

Modeling of a Building System and its Parameter Identification

Herie Park[†], Nadia Martaj^{**}, Marie Ruellan^{***}, Rachid Bennacer^{*}
and Eric Monmasson^{***}

Abstract – This study proposes a low order dynamic model of a building system in order to predict thermal behavior within a building and its energy consumption. The building system includes a thermally well-insulated room and an electric heater. It is modeled by a second order lumped *RC* thermal network based on the thermal-electrical analogy. In order to identify unknown parameters of the model, an experimental procedure is firstly detailed. Then, the different linear parametric models (*ARMA*, *ARX*, *ARMAX*, *BJ*, and *OE* models) are recalled. The parameters of the parametric models are obtained by the least square approach. The obtained parameters are interpreted to the parameters of the physically based model in accordance with their relationship. Afterwards, the obtained models are implemented in Matlab/Simulink[®] and are evaluated by the mean of the sum of absolute error (*MAE*) and the mean of the sum of square error (*MSE*) with the variable of indoor temperature of the room. Quantities of electrical energy and converted thermal energy are also compared. This study will permit a further study on Model Predictive Control adapting to the proposed model in order to reduce energy consumption of the building.

Keywords: Dynamic model, Parameter identification, Thermal network, Low energy building

1. Introduction

Nowadays, achieving low energy consumption in a building sector becomes an important issue for sustainable and eco-friendly development. More than a third of the total energy consumed in buildings is used for space heating and cooling. In order to reduce the heating and cooling energy demand of buildings, high thermal insulation of building envelopes is most often used for low energy buildings [1-2]. Since reinforcing thermal insulation prevents losing heat through the building envelopes, the auxiliary heat gains, such as solar radiation, occupant's metabolic heat, and heat dissipation of electrical equipment and appliances, and light can influence dominantly to the building's indoor thermal condition. It is related to the building energy consumption and the indoor thermal comfort of occupants [3].

To the purpose of low energy consumption within buildings, an optimal control of building thermal condition is additionally needed. As one of the techniques, Model Predictive Control (*MPC*) is suitable for the control of slow responding systems, such as buildings. Since it uses a model of the system and anticipates future loads, such as occupancy schedules, and weather forecasts of buildings, it

performs better than the algorithm which is base on measurements. It can better control the output of the system and track the set-point [4].

Most of the building energy simulation tools simulates thermal behavior of a building model, based on the fundamental laws of energy, heat, and mass transfer. The model represents all the physical components of the building system and consists of high-order differential equations. Although the high-order model can well describe its thermal behavior, it is not available for adapting the control algorithms. In order to acquire a low-order model which is simple and appropriate to control strategy, model size reduction, black-box modeling, and grey-box modeling by computational methods have been used [5].

A model size reduction method requires an initial model with a number of differential equations describing thermal behavior of the building. However, it is not easy to acquire the physical properties of existing buildings for establishing their initial models [6]. A black-box modeling method can describe the model behavior since it is based on the input-output measurements of the building system. Despite its simplicity and performance, the model is usually not available for other conditions of the building, and the parameters of the model have no physical meaning [7]. However, a gray-box modeling method is based on the physical properties of the building system. Moreover, it needs parametric identification methods in order to estimate the unknown parameters of the system. It can also interpret the physical meanings of the parameters estimated by mathematical relationship between input and output of

[†] Corresponding Author: LMT CNRS UMR 8535, Ecole Normale Supérieure de Cachan, France / Department of Electrical Engineering, Yeungnam University, Republic of Korea (park.herie@gmail.com)

^{*} LMT CNRS UMR 8535, Ecole Normale Supérieure de Cachan, France

^{**} LMT CNRS UMR 8535, Ecole Normale Supérieure de Cachan, France / EPF-Ecole d'Ingénieurs, Montpellier, France

^{***} SATIE CNRS UMR8029, University of Cergy-Pontoise, France

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the system [8].

This paper presents a gray-box modeling method for a building system with help of the correspondence between a thermal network and several linear parametric models. The building system which is modeled in this work consists of a well-insulated room and an electric heater. The well-insulated room is considered as a small-sized low energy building. In sections 2, state-of-the-art literature related to the thermal modeling of building systems is firstly introduced. Then, a physically based model of the building system used for this study is then derived. It is based on the energy balance equation and the thermal-electrical analogy. The system is modeled by a second order lumped *RC* thermal network. In order to estimate the parameters of the model, experimental set-up and parameter identification method are presented in Section 3. The thermal parameters of the model are identified by using different linear parametric models (*ARMA*, *ARX*, *ARMAX*, *BJ*, and *OE* models) with their physical interpretation. Afterwards, in Section 4, the parameters obtained by these models are matching to the parameters of the physically driven model. The designed models with different parameters are implemented in Matlab/Simulink® and evaluated. Finally, conclusions are given in Section 5.

