Outage Probability of Two-Hop Relay Networks with Related Interference

Peisheng Pan and Baoyu Zheng
College of Communication & Information Engineering, Nanjing University of Posts and Telecommunications
Nanjing, 210003 - CHINA
[e-mail: panps,zby@njupt.edu.cn]
*Corresponding author: Peisheng Pan

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Abstract

We consider a specific interference-limited wireless relay system that comprises several cooperation units (CUs) which are defined as a source and destination node pair with an associated relay node. In the wireless relay system, all source nodes simultaneously transmit their own signals and the relay node in each CU then forwards the received signal to the destination node, causing co-channel interference at both the relay node and the destination node in each CU. The co-channel interference at the relay node is closely related to that at the destination node in each CU. We first derive the end-to-end outage probability in a CU over Rayleigh slow-fading channels with interference for the decode-and-forward (DF) relaying strategy. Then, on the assumption that each CU is allocated with equal power we design an optimal power allocation between the source node and the relay node in each CU to minimize the outage probability of the investigated CU. At last, in the case that each CU is not allocated with equal power and the sum of their power is constrained, we present an optimal power allocation between CUs to minimize the sum of the outage probability of all CUs. The analytical results are verified by simulations.

Keywords: Relay networks, outage probability, interference, cooperation

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1. Introduction

Cooperative communications in wireless networks have garnered much attention due to its spatial diversity advantage [1], [2]. In cooperative communications, one or several relay nodes (RNs) can help to forward the information from a source node (SN) to a destination node (DN), especially when the direct link between a SN and a DN is blocked due to a deep fade or an intermediate wall. Extensive studies about cooperative communications such as distributed space time code, relay selection and power control [3]-[8] have been conducted in recent years in the scenario where only one SN sends messages to one DN with the help of one or several RNs. Although these previous studies have greatly helped us to understand the performance of cooperative communication systems, few scenarios have considered the case where co-channel interference exists due to the spectral reuse. Hence, there remains the need to determine the performance of cooperative communication systems in interference-limited environments. Several papers have dealt with the performance of cooperative communication systems in interference-limited environments. In [9]-[17], there have been cases where the interference exists in the relay node, destination node, or in both of them. In [9], the closed-form outage probability expressions of dual-hop relay channels for both the amplify-and-forward (AF) and the decode-and-forward (DF) relaying technologies were derived. However, they assumed that the co-channel interference exists only in the DN, and not in the RN. In [10], the author discussed the outage probability and the average bit error rate (BER) of the AF protocol with interference at the relay. In [11], the outage probability of the DF protocol was investigated with Nakagami-*m* faded multiple co-channel interferers at the relay node. In [12], the performance of the AF protocol was investigated with Nakagami-*m* faded multiple co-channel interferers at the relay node. In [13]-[17], the authors studied the outage performance of a wireless relay network where co-channel interference exists at both the relay node and the destination node. The approaches used in [9]-[17] are similar in that they assumed that there are K interferers at the node i, and the channels from the interferers to the node i are modeled as $\{h_{i,j}\}_{j=1,2,\ldots,K}$. They assumed that the interference at the relay node and that at the destination node are independent, and did not explain the source of the interferers.

Several papers have studied specific wireless relay scenarios. In [18], opportunistic relaying for cooperative diversity was studied in multicell environments where the interference from the neighboring cells is modeled as Wyner's model [21]. In [19], the behavior of an AF scheme for ad-hoc systems with inter-cluster interference was studied, and two max−min-based relay selection criteria for interference-limited systems with many relays were investigated. In [20], an outage-optimal opportunistic relaying method was suggested for interference-limited slow-fading environments. The authors considered the wireless access networks with one SN (used as the access point) and several RNs and DNs. When the SN transmits a message toward a DN, an opportunistically selected relay simultaneously forwards a previously received message to a different DN, causing interference with each other.

