q_{left} is $[x_1, x - 1]$, whereas that of q_{right} is $[x, x_2]$. Fig. 2a illustrates the idea by splitting on the horizontal attribute.

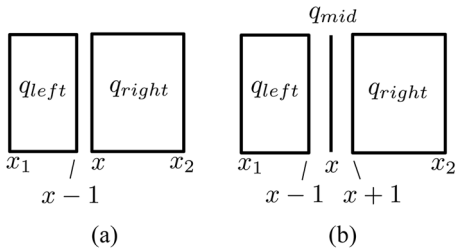

Fig. 2. Splitting: (a) 2-way and (b) 3-way.

2) **3-way split:** Let $[x_1, x_2]$ be defined as above. A *3-way split at a value* $x \in [x_1, x_2]$ partitions q into rectangles q_{left}, q_{mid}, and q_{right} as follows. On any attribute other than A_i, they have the same extent as q. Along A_i, however, the extent of q_{left} is $[x_1, x - 1]$, that of q_{mid} is $[x, x]$, and that of q_{right} is $[x + 1, x_2]$ (Fig. 2b).

In the sequel, a 2-way split will be abbreviated simply as a split. No confusion can arise as long as we always mention 3-way as referring to a 3-way split. The extent of a query q on an attribute A_i can become so short that it covers only a single value, in which case we say that A_i is *exhausted* on q. For instance, the horizontal attribute is exhausted on q_{mid} in Fig. 2b. It is easy to see that there is always a *non-exhausted* attribute on q unless q has degenerated into a point.

Binary-shrink. Next, we describe a straightforward algorithm for solving Problem 1, which will serve as the baseline approach for comparison. This algorithm, named *binary-shrink*, repeatedly performs 2-way splits until a query is resolved. Specifically, given a rectangle q, *binary-shrink* runs the rectangle (by submitting its corresponding query to the server) and finishes if q is resolved. Otherwise, the algorithm splits q on an attribute A_i that has not been exhausted, by cutting the extent $[x_1, x_2]$ of q along A_i into equally long intervals (i.e., the split is performed at $x = \lceil (x_1 + x_2)/2 \rceil$). Let q_{left}, q_{right} be the queries produced by the split. The algorithm then recurses on q_{left} and q_{right}.

It is clear that the cost of *binary-shrink* (i.e., the number of queries issued) depends on the domain sizes of the numeric attributes of $\mathbb{D}$, which can be *unbounded*. In the following subsections, we will improve this algorithm to optimality.

B. One-Dimensional Case

Before giving our ultimate algorithm for settling Problem 1 with any dimensionality d, in this subsection, we first explain how it works for $d = 1$. This will clarify the rationale behind the algorithm's efficiency, and facilitate our analysis for a general d. It is worth mentioning that the presence of only one attribute removes the need to specify the split dimension in describing a split.

Rank-shrink. Our algorithm *rank-shrink* differs from *binary-shrink* in two ways. First, when performing a 2-way split, instead of cutting the extent of a query q in half, we aim at ensuring that at least $k/4$ tuples fall in *each* of the rectangles generated by the split. Such a split, however, may not always be possible, which can happen if many tuples are identical to each other. Hence, the second difference that *rank-shrink* makes is to perform a 3-way split in such a scenario, which gives birth to a query (among the 3 created) that can be immediately resolved.

Formally, given a query q, the algorithm eventually returns $q(D)$. It starts by issuing q to the server, which returns a bag R of tuples. If q is resolved, the algorithm terminates by reporting R. Otherwise (i.e., in the event that q overflows), we sort the tuples of R in ascending order, breaking ties arbitrarily. Let o be the $(k/2)$-th tuple in the sorted order, with its A_1-value being x. Now, we count the number c of tuples in R identical to o (i.e., R has c tuples with A_1-value x), and proceed as follows:

1) Case 1: $c \le k/4$. Split q at x into q_{left} and q_{right}, each of which must contain at least $k/4$ tuples in R. To see this for q_{left} (symmetric reasoning applies to q_{right}), note there are at least $k/2 - c \ge k/4$ tuples of R *strictly* smaller than x, all of which fall in q_{left}. The case for q_{right} follows in analogy.
2) Case 2: $c > k/4$. Perform a 3-way split on q at x. Let q_{left}, q_{mid}, and q_{right} be the resulting rectangles (note that the ordering among them matters; see Section II-B). Observe that q_{mid} has degenerated into point x, and therefore, can immediately be resolved. As a technical remark, in Case 2, x might be the lower (resp. upper) bound—x cannot be both because otherwise q would be a point and therefore could not have overflowed—on the extent of q. If this happens, we simply discard q_{left} (or q_{right}) as it would have a meaningless extent.