2. Thermal Modeling of a Building System

2.1 State-of-the-art literature

On the level of modeling of a building system, a dynamic modeling is used for describing both temporal and spatial performance of the system. With the help of various information and computational calculations, more detailed and accurate results are expected [9]. As one of the dynamic modeling methods adapted to building systems, a thermal network using thermal-electrical analogy has been proposed since the mid- 1980s. The main advantage of this method is its simplicity. It is possible to express heat transfer phenomena by electrical components, add the supplementary heat sources obtained solar radiation, occupants, infiltration/ventilation and equipment, and analyze the systems with good accuracy and robustness [5, 10].

There is literature which presents methodologies to identify thermal parameters of the building systems. The parameters are obtained by experimental data and numerical calculations [10-14]. Reference [11] suggested a method to estimate the thermal parameters of building envelopes, especially the thermal capacitance of insulation materials of a wall by using a heat flow meter in laboratory. Reference [12] also estimated the thermal equivalent conductance and the thermal equivalent capacitance of a testing wall of a constructed building by conducting on experiments in situ. The parameters were obtained by treating the measured heat flux and the measured wall

temperature. These are effective methods to characterize each of the building components. However, regarding their global thermal characteristics, the measured values for each component could not be available because of the existence of different thermal dynamic interactions within the building, and the global values of the building are not able to be directly measured.

There have been several researches which present linear parameter models of the given buildings and that obtain their parameters by numerically treating the experimental results. Reference [13] was established several parametric models (*ARX*, *ARMAX*, *BJ*, and *OE* models) to identify the thermal behavior of an office and provided reasonably good predictions of indoor temperature and relative humidity. Moreover, [14] applied an *ARX* model and a neural network *ARX* model to the prediction of indoor temperature and relative humidity of an unoccupied residential building. These black-box modeling approaches do not require the physical properties and relying physical laws of the system to predict the system behavior, whereas the obtained parameters are not corresponding to any physical values of the system and only shows the mathematical relationship between inputs and outputs of the system.

In this paper, the authors recall several empirically designed parametric models of a building system and estimate their parameters by the least square approach. However, unlikely to the previous works, the estimated parameters of this work are corresponding to the parameters of a physically driven thermal model which are proposed in the following subsection. Therefore, the physically interpreted values are able to be directly applied to the proposed thermal model for a validation and provide global thermal characteristics of the system.

2.2 Physically driven model of a building system

We propose a thermal network model of a building system. This modeling approach uses the analogy between a thermal system and an electrical system. The thermal-electrical analogy is detailed in Table 1.

This dynamic thermal network model of a building system is physically based on the first law of thermodynamics and is deduced as follows:

$$\dot{x} = Ax + Bu \quad (1)$$

Table 1. Thermal-electrical analogy

	Thermal		Electrical	
	Parameter	Unit	Parameter	Unit
Source	Temperature T	[K]	Voltage V	[V]
	Heat flux Φ	[W], [J/s]	Current I	[A], [C/s]
Element	Conductivity k	[W/K/m]	Conductivity σ	[A/V/m]
	Stored heat Q	[J]	Stored charge q	[C]
	Thermal resistance R_{ther}	[K/W]	Electrical resistance R_{elec}	[Ω], [V/A]
	Thermal capacitance C_{ther}	[J/K]	Electrical capacitance C_{elec}	[F], [C/V]

$$y = Cx + Du$$

where

$$x = [T_m \ T_{m-1} \ T_{m-2} \ \dots \ T_1]^T$$

$$u = [\Phi_n \ \Phi_{n-1} \ \Phi_{n-2} \ \dots \ \Phi_1 \ T_0]^T$$

$$A = \begin{bmatrix} \frac{1}{R_m C_m} & 0 & 0 & 0 \dots 0 & 0 \\ \frac{1}{R_m C_{m-1}} \left(\frac{1}{R_m} + \frac{1}{R_{m-1}} \right) \frac{1}{C_{m-1}} & 0 & 0 \dots 0 & 0 \\ 0 & \frac{1}{R_{m-1} C_{m-2}} & \left(\frac{1}{R_{m-1}} + \frac{1}{R_{m-2}} \right) \frac{1}{C_{m-2}} & 0 \dots 0 \\ \vdots & \vdots & \vdots & \ddots \vdots \\ 0 & 0 & 0 & 0 \dots \frac{1}{R_2 C_1} \left(\frac{1}{R_2} + \frac{1}{R_1} \right) \frac{1}{C_1} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{C_m} & 0 & 0 & 0 \dots 0 & \frac{1}{R_m C_m} \\ 0 & \frac{1}{C_{m-1}} & 0 & 0 \dots 0 & \frac{1}{R_{m-1} C_{m-1}} \\ 0 & 0 & \frac{1}{C_{m-2}} & 0 \dots 0 & \frac{1}{R_{m-2} C_{m-2}} \\ \vdots & \vdots & \vdots & \ddots \vdots & \vdots \\ 0 & 0 & 0 & 0 \dots \frac{1}{C_1} & \frac{1}{R_1 C_1} \end{bmatrix}$$

where x is the state vector, u is the input vector, y is the output vector. A , B , C , and D are the matrices of the model, T is the temperature [K]. Φ is the heat flux [W]. R and C in the matrices are the thermal resistance [K/W] and capacitance [J/K], respectively. The indexes m , n are the numbers of the temperature nodes and the heat sources, respectively.