In this paper, we investigate a specific interference-limited wireless relay scenario that comprises several cooperation units (CU). Each CU is defined as one source and destination node pair with the help of an associated relay node over Rayleigh fading channels. We assume that the direct link between any source node and destination node is blocked due to a deep fade or an intermediate wall. The SN in each CU transmits its signals in the first time slot, and the RN then relays the signals to the DN in the second time slot. The interference affects both

SN-RN links and RN-DN links. In this scenario, we observe that the interference at the relay node of a CU is related to the interference at the destination node of the CU. On the contrary, most previous studies assume that the interference at the relay and that at the destination are independent. Motivated by this point, we study the end-to-end outage probability performance in a CU over slow-fading channels in such a scenario. We derive the end-to-end outage probability in a CU and the signal-to-interference-plus-noise ratio (SINR) distribution in closed form. Then, when each CU is allocated with equal power, we derive an optimal power allocation between the source node and the relay node in each CU to minimize the outage probability of the investigated CU. At last, in the case that each CU is not allocated with equal power and the sum of their power is constrained, we derive an optimal power allocation between CUs to minimize the sum of the outage probability of all CUs. In simulations, we discuss the impact of interferences from other CUs on the outage probability of the investigated CU, the optimal power allocation between the source node and the relay node in each CU, and the optimal power allocation between CUs.

The remainder of the paper is organized as follows. Section 2 describes the system model of an interference-limited wireless relay network. Section 3 analyzes the outage probability of the end-to-end in a CU over Rayleigh fading channels , derives the optimal power allocation between the source node and the relay node in each CU, and determines the optimal power allocation between CUs. Section 4 shows and discusses numerical results. Finally, Section 5 concludes the paper.

2. System Model

We consider a half-duplex dual-hop wireless network that includes *K* CUs, as shown in **Fig. 1**

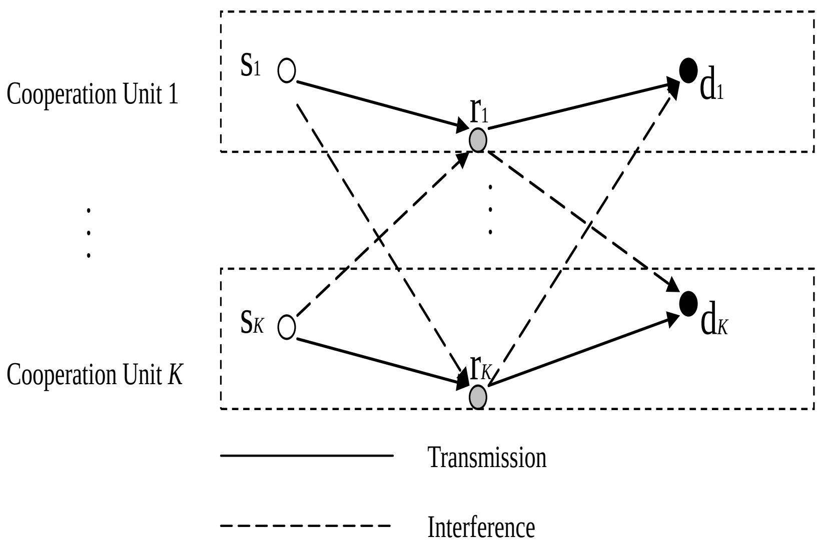

Fig. 1. Half-duplex dual-hop wireless network scenario that includes *K* cooperation units.

Each CU comprises an SN, an RN, and a DN. We assume that the direct link between any

SN and DN in relay networks is blocked due to a deep fade or an intermediate wall, and all CUs are precisely synchronized. During the first hop, the SN in each CU transmits a signal to its corresponding RN, causing interference to RNs in other CUs. The signal received at the relay node in the CU *i* can be expressed as

$$y_{R_i} = h_{S_i R_i} x_{S_i} + \sum_{j=1, j\neq i}^{K} h_{S_j R_i} x_{S_j} + n_{R_i} \tag{1}$$

where $h_{S_i R_i}$ is the channel between the source node S_i and the relay node R_i in the CU *i*, $h_{S_j R_i}$ is the channel between the source node S_j in the CU *j* and the relay node R_i in the CU *i*, x_{S_i} is the signal transmitted by the source node S_i with $\mathrm{E}\left\{\left|x_{S_i}\right|^2\right\} = P_{S_i}$ and n_{R_i} is an additive white Gaussian noise with an average power of N_0 at the relay node R_i. Clearly, the second part in Equation (1) is the interference to the relay node R_i from other CUs. We assume that $h_{S_i R_j} (i, j = 1, 2, ..., K)$ is an independent zero-mean complex Gaussian random variable with variance $\lambda_{S_i R_j}$, denoting the Rayleigh fading channel.