In either case, we are left with at most two queries (i.e., q_{left} and q_{right}) to further process. The algorithm handles each of them recursively in the same manner.

Example. We use the dataset D in Fig. 3a to demonstrate the algorithm. Let $k = 4$. The first query is $q_1 = (-\infty, \infty)$. Suppose that the server responds by returning $R_1 = \{t_4, t_6, t_7, t_8\}$ and a signal that q_1 overflows. The $(k/2) = 2$nd smallest tuple in R_1 is t_6 (after random tie breaking), whose value is $x = 55$. As R_1 has $c = 3$ tuples with value 55 and $c > k/4 = 1$, we perform a 3-way split on q_1 at 55, generating $q_2 = (-\infty, 54]$, $q_3 = [55, 55]$, and $q_4 = [56, \infty)$. As q_3 has degenerated into a point, it is resolved immediately, fetching t_6, t_7, and t_8. These tuples have already been extracted before, but this time they come with an extra fact that no more tuple can exist at point 55.

Consider q_2. Suppose that the server's response is $R_2 = \{t_1, t_2, t_4, t_5\}$, plus an overflow signal. Hence, $x = 20$ and $c = 1$. Thus, a 2-way split on q_2 at 20 creates $q_5 = (-\infty, 19]$ and $q_6 = [20, 54]$. Queries q_4, q_5, and q_6 are all resolved.

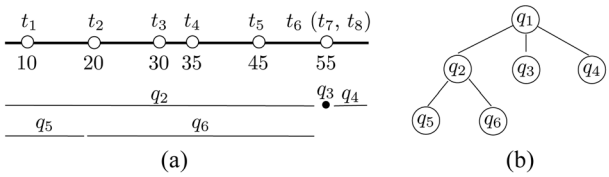

Fig. 3. Illustration of 1d *rank-shrink*: (a) dataset D and queries and (b) recursion tree.

Analysis. The lemma below bounds the cost of *rank-shrink*.

Lemma 1. *When $d = 1$, rank-shrink requires $O(n/k)$ queries.*

Proof. The main tool used by our proof is a *recursion tree* T that captures the spawning relationships of the queries performed by *rank-shrink*. Specifically, each node of T represents a query. Node u is the parent of node u' if query u' is created by a 2-way or 3-way split of query u. Each internal node thus has 2 or 3 child nodes. Fig. 3b shows the recursion tree for the queries performed in our earlier example on Fig. 3a.

We focus on bounding the number of leaves in T because it dominates the number of internal nodes. Observe that each leaf v corresponds to a *disjoint* interval in $dom(A_1)$, due to the way splits are carried out. There are three types of v:

- Type-1: the query represented by v is immediately resolved in a 3-way split (i.e., q_{mid} in Case 2). The interval of v contains at least $k/4$ identical tuples in D.
- Type-2: query v is not type-1, but also covers at least $k/4$ tuples in D.
- Type-3: query v covers less than $k/4$ tuples in D.

For example, among the leaf nodes in Fig. 3, q_3 is of type-1, q_5 and q_6 are of type-2, and q_4 is of type-3.

As the intervals of various leaves cover disjoint bags of tuples, the number of type-1 and type-2 leaves is at $\frac{n}{k/4} = 4n/k$. Each leaf of type-3 must have a sibling in T that is a type-2 leaf (in Fig. 3, such a sibling of q_4 is q_3). In contrast, a type-2 leaf has at most 2 siblings. It thus follows that there are at most twice as many type-3 leaves as type-2, i.e., the number of type-3 leaves is no more than $8n/k$. This completes the proof.

This analysis implies that quite loosely, T has no more than $4n/k + 8n/k = 12n/k$ leaves. Thus, there cannot be more than this number of internal nodes in T. □

C. Rank-Shrink for Higher Dimensionality

We are now ready to extend *rank-shrink* to handle any $d > 1$. In addition to the ideas exhibited in the preceding subsection, we also apply an inductive approach, which

involves converting the d-dimensional problem to several $(d-1)$-dimensional ones. Our discussion below assumes that the $(d-1)$-dimensional problem has already been settled by *rank-shrink*.