From the above model, an overall structure of an electric heater and a simplified well-insulated room is developed by a second order lumped RC ($2R2C$) thermal network.

$$\begin{bmatrix} \dot{T}_{ap} \\ \dot{T}_i \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_{ap} C_{ap}} & \frac{1}{R_{ap} C_{ap}} \\ \frac{1}{R_{ap} C_{th}} & -\frac{R_{ap} + R_{th}}{R_{ap} R_{th} C_{th}} \end{bmatrix} \begin{bmatrix} T_{ap} \\ T_i \end{bmatrix} + \begin{bmatrix} \frac{1}{C_{ap}} & 0 \\ 0 & \frac{1}{R_{th} C_{th}} \end{bmatrix} \begin{bmatrix} P_{elec} \\ T_e \end{bmatrix} \quad (2)$$

where indexes ap , th , i , and e represent the appliance model (electric heater), the thermal model of the building, the interior and the exterior of the building, respectively. P_{elec} is the electrical power [W] which is supplied to the

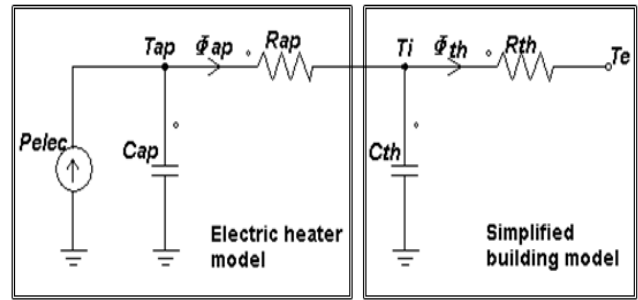


Fig. 1. An overall structure of a simplified well-insulated building and an electric heater using thermal network

electrical appliance, which is a heater in this work.

Fig. 1 illustrates an equivalent electrical circuit which corresponds to Eq. (2). The electric heater model composes of $1R1C$ thermal parameters (R_{ap} , C_{ap}) and a power source P_{elec} . In addition, the simplified well-insulated room model consists of another first order thermal parameters (R_{th} , C_{th}) and the temperature T_e .

The temperatures of the heater and the room depend on the power consumption of the heater, the exterior temperature of the building, and the thermal parameters of the room and the heater. Among these quantities, the temperatures and the power consumption of the heater are measurable. From the physical laws and the measurable input and output data, the unknown thermal parameters can be obtained. As a consequence, the experiment design is necessary to collect the measurable data. The following section firstly describes the experimental procedure. Then, several different linear parametric models based on the measurement are presented.

3. Parameter Identification

3.1 Description of experiment

3.1.1. Electric heater

The heater used in this work is an electric convector (Model: DeLonghi HS20F, 230 [V], 50~60 [Hz]). Heat is produced by Joule effect and mainly transferred by convection and radiation. The heater has five heating levels correspond to the desired temperature ranges. A dial for selecting the level is manually controlled by users. It is connected to a bimetallic heat sensor. The sensor makes the heater switched on and off. The nominal power for heating is 1,000 [W]. The heater was placed in the well-insulated room during the whole period of the measurement.

3.1.2. Well-insulated room

The test room is located in the University of Cergy-Pontoise, at Neuville, France. This room (size: 4 x 2.4 x 4

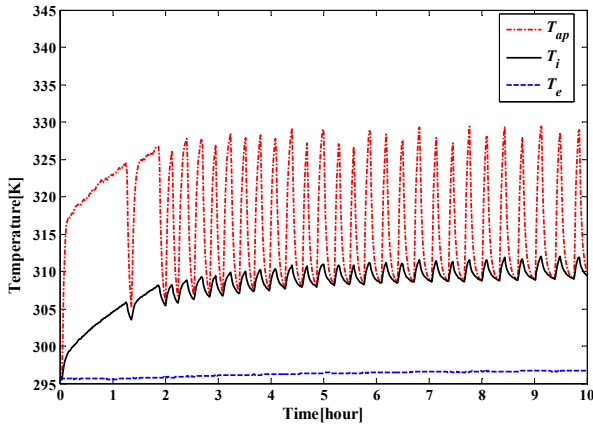


Fig. 2. An example of the measured data of $T_{ap}(t)$, $T_i(t)$, and $T_e(t)$

[m³]), of which wall is made up polyurethane and stainless steel sheet, is one of the well-insulated rooms in the site. It has a door (length: 0.9 [m], height: 1.9 [m]) with the same materials to the wall, and a small window (length: 0.3 [m], height: 0.5 [m]). The thermal characteristics of the room were already identified in ref. [3]. They are equal to: $R_{th}=57.8$ [mK/W], $C_{th}=654$ [kJ/K], and $\tau_{th}=3.78 \times 10^4$ [s]. The details of the room are also given in ref. [3].