During the second hop, the relay node in each CU forwards the message to the destination node with a decode-and-forward (DF) strategy. The received signal at the destination node in the CU *i* is given by

$$y_{D_i} = h_{R_i D_i} x_{R_i} + \sum_{j=1, j\neq i}^{K} h_{R_j D_i} x_{R_j} + n_{D_i} \tag{2}$$

where $h_{R_i D_i}$ is the channel between the relay node R_i and the destination node D_i in the CU *i*, $h_{R_j D_i}$ is the channel between the relay node R_j in the CU *j* and the destination node D_i in the CU *i*, x_{R_i} is the signal transmitted by the rely node R_i with $\mathrm{E}\left\{\left|x_{R_i}\right|^2\right\} = P_{R_i}$, n_{D_i} is an additive white Gaussian noise (AWGN) with an average power of N_0 at the DN D_i. The second part in Equation (2) is the interference at D_i due to other CUs. We assume that $h_{R_i D_j} (i, j = 1, 2, ..., K)$ is an independent zero-mean complex Gaussian random variable with variance $\lambda_{R_i D_j}$, denoting the Rayleigh fading channel. First, we assume that each CU is allocated with equal power $P_1 = P_2 = ... = P_K = P$. The source power constraint in the CU i is given by

$$P_{S_i} = \zeta_i P \tag{3}$$

where $\zeta_i \in (0,1)$ denotes the fraction of the total power P allocated to the source node S_i in the CU *i*. The relay power constraint in the CU i is given by

$$P_{R_i} = (1-\zeta_i)P. \tag{4}$$

The interference at the relay node of CU *i* is related to the interference at the DN of CU *i*.

3. Outage Probability

In this section, we respectively analyze the outage probability between the SN S_i and the RN R_i and the outage probability between R_i and D_i in the CU i by using the following theorem. Then, we present some results of the outage probability for the DF relaying strategy in the CU i. At last, we determine the optimal power allocation between the SN and the RN in each CU and the optimal power allocation between CUs.

We consider the CU i that comprises S_i, R_i and D_i. The mutual information between S_i and R_i is given by

$$I_{S_iR_i} = \frac{1}{2}\log_2(1+\gamma_{R_i}) \tag{5}$$

where γ_{R_i}, which denotes the SINR in the relay node R_i, can be expressed as

$$\gamma_{R_i} = \frac{P_{S_i}\left|h_{S_iR_i}\right|^2}{N_0 + \sum_{j=1, j\neq i}^{K} P_{S_j}\left|h_{S_jR_i}\right|^2} \tag{6}$$

The mutual information between R_i and D_i is given by

$$I_{R_iD_i} = \frac{1}{2}\log_2(1+\gamma_{D_i}). \tag{7}$$

where γ_{D_i}, denoting the SINR in the destination node D_i, can be expressed as

$$\gamma_{D_i} = \frac{P_{R_i}\left|h_{R_iD_i}\right|^2}{N_0 + \sum_{j=1, j\neq i}^{K} P_{R_j}\left|h_{R_jD_i}\right|^2}\ . \tag{8}$$

The outage probability with the DF relaying strategy in the CU i can be written as

$$\begin{aligned}\Pr_{out}^{i} &= \Pr(I_{S_iD_i} \le R\} \\ &= \Pr(\min\left\{I_{S_iR_i}, I_{R_iD_i}\right\} \le R\} \\ &= 1-(1-\Pr(I_{S_iR_i} \le R))(1-\Pr(I_{R_iD_i} \le R))\end{aligned} \tag{9}$$

where R denotes the target spectral efficiency between S_i and D_i in bps/Hz. From Equations (5) and (6), we have

$$\Pr(I_{S_iR_i} \le R) = \Pr(\frac{1}{2}\log_2(1+\gamma_{R_i}) \le R) = \Pr(\frac{\left|h_{S_iR_i}\right|^2}{1+\sum_{j=1, j\neq i}^{K}\frac{\left|h_{S_jR_i}\right|^2 P_{S_j}}{N_0}} \le \frac{2^{2R}-1}{\frac{P_{S_i}}{N_0}}). \tag{10}$$