Given a query q, the algorithm (as in 1d) sets out to solicit the server's response R, and finishes if q is resolved. Otherwise, it examines whether A_1 is exhausted in q, and whether the extent of q on A_1 has only 1 value x in $dom(A_1)$. If so, we can then focus on attributes A_2, ..., A_d. This is a $(d-1)$-dimensional version of Problem 1, in the $(d-1)$-dimensional subspace covered by the extents of q on A_2, ..., A_d, eliminating A_1 by fixing it to x. Hence, we invoke *rank-shrink* to solve it.

Consider that A_1 is not exhausted on q. Similar to the 1d algorithm, we will split q such that either every resulting rectangle covers at least $k/4$ tuples in R, or one of them can be immediately solved as a $(d-1)$-dimensional problem. The splitting proceeds exactly as described in Cases 1 and 2 of Section II. The only difference is that the rectangle q_{mid} in Case is not a point, but instead, a rectangle on which A_1 has been exhausted. Hence, q_{mid} is processed as a $(d-1)$-dimensional problem with *rank-shrink*.

As with the 1d case, the algorithm recurses on q_{left} and q_{right} (provided that they have not been discarded for having a meaningless extent on A_1).

Example. We demonstrate the algorithm using the 2d dataset in Fig. 4, where D has 10 tuples t_1, ..., t_{10}. Let $k = 4$. The first query q_1 issued covers the entire data space. Suppose that the server responds with $R_1 = \{t_4, t_7, t_8, t_9\}$ and an overflow signal. We split q_1 3-ways at $A_1 = 80$ into q_2, q_3, and q_4, whose rectangles can be found in Fig. 4. The A_1-extents of q_2, q_3, *and* q_4 are $(-\infty, 79]$, $[80, 80]$, and $[81, \infty)$, respectively, while their A_2-extents are all $(-\infty, \infty)$. Note that A_1 is exhausted on q_2; alternatively, we can see that q_2 is equivalent to a 1d query on the vertical line $A_1 = 80$. Hence, q_2 is recursively settled by our 1d algorithm (requiring 3 queries, which can be verified easily).

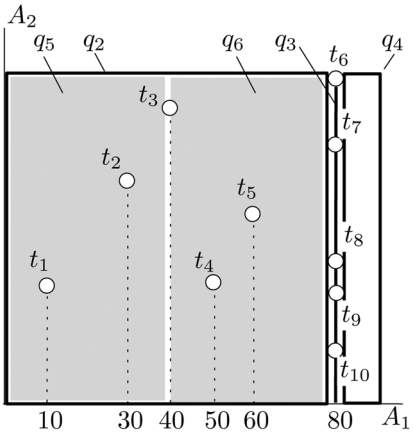

Fig. 4. Illustration of 2d *rank-shrink*.

Suppose that the server's response to q_2 is $R_2 = \{t_2, t_3, t_4, t_5\}$ and an overflow signal. Accordingly, q_2 is split into q_5 and q_6 at $A_1 = 40$, whose rectangles are also shown in Fig. 4. Finally, q_4, q_5, and q_6 are all resolved.

Analysis. We have the lemma below for general d:

LEMMA 2. *Rank-shrink performs $O(dn/k)$ queries.*

Proof. The case $d = 1$ has been proven in Lemma 1.

Next, assuming that *rank-shrink* issues at most $\alpha(d-1)n/k$ queries for solving a $(d-1)$-dimensional problem with n tuples (where α is a positive constant), we will show that the cost is at most $\alpha dn/k$ for dimensionality d.

Again, our argument leverages a recursion tree T. As before, each node of T is a query, such that node u parents node u', if query u' was created from splitting u. We make a query v a leaf of T as soon as one of the following occurs:

- v is resolved. We associate v with a *weight* set to 1.
- A_1 is exhausted on rectangle v. Recall that such a query is solved as a $(d-1)$-dimensional problem. We associate v with a weight, equal to the cost for *rank-shrink* for that problem.

For our earlier example in Fig. 4, the recursion tree T happens to be the same as the one in Fig. 3b. The difference is that each leaf has a weight. Specifically, the weight of q_3 is 3 (i.e., the cost of solving the 1d query at the vertical line $A_1 = 80$ in Fig. 4), and the weights of the other leaves are 1.

Therefore, the total cost of *rank-shrink* on the d-dimensional problem is equal to the total number of internal nodes in T, plus the total weight of all the leaves.

As the A_1-extents of the leaves' rectangles have no overlap, their rectangles cover disjoint tuples. Let us classify the leaves into type-1, -2, and -3, as in the proof of Lemma 1, by adapting the definition of type-1 in a straightforward fashion: v is of this type if it is the middle node q_{mid} from a 3-way split. Each type-leaf has weight 1 (as its corresponding query must be resolved). As proved in Lemma 1, the number of them is no more than $8n/k$.