3.1.3. Measured data

The load profile $P_{elec}(t)$ of the heater is acquired by NZR Standby-Energy-Monitor 16 (NZR SEM 16). This device measures and stores the following signatures of the electrical appliance: current, voltage, active power, energy consumption, energy costs, and maximal/minimal power during the measurement. For the temperature measurement, the room is equipped with a temperature acquisition device and twenty K-type thermocouples. The standard deviation of thermocouples is about 0.03 [K] at stable conditions. Two thermocouples are positioned beyond the electric heater for measuring $T_{ap}(t)$ during the test. Eighteen thermocouples are positioned for measuring $T_i(t)$ and $T_e(t)$ and the details of the position of these thermocouples are described in [3]. Each quantity is measured each 60 [s] and stored into a host computer.

The measurement was carried out during a week. Before starting the experiment, the heater was placed in the well-insulated room in order to have the same initial temperatures between the heater and the room $T_{ap}(0)=T_i(0)$. During the first 72 [h] of the experiment, the heater is operating. Then it is off for rest of the experiment. An example of the measured data of $T_{ap}(t)$, $T_i(t)$, and $T_e(t)$ is illustrated in Fig. 2.

3.2. Description of parametric identification

3.2.1. Parametric models

Parametric models provide a compact form of a system

Table 2. The different model and the selection of the polynomials

	A_l	B_l	C_l	D_l	F_l
ARMA	√	√	-	-	1
ARX	√	√	1	1	1
ARMAX	√	√	√	1	1
BJ	1	√	1	1	√
OE	1	√	√	√	√

1: fixed as 1, √: chosen freely, -: no value

using predefined polynomials, $A_l(q)$, $B_l(q)$, $C_l(q)$, $D_l(q)$, or $F_l(q)$ in the shift operator q . The general form of the models is as follows [15].

$$A_l(q)y(k) = \frac{B_l(q)}{F_l(q)}x(k) + \frac{C_l(q)}{D_l(q)}e(k) \quad (3)$$

where x , y are the input and the output of a system, respectively. e is the stochastic error. The error accounts for the zero mean white noise which has a normal distribution, a zero mean, and a constant covariance, or the disturbance of the system.

The polynomials of the structure are defined as:

$$A_l(q) = 1 + a_1q^{-1} + a_2q^{-2} + \dots + a_{n_a}q^{-n_a} \quad (4)$$

$$B_l(q) = b_0 + b_1q^{-1} + b_2q^{-2} + \dots + b_{n_b}q^{-n_b} \quad (5)$$

$$C_l(q) = 1 + c_1q^{-1} + c_2q^{-2} + \dots + c_{n_c}q^{-n_c} \quad (6)$$

$$D_l(q) = 1 + d_1q^{-1} + d_2q^{-2} + \dots + d_{n_d}q^{-n_d} \quad (7)$$

$$F_l(q) = 1 + f_1q^{-1} + f_2q^{-2} + \dots + f_{n_f}q^{-n_f} \quad (8)$$

By selecting one or more polynomials, the different model structures are obtained including *ARMA* (Auto-Regressive Moving Average) model, *ARX* (Auto-Regressive with eXogenous input) model, *ARMAX* (Auto-Regressive Moving Average with eXogenous input) model, *BJ* (Box-Jenkins) model, and *OE* (Output Error) model. Table 2 shows the different model structures and the selection of the polynomials.

3.2.2. Least Square Estimation

The parameters of the given models are obtained by least square estimation [15]. The criterion of least square estimation, J_N is expressed as

$$J_N(\theta) = \frac{1}{N} \sum_{k=1}^N \varepsilon^2(k, \theta) = \frac{1}{N} \sum_{k=1}^N [y(k) - \hat{y}(k, \theta)]^2 \quad (9)$$

where N is the number of data, ε is the difference between measured data y and estimated data \hat{y} . θ is the parameter vector. The estimated data \hat{y} is expressed as

$$\hat{y}(k, \theta) = \varphi^T(k) \theta \quad (10)$$

where $\varphi^T(k)$ is the transition matrix of the regression vector. The optimal parameter vector $\hat{\theta}_N$ of which criterion is minimized is then