Theorem 1: If U_1 is an exponential random variable with $E\{U_1\}=\lambda_1$, U_2 is a sum of N statistically independent exponential random variables, that is $U_2=\sum_{j=1}^{N}Y_j$ with $\mathrm{E}\{Y_j\}=\mu_j$, then the cumulative distribution function (c.d.f.) of the random variable $U=\frac{U_1}{1+U_2}$ can be obtained as

$$F_U(x)=1-\exp(-\frac{x}{\lambda_1})\sum_{i=1}^{\eta(\mathbf{Q})}\sum_{j=1}^{\tau_i(\mathbf{Q})}\chi_{i,j}(\mathbf{Q})(1+\frac{x\mu_{<i>}}{\lambda_1})^{-j} \tag{11}$$

where $\mathbf{Q}=diag(\mu_1,\mu_2,...,\mu_N)$, $\eta(\mathbf{Q})$ is the number of distinct diagonal elements of the diagonal matrix $\mathbf{Q}$, $\mu_{<1>}>\mu_{<2>}>...>\mu_{<\eta(\mathbf{Q})>}$ are the distinct diagonal elements in decreasing order, $\tau_i(\mathbf{Q})$ is the multiplicity of $\mu_{<i>}$, and $\chi_{i,j}(\mathbf{Q})$ is the (*i, j*)th characteristic coefficient of $\mathbf{Q}$ [22].

Proof: See Appendix I.

From the definition of $\chi_{i,j}(\mathbf{Q})$ [22], we have Equation (12)

$$\chi_{i,j}(\mathbf{Q})=\frac{(-1)^{\varpi_{i,j}}}{\mu_{<i>}^{\varpi_{i,j}}}\sum_{\substack{k_1+k_2+...+k_{\eta(\mathbf{Q})}=\varpi_{i,j}\\ k_l\geq 0 \ for\ \forall\, l\neq i\\ k_i=0}}\prod_{\substack{l=1\\ l\neq i}}^{\eta(\mathbf{Q})}\binom{\tau_l(\mathbf{Q})+k_l-1}{k_l}\frac{\mu_{<l>}^{k_l}}{\left(1-\frac{\mu_{<l>}}{\mu_{<i>}}\right)}. \tag{12}$$

where $\omega_{i,j}=\tau_i(\mathbf{Q})-j$. Two results can be obtained from the definition of $\chi_{i,j}(\mathbf{Q})$. The first result is

$$\chi_{i,j}(\mathbf{Q})=\begin{cases}0, if\ i=2,3,...,n;\ j=1,2,...,n\\ 1, if\ i=1,\ j=n\end{cases} \tag{13}$$

where $\mathbf{Q}=\mathbf{I}_n$ and the second result is

$$\chi_{i,j}(\mathbf{Q})=\begin{cases}\dfrac{\mu_{<i>}^{n-1}}{\prod_{\substack{l=1\\ l\neq i}}^{N}(\mu_{<i>}-\mu_{<l>})}, if\ \ j=1\\ 0, otherwise\end{cases} \tag{14}$$

where $\mathbf{Q}=diag(\mu_1,\mu_2,...,\mu_n)$, $\mu_i\neq\mu_j\,(i\neq j)$, $\mu_{<1>}>\mu_{<2>}>...>\mu_{<n>}$ are the distinct diagonal elements in decreasing order.

From theorem 1, if all $\mu_j\ (j=1,2,...,N)$ are the same and equal to μ, the c.d.f. of U is simplified as

$$F_U(x)=1-\exp(-\frac{x}{\lambda_1})(1+\frac{x\mu}{\lambda_1})^{-N}. \tag{15}$$

Given the above facts, we then have the outage probability $\Pr(I_{S_iR_i} \le R)$ as follows

$$\Pr(I_{S_iR_i} \le R) = 1 - \exp(-\frac{(2^{2R}-1)N_0}{\lambda_{S_iR_i} P_{S_i}}) \sum_{i=1}^{\eta(\mathbf{Q})} \sum_{j=1}^{\tau_i(\mathbf{Q})} \chi_{i,j}(\mathbf{Q})(1 + \frac{(2^{2R}-1)N_0 \mu_{<i>}}{\lambda_{S_iR_i} P_{S_i}})^{-j} \quad (16)$$

where $\mathbf{Q} = diag(\mu_1, \mu_2, ..., \mu_{K-1}),\ \mu_j = \frac{P_{S_j} \lambda_{S_jR_i}}{N_0}$ $(j = 1, 2, ..., K, j \ne i)$.