Let v_1,..., v_β be all the type-1 and type-2 nodes (i.e., suppose the number of them is β). Assume that node v_i contains n_i tuples of D. It holds that $\sum_{i=1}^{\beta} n_i \le |D| = n$. The weight of v_i, by our inductive assumption, is at most $\alpha(d-1)n_i/k$. Hence, the total weight of all the type-1 and type-2 nodes does not exceed $\alpha(d-1)n/k$.

The same argument in the proof of Lemma 1 shows that T has less than $12n/k$ internal nodes. Thus, summarizing the above analysis, the cost of d-dimensional *rank-shrink* is no more than: $\frac{12n}{k} + \frac{8n}{k} + \alpha(d-1)\frac{n}{k} = (20 + \alpha(d-1))\frac{n}{k}$. To complete our inductive proof, we want $(20 + \alpha(d-1))\frac{n}{k}$ to be bounded from above by $\alpha dn/k$. This is true for any $\alpha \ge 20$. □

Remark. This concludes the proof of the first bullet of Theorem 1. When d is a fixed value (as is true in prac-

tice), the time complexity in Lemma 2 becomes $O(n/k)$, asymptotically matching the trivial lower bound n/k. A natural question at this point of whether there is an algorithm that can still guarantee cost $O(n/k)$ if d is not constant, Next, we will show that this is impossible.

D. A Lower Bound

The objective of this subsection is to establish:

THEOREM 3. *Let k, d, and m be arbitrary positive integers such that $d \le k$. There is a dataset D (in a numeric data space) with $n = m(k + d)$ tuples such that any algorithm must use at least dm queries to solve Problem 1 on D.*

It is therefore impossible to improve our algorithm *rank-shrink* (see Lemma 2) by more than a constant factor in the worst case, as shown below:

COROLLARY 1. *In a numeric data space, no algorithm can guarantee solving Problem 1 with $o(dn/k)$ queries.*

Proof. If there existed such an algorithm, let us use it on the inputs in Theorem 3. The cost is $o(dn/k) = o(dm(k+d)/k)$ which, due to $d \le k$, is $o(dm)$, causing a contradiction. □

We now proceed to prove Theorem 3 using a hard dataset D, as illustrated in Fig. 5. The domain of each attribute is the set of integers from 1 to m +1, or $\mathbb{D} = [1, m + 1]^d$. D has m groups of $d + k$ tuples. Specifically, the i-th group ($1 \le i \le m$) has k tuples at the point (i, ..., i), taking value i on all attributes. We call them *diagonal tuples.* Furthermore, for each $j \in [1, d]$, group i also has a tuple that takes value $i + 1$ on attribute A_j, and i on all other attributes. Such a tuple is referred to as a *non-diagonal* tuple. Overall, D has km diagonal and dm non-diagonal tuples.

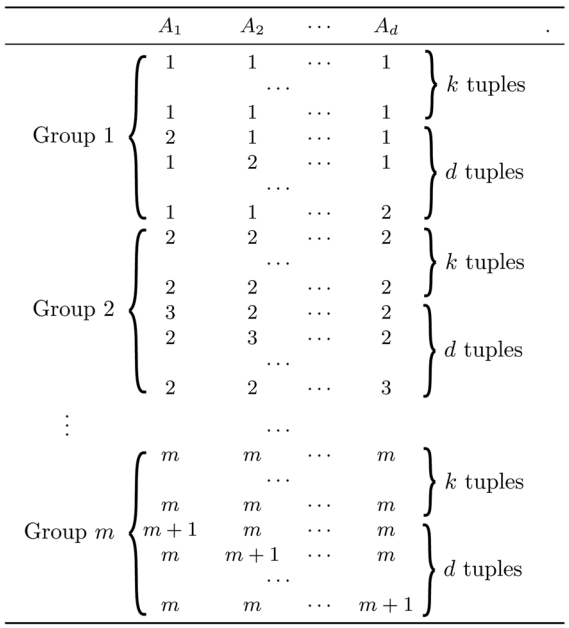

Fig. 5. A hard numeric dataset.

Let S be the set of dm points in $\mathbb{D}$ that are equivalent to the dm non-diagonal tuples in D, respectively (i.e., each point in S corresponds to a distinct non-diagonal tuple). As explained in Section II-A, each query can be regarded as an axis-parallel rectangle in $\mathbb{D}$. With this correspondence in mind, we observe the following for any algorithm that correctly solves Problem 1 on D.