$$\hat{\theta}_N = \arg \min J_N(\theta) \quad (11)$$

3.2.3. Physical interpretation of the parameters

The structure of the building system introduced in Section 2 is described in frequency domain (s-domain) using Laplace transformation. The transfer functions of the structure are expressed as

$$\frac{T_1(s)}{P_{elec}(s)} = \frac{R_{ap}R_{th}C_{th}s + R_{ap} + R_{th}}{R_{ap}R_{th}C_{ap}C_{th}s^2 + (R_{ap}C_{ap} + R_{th}C_{ap} + R_{th}C_{th})s + 1} \quad (12)$$

$$\frac{T_2(s)}{P_{elec}(s)} = \frac{R_{th}}{R_{ap}R_{th}C_{ap}C_{th}s^2 + (R_{ap}C_{ap} + R_{th}C_{ap} + R_{th}C_{th})s + 1} \quad (13)$$

where T_1 is the difference between T_{ap} and T_i , and that T_2 is the difference between T_i and T_e . It is supposed that the initial temperatures are the same ($T_{ap}(0)=T_i(0)=T_e(0)$). These functions can be represented in discrete-time domain by using Euler method. The structure is then expressed by using the shift operator q as follows:

$$\frac{T_1(q)}{P_{elec}(q)} = \frac{\frac{\alpha_4}{\alpha_1} + \frac{\alpha_5}{\alpha_1} q^{-1}}{1 + \frac{\alpha_2}{\alpha_1} q^{-1} + \frac{\alpha_3}{\alpha_1} q^{-2}} \quad (14)$$

$$\frac{T_2(q)}{P_{elec}(q)} = \frac{\frac{\alpha_6}{\alpha_1}}{1 + \frac{\alpha_2}{\alpha_1} q^{-1} + \frac{\alpha_3}{\alpha_1} q^{-2}} \quad (15)$$

where

$$\begin{aligned} \alpha_1 &= R_{ap}R_{th}C_{ap}C_{th} + [(R_{ap} + R_{th})C_{ap} + R_{ap}C_{th}]T_S + T_S^2 \\ \alpha_2 &= -2R_{ap}R_{th}C_{ap}C_{th} - [(R_{ap} + R_{th})C_{ap} + R_{ap}C_{th}]T_S \\ \alpha_3 &= R_{ap}R_{th}C_{ap}C_{th} \end{aligned}$$

$$\alpha_4 = R_{ap}R_{th}C_{th}T_S + (R_{ap} + R_{th})T_S^2$$

$$\alpha_5 = -R_{ap}R_{th}C_{th}T_S$$

$$\alpha_6 = -R_{th}T_S^2$$

Eqs. (14-15) have the same structure with Eq. (3) which represents the general form of considering parametric models. Consequently, as comparing the parameters of their structures to each parametric model' one, the relation between the physical parameters R_{ap} , C_{ap} , R_{th} , C_{th} and the coefficients of the parametric models is revealed. Regarding T_{ap} and T_i , it yields the physical interpretation of the coefficients a_1 , a_2 , b_0 and b_1 as follows:

$$a_1 = \frac{\alpha_2}{\alpha_1} = \frac{-2R_{ap}R_{th}C_{ap}C_{th} - [(R_{ap} + R_{th})C_{ap} + R_{ap}C_{th}]T_S}{R_{ap}R_{th}C_{ap}C_{th} + [(R_{ap} + R_{th})C_{ap} + R_{ap}C_{th}]T_S + T_S^2}$$

$$a_2 = \frac{\alpha_3}{\alpha_1} = \frac{R_{ap}R_{th}C_{ap}C_{th}}{R_{ap}R_{th}C_{ap}C_{th} + [(R_{ap} + R_{th})C_{ap} + R_{ap}C_{th}]T_S + T_S^2}$$

$$b_0 = \frac{\alpha_4}{\alpha_1} = \frac{R_{ap}R_{th}C_{th}T_S + (R_{ap} + R_{th})T_S^2}{R_{ap}R_{th}C_{ap}C_{th} + [(R_{ap} + R_{th})C_{ap} + R_{ap}C_{th}]T_S + T_S^2}$$

$$b_1 = \frac{\alpha_5}{\alpha_1} = \frac{-R_{ap}R_{th}C_{th}T_S}{R_{ap}R_{th}C_{ap}C_{th} + [(R_{ap} + R_{th})C_{ap} + R_{ap}C_{th}]T_S + T_S^2}$$

Accordingly, the parameters of the physically based model can be obtained as solving the above accordance after estimating the coefficients of the parametric models and can be directly applied to the proposed model of the considered building system.

4. Case Study

4.1. Different models

The proposed identification procedure is applied to identify thermal parameters of a simplified well-insulated room and an electric heater. The used different parametric models in this work are *ARMA*, *ARX*, *ARMAX*, *BJ*, and *OE* models. The input of the models is P_{elec} . The output of the models is T_1 , which is the difference between T_{ap} and T_i . These input and output data were measured during a week as mentioned in section 3.1.3.