Similarly, we can obtain the outage probability $\Pr(I_{R_iD_i} \le R)$ as

$$\Pr(I_{R_iD_i} \le R) = 1 - \exp(-\frac{(2^{2R}-1)N_0}{\lambda_{R_iD_i} P_{R_i}}) \sum_{i=1}^{\eta(\mathbf{V})} \sum_{j=1}^{\tau_i(\mathbf{V})} \chi_{i,j}(\mathbf{V})(1 + \frac{(2^{2R}-1)N_0 \nu_{<i>}}{\lambda_{R_iD_i} P_{R_i}})^{-j} \quad (17)$$

where $\mathbf{V} = diag(\nu_1, \nu_2, ..., \nu_{K-1}),\ \nu_j = \frac{P_{R_j} \lambda_{R_jD_i}}{N_0}$ $(j = 1, 2, ..., K, j \ne i)$.

We then obtain the outage probability of the CU *i* from Equation (9), (16) and (17) as

$$\begin{aligned} \Pr_{out}^{i} = 1 - \exp(-(&\frac{(2^{2R}-1)N_0}{\lambda_{S_iR_i} P_{S_i}} + \frac{(2^{2R}-1)N_0}{\lambda_{R_iD_i} P_{R_i}})) \\ &\times [\sum_{i=1}^{\eta(\mathbf{Q})} \sum_{j=1}^{\tau_i(\mathbf{Q})} \chi_{i,j}(\mathbf{Q})(1 + \frac{(2^{2R}-1)N_0 \mu_{<i>}}{\lambda_{S_iR_i} P_{S_i}})^{-j}] \\ &\times [\sum_{i=1}^{\eta(\mathbf{V})} \sum_{j=1}^{\tau_i(\mathbf{V})} \chi_{i,j}(\mathbf{V})(1 + \frac{(2^{2R}-1)N_0 \nu_{<i>}}{\lambda_{R_iD_i} P_{R_i}})^{-j}]. \end{aligned} \quad (18)$$

Here, we study a special case where each CU allocates the same power to its source node $\zeta_i = \zeta$ $(i = 1, 2, ..., K)$, each channel between the source node S_j in the CU *j* and the relay node R_i in the CU *i* has the same variance $\lambda_{S_jR_i} = \lambda_1$, and each channel between the relay node R_j in the CU *j* and the destination node D_i in the CU *i* has the same variance $\lambda_{R_jD_i} = \lambda_2$. From Equations (13) and (16), we have

$$\Pr(I_{S_iR_i} \le R) = 1 - \exp(-\frac{(2^{2R}-1)}{\lambda_{S_iR_i} \rho \zeta})(1 + \frac{(2^{2R}-1)\lambda_1}{\lambda_{S_iR_i}})^{-(K-1)} \quad (19)$$

where $\rho = \frac{P}{N_0}$. Similarly, we have

$$\Pr(I_{R_iD_i} \le R) = 1 - \exp(-\frac{(2^{2R}-1)}{\lambda_{R_iD_i} \rho (1-\zeta)})(1 + \frac{(2^{2R}-1)\lambda_2}{\lambda_{R_iD_i}})^{-(K-1)} \quad (20)$$

Therefore, the outage probability of the CU *i* becomes

$$\Pr_{out}^{i}=1-\exp(-(\frac{c_1}{\rho\zeta}+\frac{c_2}{\rho(1-\zeta)}))[(1+c_1\lambda_1)(1+c_2\lambda_2)]^{-(K-1)} \tag{21}$$

where $c_1=\frac{(2^{2R}-1)}{\lambda_{S_iR_i}}$, $c_2=\frac{(2^{2R}-1)}{\lambda_{R_iD_i}}$.

Remark 1: *Using (21), if* $\rho\to\infty$*, then*

$$\Pr_{out}^{i}=1-[(1+c_1\lambda_1)(1+c_2\lambda_2)]^{-(K-1)} \tag{22}$$

It can be seen from Equation (22) that the outage probability of CU i becomes constant as $\rho\to\infty$*. This implies that we cannot always reduce the outage probability of CU i by increasing SNR steadily. As the SNR increases, the interference at each CU is also increased.*

Remark 2: *It is observed from Equation (22) that the outage probability of CU* i *becomes large as* K *becomes large. This result is quite intuitive since in wireless networks, a larger number of CUs will cause more interference with each other.*

Remark 3: *It is observed from Equation (22) that the outage probability of CU i becomes small when* $\lambda_{S_iR_i}$ *and* $\lambda_{R_iD_i}$ *become larger, i.e.,* c_1 *and* c_2 *become smaller. Therefore, the good channel condition between the SN and RN (the RN and DN) in a CU is beneficial to reduce the outage probability of this CU.*