LEMMA 3. *When the algorithm terminates, each point in S must be covered by a distinct resolved query already performed.*

Proof. Every point $p \in S$ must be covered by a resolved query. Otherwise, p is either never covered by any query, or covered by only overflowing queries. In the former case, the tuple of D at p could not have been retrieved, whereas in the latter, the algorithm could not rule out the possibility that D had more than one tuple at p. In neither case could the algorithm have terminated.

Next, we show that no resolved query q covers more than one point in S. Otherwise, assume that q contains p_1 and p_2 in S, in which case q fully encloses the minimum bounding rectangle, denoted as r, of p_1 and p_2. Without loss of generality, suppose that p_1 (p_j) is from group i (j) such that $i \le j$. If $i = j$, then r contains the point (i, ..., i), in which case at least $k + 2$ tuples satisfy q (i.e., p_1, p_2, and the k diagonal tuples from group i). Alternatively, consider $i < j$. In this scenario, the coordinate of p_1 is at most $i + 1 \le j$ on all attributes, while the coordinate of p_2 is at least j on all attributes. Thus, r contains the point (j, ..., j), causing at least $k + 2$ tuples to satisfy q (i.e., p_1, p_2, and the k diagonal tuples from group j). Therefore, q must overflow in any case, which is a contradiction. □

The lemma indicates that at least $|S| = dm$ queries must be performed, which validates Theorem 3.

III. MAPREDUCE

As explained earlier, a MapReduce algorithm proceeds in *rounds*, where each round has three phases: *map*, *shuffle*, and *reduce*. As all machines execute a program in the same way, next we focus on one specific machine $\mathcal{M}$.

Map. In this phase, $\mathcal{M}$ generates a list of key-value pairs (k, v) from its local storage. While the key k is usually numeric, the value v can contain arbitrary information. The pair (k, v) will be transmitted to another machine in the shuffle phase, such that the recipient machine is determined *solely* by k, as will be clarified shortly.

Shuffle. Let L be the list of key-value pairs that all the machines produced in the map phase. The shuffle phase distributes L across the machines adhering to the constraint that pairs with the same key must be delivered to

the same machine. That is, if (k, v_1), (k, v_2),..., (k, v_x) are the pairs in L having a common key k, all of them will arrive at an identical machine.

Reduce. $\mathcal{M}$ incorporates the key-value pairs received from the previous phase into its local storage. Then, it carries out whatever processing is needed on its local data. After all machines have completed the reduce phase, the current round terminates.

Discussion. It is clear that the machines communicate only in the shuffle phase, whereas in the other phases each machine executes the algorithm sequentially, focusing on its own storage. Overall, parallel computing happens mainly in the reduce phase. The major role of the map and shuffle phases is to swap data among the machines, so that computation can take place on different combinations of objects.

Simplified view of our algorithms. Let us number the t machines of the MapReduce system arbitrarily from 1 to t. In the map phase, all our algorithms will adopt the convention that $\mathcal{M}$ *generates a key-value pair* (k, v) *if and only if it wants to send* v *to machine* k. In other words, the key field is explicitly the ID of the recipient machine.

This convention admits a conceptually simpler modeling. In describing our algorithms, we will combine the map and shuffle phases into one called *map-shuffle. In the map-shuffle phase,* $\mathcal{M}$ *delivers* v *to machine* k, which means that $\mathcal{M}$ creates (k, v) in the map phase, which is then transmitted to machine k in the shuffle phase. The equivalence also explains why the simplification is only at the logical level, while physically, all our algorithms are still implemented in the standard MapReduce paradigm.

Statelessness for fault tolerance. Some MapReduce implementations (e.g., Hadoop) require that at the end of a round, each machine should send all the data in its storage to a *distributed file system* (DFS), which, in our context, can be understood as a "disk in the cloud" that guarantees consistent storage (i.e., it never fails). The objective is to improve the system's robustness in the scenario where a machine collapses during the algorithm's execution. In such a case, the system can replace this machine with another one, ask the new machine to load the storage of the old machine at the end of the previous round, and re-do the current round (where the machine failure occurred). Such a system is called *stateless,* because intuitively, no machine is responsible for remembering any state of the algorithm [6].

The four minimality conditions defined in Section I ensure efficient enforcement of statelessness. In particular, a *minimum footprint* guarantees that at each round, every machine sends $O(m)$ words to the DFS, which is still consistent with *bounded traffic.*

In the *sorting problem*, the input is a set S of n objects from an ordered domain. For simplicity, we assume that objects are real values, because our discussion easily generalizes to other ordered domains. Let $\mathcal{M}_1$,..., $\mathcal{M}_t$ denote the machines in the MapReduce system. Initially, S is distributed across these machines, each storing $O(m)$ objects, where $m = n/t$. At the end of sorting, all objects in $\mathcal{M}_i$ must precede those in $\mathcal{M}_j$ for any $1 \le i < j \le t$.