Table 3. The fitted parameters of parametric models

	a_1	a_2	b_0	b_1	c_1	c_2	d_1	d_2	f_1	f_2
ARMA	-1.546	0.5692	0.0061	-0.0048	-	-	-	-	1	1
ARX	-1.535	0.5356	0.0087	-0.0086	1	1	1	1	1	1
ARMAX	-1.519	0.5202	0.0091	-0.0090	-0.7962	-0.1921	1	1	1	1
BJ	1	1	0.0091	-0.0090	1	1	1	1	-1.519	0.5202
OE	1	1	0.0091	-0.0090	1.214	0.2638	0.4879	-0.3646	-1.519	0.5202

- : no value

Table 4. Identified thermal parameters of overall structure

	Electric heater model		Simplified room model	
	R_{ap} [mK/W]	C_{ap} [kJ/K]	R_{th} [mK/W]	C_{th} [kJ/K]
ARMA	18.5	7.11	59.5	15.2
ARX	18.8	3.71	57.2	547.2
ARMAX	19.1	3.44	56.9	656.5
BJ	19.1	3.44	56.9	656.6
OE	19.1	3.44	56.9	656.6

By using experimental results, the fitted parameters of each parametric model are obtained and detailed in Table 3. These parameters are matching to the thermal parameters of the introduced system of the simplified room and the electric heater. By the correspondence which is stated in Section 3.2.3, matching results of the models are calculated and listed in Table 4.

The thermal parameters are only dependant on the plant process of each model. In other words, the parameters of the noise functions namely, c_1 , c_2 , d_1 , and d_2 are not taken into the matching calculations. However, since the plant processes of *ARMAX*, *BJ*, and *OE* models are the same in this case, the obtained thermal parameters of the models are also identical. This result comes from that the amplitude of T_i is relatively much larger than the disturbances.

From the obtained parameters, the dynamics of the structure can be known. Since the structure is modeled by *2R2C*, it has two time constants, τ_{ap} and τ_{th} where

$$\tau_{ap} = R_{ap} \cdot C_{ap} \quad (16)$$

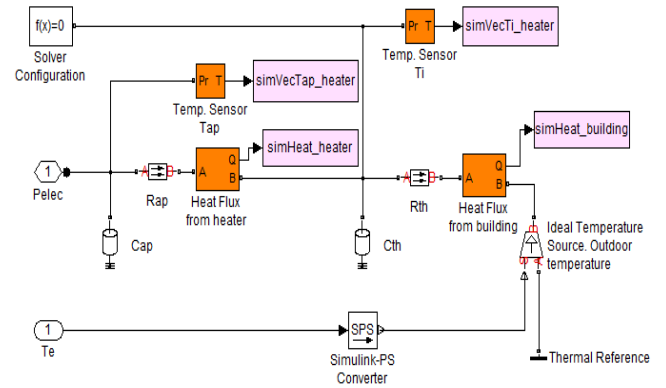
$$\tau_{th} = R_{th} \cdot C_{th} \quad (17)$$

Each time constant of the models are then calculated. The ratios of τ_{ap} and τ_{th} of each parametric model (*ARMA*, *ARX* and *ARMAX* model) are respectively 0.14, 0.0022, and 0.0017. It shows that the thermal dynamics of the heater is much more rapid than the building's one.

4.2. Simulation results

To validate the presented parameter identification method, the overall structure modeled by *2R2C* was implemented in Matlab/Simulink® as shown in Fig. 3. The conditions of the simulation, such as the operation period of the electric heater, its power consumption and the outdoor temperature of the room, are the same to the experimental one.

Under these conditions, the parameters obtained by *ARMA*, *ARX*, and *ARMAX* models were separately selected for the simulation. Since the thermal parameters of *ARMAX*, *BJ*, and *OE* are the same as shown in Table 4, we only took the values of *ARMAX* as the representative of *ARMAX*, *BJ*, and *OE* models. The temperature of the heater T_{ap} and the heat flux from the heater Φ_{ap} were simulated and compared

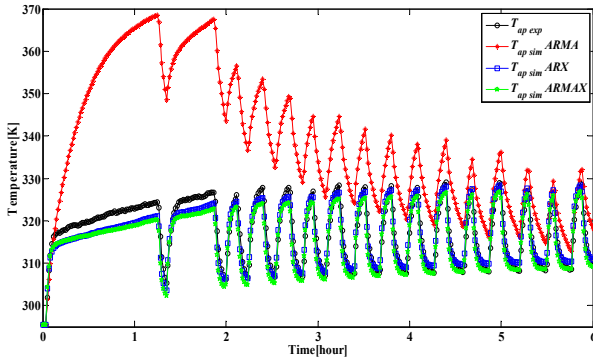

Fig. 3. Block diagram of the overall structure implementation in Matlab/ Simulink®

to the measured data.