A. TeraSort

Parameterized by $\rho \in (0,1]$, *TeraSort* [4] runs as follows:

Round 1. *Map-shuffle*(ρ)

Every $\mathcal{M}_i$ $(1 \le i \le t)$ samples each object from its local storage with probability ρ independently. It sends all the sampled objects to $\mathcal{M}_1$.

Reduce (only on $\mathcal{M}_1$)

1. Let S_{samp} be the set of samples received by $\mathcal{M}_1$, and $s = |S_{samp}|$.
2. Sort S_{samp}, and pick $b_1,..., b_{t-1}$, where b_i is the $i\lceil s/t \rceil$-th smallest object in S_{samp}, for $1 \le i \le t - 1$. Each b_i is a *boundary object*.

Round 2. *Map-shuffle* (assumption: $b_1,..., b_{t-1}$ have been sent to all machines)

Every $\mathcal{M}_i$ sends the objects in $(b_{j-1}, b_j]$ from its local storage to $\mathcal{M}_j$, for each $1 \le j \le t$, where $b_0 = -\infty$ and $b_t = \infty$ are dummy boundary objects.

Reduce:

Every $\mathcal{M}_i$ sorts the objects received in the previous phase.

For convenience, the procedure above sometimes asks a machine $\mathcal{M}$ to send data to itself. Needless to say, such data "transfer" occurs internally in $\mathcal{M}$, with no network transmission. Also, note the assumption at the map-shuffle phase of Round 2, which we call the *broadcast assumption*, and will deal with later in Section III-C.

In O'Malley's study [4], ρ was left as an open parameter. Next, we analyze the setting of this value to make *TeraSort* a minimal algorithm.

B. Choice of ρ

Define $S_i = S \cap (b_{i-1}, b_i]$, for $1 \le i \le t$. In Round 2, all the objects in S_i are gathered by $\mathcal{M}_i$, which sorts them in the reduce phase. For *TeraSort* to be minimal, the following must hold:

$\mathcal{P}_1$. $s = O(m)$.
$\mathcal{P}_2$. $|S_i| = O(m)$ for all $1 \le i \le t$.

Specifically, $\mathcal{P}_1$ is necessary because $\mathcal{M}_1$ receives $O(s)$ objects over the network in the map-shuffle phase of

Round 1, which has to be $O(m)$ to satisfy *bounded net-traffic* (see Section I). $\mathcal{P}_2$ is necessary because $\mathcal{M}_i$ must receive and store $O(|S_i|)$ words in Round 2, which needs to be $O(m)$ to qualify as *bounded net-traffic* with a *minimum footprint*.

We now establish an important fact about *TeraSort:*

THEOREM 4. *When $m \geq t \ln(nt)$, $\mathcal{P}_1$ and $\mathcal{P}_2$ hold simultaneously with a probability of at least $1 - O(\frac{1}{n})$ by setting $\rho = \frac{1}{m}\ln(nt)$.*

Proof. We will consider $t \geq 9$, because otherwise, $m = \Omega(n)$, in which case $\mathcal{P}_1$ and $\mathcal{P}_2$ hold trivially. Our proof is based on the Chernoff bound—let $X_1,\ldots, X_n$ be independent Bernoulli variables with $\mathbf{Pr}[X_i = 1] = p_i$, for $1 \leq i \leq n$. Set $X = \sum_{i=1}^{n} X_i$ and $\mu = \mathbf{E}[X] = \sum_{i=1}^{n} p_i$. The Chernoff bound states (i) for any $0 < \alpha < 1$, $\mathbf{Pr}[X \geq (1+\alpha)\mu] \leq \exp(-\alpha^2\mu/3)$ while $\mathbf{Pr}[X \leq (1-\alpha)\mu] \leq \exp(-\alpha^2\mu/3)$, and (ii) $\mathbf{Pr}[X \geq 6\mu] \leq 2^{-6\mu}$—and an interesting bucketing argument.

First, it is easy to see that $\mathbf{E}[s] = m\rho t = t\ln(nt)$.

A simple application of the Chernoff bound results in:

$$\mathbf{Pr}[s \geq 1.6 \cdot t\ln(nt)] \leq \exp(-0.12 \cdot t\ln(nt)) \leq 1/n$$

where the last inequality uses the fact that $t \geq 9$. This implies that $\mathcal{P}_1$ can fail with a probability of at most $1/n$. Next, we analyze $\mathcal{P}_2$ under the event $s < 1.6t\ln(nt) = O(m)$.

Imagine that S has been sorted in ascending order. We divide the sorted list into $\lfloor t/8 \rfloor$ sub-lists as evenly as possible, and call each sub-list a *bucket*. Each bucket has between $8n/t = 8m$ and $16m$ objects. We observe that $\mathcal{P}_2$ *holds if every bucket covers at least one boundary object*. To understand why, note that under this condition, no bucket can fall between two consecutive boundary objects (counting also the dummy ones—if there was one, the bucket would not be able to cover any boundary object). Hence, every S_i, $1 \leq i \leq t$, can contain objects in at most 2 buckets, i.e., $|S_i| \leq 32m = O(m)$.

A bucket β definitely includes a boundary object if β covers more than $1.6\ln(nt) > s/t$ samples (i.e., objects from S_{samp}), as a boundary object is taken every $\lceil s/t \rceil$ consecutive samples. Let $|\beta| \geq 8m$ be the number of objects in β. Random variable x_j, $1 \leq j \leq |\beta|$ is defined to be 1 if the j-th object in β is sampled, and 0 otherwise. Define:

$$X = \sum_{j=1}^{|\beta|} x_j = |\beta \cap S_{samp}|.$$

Clearly, $\mathbf{E}[X] \geq 8m\rho = 8\ln(nt)$. We have:

$$\begin{aligned}
\mathbf{Pr}[X \leq 1.6\ln(nt)] &= \mathbf{Pr}[X \leq (1-4/5)\,8\ln(nt)] \\
&\leq \mathbf{Pr}[X \leq (1-4/5)\mathbf{E}[X]] \\
\text{(by Chernoff)} &\leq \exp\left(-\frac{16}{25}\frac{\mathbf{E}[X]}{3}\right) \\
&\leq \exp\left(-\frac{16}{25}\cdot\frac{8\ln(nt)}{3}\right) \\
&\leq \exp(-\ln(nt)) \\
&\leq 1/(nt).
\end{aligned}$$

We say that β *fails* if it covers no boundary object. The above derivation shows that β fails with a probability of at most $1/(nt)$. As there are at most $t/8$ buckets, the probability that at least one bucket fails is at most $1/(8n)$. Hence, $\mathcal{P}_2$ can be violated with a probability of at most $1/(8n)$ under the event $s < 1.6t\ln(nt)$, i.e., at most $9/8n$ overall.

Therefore, $\mathcal{P}_1$ and $\mathcal{P}_2$ hold at the same time with a probability of at least $1 - 17/(8n)$. □

Discussion. For large n, the success probability $1 - O(1/n)$ in Theorem 4 is so high that the failure probability $O(1/n)$ is negligible, i.e., $\mathcal{P}_1$ and $\mathcal{P}_2$ are almost never violated.

The condition about m in Theorem 4 is tight within a logarithmic factor, because $m \geq t$ is an implicit condition for *TeraSort* to work, with both the reduce phase of Round 1 and the map-shuffle phase of Round 2 requiring a machine to store $t - 1$ boundary objects.

In reality, typically, $m \gg t$, and the memory size of a machine is significantly greater than the number of machines. More specifically, m is on the order of at least 10^6 (this is using only a few megabytes per machine), while t is on the order of 10^4 or lower. Therefore, $m \geq t\ln(nt)$ is a (very) reasonable assumption, which explains why *TeraSort* has excellent efficiency in practice.

Minimality. We now establish the minimality of *TeraSort,* temporarily ignoring how to fulfill the broadcast assumption. Properties $\mathcal{P}_1$ and $\mathcal{P}_2$ indicate that each machine needs to store only $O(m)$ objects at any time, consistent with a *minimum footprint*. Regarding the network cost, a machine $\mathcal{M}$ in each round sends only objects that were already on $\mathcal{M}$ when the algorithm started. Hence, $\mathcal{M}$ sends $O(m)$ network data per round. Furthermore, $\mathcal{M}_1$ receives only $O(m)$ objects by $\mathcal{P}_1$. Therefore, *bounded-bandwidth* is fulfilled. *Constant round* is obviously satisfied. Finally, the computation time of each machine $\mathcal{M}_i$ $(1 \leq i \leq t)$ is dominated by the cost of sorting S_i in Round 2, i.e., $O(m\log m) = O(\frac{n}{t}\log n)$ by $\mathcal{P}_2$. As this is $1/t$ of the $O(n\log n)$ time of a sequential algorithm, *optimal computation* is also achieved.