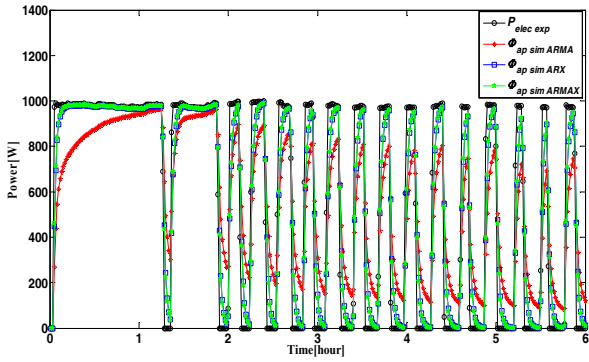
The measured data and the simulation results of T_{ap} with respect to the measured P_{elec} ($P_{elec\ exp}$) and the estimated heat fluxes of the heater $\Phi_{ap\ sim}$ are illustrated in Figs. 4-5. As mentioned above, the system has two thermal dynamics. The first one is the dynamics of the heater and another one is the dynamics of the building. Because of the faster dynamics of the heater, T_{ap} rapidly changes at the moment when the electric heater turns on and off. At the same time, there exists the heat which is charged and discharged in the heater. It is explained by the existence of the thermal capacity of the heater. Moreover, T_{ap} globally rises until the building temperature arrives at steady-state due to the slower dynamics of the building.

The comparisons of the results are also accomplished in both cases of transient and steady-states. During the transient state (see Fig. 4), T_{ap} of *ARMAX* model is lower than the others because its time constant of the room is the biggest. It is known that it takes more time to reach to the steady-state if the time constant is bigger. On the same principle, T_{ap} simulated by *ARMA* parameters rises much more rapidly than the others since the estimated time constant of the room by *ARMA* model is smaller.

During steady-state (see Fig. 5), there is no big difference of T_{ap} between *ARX* and *ARMAX* models. It is explained by the faster dynamics of the heater. τ_{ap} calculated by *ARX* and *ARMAX* models are respectively 70 and 64 [s]. From this, it is known that the duration while the heater is on (during about 6~8 [min]) is enough long to let T_{ap} of these two models reach to the reference temperature. Moreover, T_{ap} of *ARMA* model becomes more stable and follows better the experimental result during this state. However its rising and descending speeds are relatively slower than *ARX* and *ARMAX* models', contrarily to the transient state. It is caused by τ_{ap} of *ARMA* model. It is about twice of τ_{ap} of *ARX* and *ARMAX* models. In addition, the value of C_{th} has no more effect to T_{ap} at this state. The amplitude of T_{ap} depends on the R_{ap} , C_{ap} , and R_{th} .

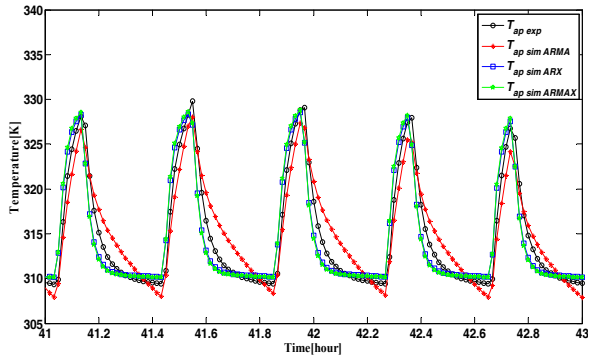


(a)

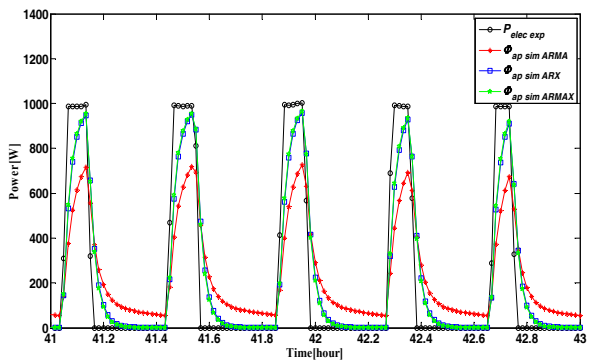


(b)

Fig. 4. Comparison of measured and estimated data in transient state (a) T_{ap} , (b) P_{elec} and Φ_{ap}



(a)



(b)

Fig. 5. Comparison of measured and estimated data in pseudo steady-state (a) T_{ap} , (b) P_{elec} and Φ_{ap}

Table 5. Evaluation of models

	MAE	MSE
ARMA	2.48	28.62
ARX	0.85	2.02
ARMAX	0.78	2.24

4.3 Evaluation of models

To evaluate the models, the mean of the sum of absolute error (MAE) and the mean of the sum of square error (MSE) were used [16]:

$$MAE = \frac{1}{N} \sum_{k=1}^N |y(k) - \hat{y}(k)| \quad (18)$$

$$MSE = \frac{1}{N} \sum_{k=1}^N [y(k) - \hat{y}(k)]^2 \quad (19)$$

These evaluation values of $ARMA$, ARX , and $ARMAX$ models were calculated. The observed variable is $T_i(t)$. The details of the results are listed in Table 5.