C. Removing the Broadcast Assumption

Before Round 2 of *TeraSort*, $\mathcal{M}_1$ needs to *broadcast* the boundary objects $b_1,\ldots, b_{t-1}$ to the other machines. We have to be careful because a naive solution would ask $\mathcal{M}_1$ to send $O(t)$ words to every other machine, and hence, incur $O(t^2)$ network traffic overall. This not only requires one more round, but also violates *bounded net-traffic* if t exceeds $\sqrt{m}$ by a non-constant factor.

In O'Malley's study [4], this issue was circumvented by assuming that all the machines can access a distributed file system. In this scenario, $\mathcal{M}_1$ can simply write the boundary objects to a file on that system, after which each $\mathcal{M}_i$, $2 \leq i \leq t$, obtains them from the file. In other words, a brute-force *file-accessing step* is inserted between

the two rounds. This is allowed by the current Hadoop implementation (on which *TeraSort* was based [4]).

Technically, however, the above approach destroys the elegance of *TeraSort*, because it requires that, besides sending key-value pairs to each other, the machines should also communicate via a distributed file. This implies that the machines are not share-*nothing*, because they are essentially sharing the file. Furthermore, as far as this paper is concerned, the artifact is inconsistent with the definition of minimal algorithms. As sorting lingers in all the problems to be discussed later, we are motivated to remove the artifact to keep our analytical framework clean.

We now provide an elegant remedy, which allows *TeraSort* to still terminate in 2 rounds, and retain its minimality. The idea is to give *all* machines a copy of S_{samp}. Specifically, we modify Round 1 of *TeraSort* as:

Round 1. *Map-shuffle*(ρ)

After sampling as in *TeraSort,* each $\mathcal{M}_i$ sends its sampled objects to all machines (not just to $\mathcal{M}_1$).

Reduce

This is the same as in *TeraSort*, but performed on all machines (not just on $\mathcal{M}_1$).

Round 2 still proceeds as before. The correctness follows from the fact that in the reduce phase, every machine picks boundary objects in exactly the same way from an identical S_{samp}. Therefore, all machines will obtain the same boundary objects, thus eliminating the need for broadcasting. Henceforth, we will call the modified algorithm *pure TeraSort*.

At first glance, the new map-shuffle phase of Round 1 may seem to require a machine $\mathcal{M}$ to send out considerable data, because every sample necessitates $O(t)$ words of network traffic (i.e., $O(1)$ to every other machine). However, as every object is sampled with probability $\rho = \frac{1}{m}\ln(nt)$, the number of words sent by $\mathcal{M}$ is only $O(m \cdot t \cdot \rho) = O(t \ln(nt))$ in expectation. The lemma below gives a much stronger fact:

LEMMA 4. *With a probability of at least* $1 - \frac{1}{n}$*, every machine sends* $O(t \ln(nt))$ *words over the network in Round 1 of pure TeraSort.*

Proof. Consider an arbitrary machine $\mathcal{M}$. Let random variable X be the number of objects sampled from $\mathcal{M}$. Hence, $\mathbf{E}[X] = m\rho = \ln(nt)$. A straightforward application of the Chernoff bound gives:

$$\mathbf{Pr}[X \geq 6\ln(nt)] \leq 2^{-6\ln(nt)} \leq 1/(nt).$$

Hence, $\mathcal{M}$ sends more than $O(t \ln(nt))$ words in Round 1 with a probability of at most $1/(nt)$. By union bound, the probability that this is true for all t machines is at least $1 - 1/n$. □

Combining the above lemma with Theorem 4 and the minimality analysis in Section III-B, we can see that *pure TeraSort* is a minimal algorithm with a probability of at least $1 - O(1/n)$ when $m \geq t \ln(nt)$.

We close this section by pointing out that the fix of *TeraSort* is of mainly theoretical concerns. The fix serves the purpose of convincing the reader that the broadcast assumption is not a technical "loose end" in achieving minimality. In practice, *TeraSort* has nearly the same performance as our *pure* version, at least on Hadoop, where (as mentioned before) the brute-force approach of *TeraSort* is well supported.

IV. FINAL REMARKS

We have obtained a non-trivial glimpse at the results from research during an appointment with the KAIST. Interested readers are referred previous studies [1, 5] for additional details, including full surveys of the literature. Due to space constraints, in this paper, other results have not been described, but can be found online at `http://www.cse.cuhk.edu.hk/~taoyf`.

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