As comparing each evaluation criterion, $ARMA$ model has the worst performance. It is especially due to the over-estimated T_i during transient-state. $ARMA$ model does not consider any disturbance of the system and only searches the least squares error from the plant process. It tends to identify the parameters from the steady-state, of which duration is long. Hence, it leads an over-estimated τ_{ap} and an under-estimated τ_{th} . On the contrary, ARX model and $ARMAX$ model of which noise always exists on the systems are more realistic and better performed. Between two models, ARX model is more effective than $ARMAX$ model which has the same plant process to BJ and OE models in this study. Moreover, the calculated total electrical energy consumption of the heater during the measurement period is 68.52 MJ. It was verified that the simulated total thermal energies of all models are identical to the electrical energy with 0.99 of the ratios.

6. Conclusion

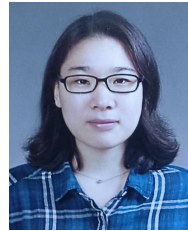
This paper introduced a method to establish a low order model of a building system. The system includes a simplified well-insulated room and an electric heater. It was modeled by a second order lumped RC thermal network. Based on the model structure and the experimental results, the parameters of different linear parametric models ($ARMA$, ARX , $ARMAX$, BJ , and OE models) were obtained. The corresponding parameters of each parametric model were converted to the parameters of the physically based model. Finally, the designed models were implemented in Matlab/Simulink® and were evaluated by MAE and MSE . As a result, the order of performance is as follows: $ARMA < ARMAX, BJ, OE < ARX$. The simulated

thermal energy of each model was identical to the consumed electrical energy with 0.99 of the ratios.

This study provided a low order model structure of a building system and its parameters. It contributes to predict thermal behavior and energy consumption of the building. Moreover, it permits a further study on the *MPC* control method adapting in the proposed model in order to reduce heating and cooling energy of the building.

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Herie Park She received the B.S. degrees in electrical engineering from University of Cergy-Pontoise, France in 2006 and Yeungnam University, Korea in 2007, and the M.S. degree in electrical engineering from Yeungnam University in 2009. She received her Ph.D degrees in Electrical Engineering

from University of Cergy-Pontoise and Yeungnam University in 2013. She is currently a Post-Doc at Ecole Normale supérieure de Cachan, France. Her research interests are thermal-electrical modeling of energy systems, energy management of smart buildings, and electrical discharge phenomena.



Nadia Martaj obtained the BS in Energetics in 2000 from Casablanca Technological University in Morocco. Then she enrolled in Paris Ouest University (Paris 10), where she obtained the M.S. and the Ph.D. degrees in thermal and mechanical engineering. Since 2012, she has been an assistant

professor at EPF-Ecole d'Ingénieurs, Montpellier, France. Her fields of research cover simulation of building thermal performance, stirling engines, finite dimension thermodynamics, energetical/entropical/exergetical optimization and refrigerating machines.



Marie Ruellan She received the Ing. degree from INSA Strasbourg, France in 2004 and the Ph.D degree from ENS Cachan, France in 2007. She is currently an assistant professor at University of Cergy-Pontoise, France since 2008. She is also with the Systèmes et Applications des Technologies de l'Information et de l'Energie laboratory (SATIE, UMR CNRS8029). Her current research interests are energy management and energetic systems optimization in the building sector



Rachid Bennacer He is an Engin-eer in Mechanical field (1989), and he got his Ph.D degree at Pierre et Marie Curie University (Paris 6), France in 1993. He is currently a senior professor at Ecole Normale supérieure de Cachan, France. His Research field covers several domains as material science, energy system, pollution and renewable energy with an expertise in convection-diffusion problems in porous media. He published more than 120 international journals.



Eric Monmasson He received the Ing. and Ph.D. degrees from Ecole Nationale Supérieure d'Ingénieurs d'Electrotechnique, Electronique, Informatique et Hydraulique de Toulouse (ENSEEIH), Toulouse, France, in 1989 and 1993, respectively. He is currently a full professor at University of Cergy-Pontoise, France. He is also with the Systèmes et Applications des Technologies de l'Information et de l'Energie laboratory (SATIE, UMR CNRS8029). His current research interests include the advanced control of electrical motors and generators and the use of FPGAs for industrial control systems. He is the author or coauthor of 3 books and more than 150 scientific papers.