Remark 4: *Using the following expressions, we try to find* ζ *that minimizes the outage probability of CU* i.

$$\frac{\mathrm{d}\,f(\zeta)}{\mathrm{d}\zeta}=0 \tag{23}$$

where $f(\zeta)=\frac{c_1}{\rho\zeta}+\frac{c_2}{\rho(1-\zeta)}$. Solving (23), we have

$$\zeta_{opt}=\begin{cases}\frac{1}{2},\ if\ c_1=c_2\\ \frac{c_1-\sqrt{c_1c_2}}{c_1-c_2},\ otherwise\end{cases} \tag{24}$$

This implies that if the channel condition between the SN and RN is as good as the channel condition between the RN and DN in a CU, i.e., $\lambda_{S_iR_i}=\lambda_{R_iD_i}$*, the optimal power allocation method should evenly distribute the total power between the SN and RN in each CU.*

Next, we will discuss the case that each CU is not allocated with equal power. We assume that CU *i* is allocated with the power P_i, and the sum power constraint is

$$\sum_{i=1}^{K}P_i=P_{total} \tag{25}$$

We assume that the power factor ζ of each CU is the same, each channel between the source node S_j in the CU j and the relay node R_i in the CU i has the same variance

$\lambda_{S_jR_i} = \lambda_1 (i, j = 1, 2, ..., K, i \neq j)$, each channel between the relay node R_j in the CU *j* and the destination node D_i in the CU *i* has the same variance $\lambda_{R_jD_i} = \lambda_2 (i, j = 1, 2, ..., K, i \neq j)$, each channel between the source node S_i and the relay node R_i in the CU *i* has the same variance $\lambda_{S_iR_i} = \lambda_3 (i = 1, 2, ..., K)$, and each channel between the relay node R_i the destination node D_i in the CU *i* has the same variance $\lambda_{R_iD_i} = \lambda_4 (i = 1, 2, ..., K)$. Similarly, the outage probability of the CU i becomes

$$\Pr_{out}^{i} = 1 - \exp(-(\frac{c_1}{\rho_i \zeta} + \frac{c_2}{\rho_i (1-\zeta)}))[(1 + c_1\lambda_1)(1 + c_2\lambda_2)]^{-(K-1)} \tag{26}$$

where $c_1 = \frac{(2^{2R} - 1)}{\lambda_3}$, $c_2 = \frac{(2^{2R} - 1)}{\lambda_4}$, $\rho_i = \frac{P_i}{N_0}$. Our design objective is to minimize the sum of the outage probability of all CUs under the total power constraint. The optimazition problem can be formulated as

$$\min_{P_1, P_2, ..., P_K} \sum_{i=1}^{K} \Pr_{out}^{i} \tag{27a}$$

$$\text{Subject to } \sum_{i=1}^{K} P_i = P_{total} \tag{27b}$$

The above optimazition problem can be solved by Lagrangian

$$\begin{aligned} L(P_1, P_2, ..., P_K, \lambda) = &\sum_{i=1}^{K} (1 - \exp(-(\frac{c_1}{\rho_i \zeta} + \frac{c_2}{\rho_i (1-\zeta)}))((1 + c_1\lambda_1)(1 + c_2\lambda_2))^{-(K-1)}) \\ &+ \lambda \sum_{i=1}^{K} P_i \end{aligned} \tag{28}$$

Where λ is the Lagrange multiplier associated with the constraint (27b). Making the derivative of the Lagrange with repect to P_i gives

$$\frac{\partial L}{\partial P_i} = -((1 + c_1\lambda_1)(1 + c_2\lambda_2))^{-(K-1)} (\frac{c_1 N_0}{\zeta} + \frac{c_2 N_0}{1-\zeta}) \exp(-(\frac{c_1 N_0}{P_i \zeta} + \frac{c_2 N_0}{P_i (1-\zeta)})) P_i^{-2} + \lambda \tag{29}$$

Letting $\frac{\partial L}{\partial P_i} = 0$, the optimal $\{P_i\}$ that can minimize (27a) can be obtained by

$$\begin{cases} \sum_{i=1}^{K} P_i = P_{total} \\ \dfrac{((1 + c_1\lambda_1)(1 + c_2\lambda_2))^{-(K-1)} (\frac{c_1 N_0}{\zeta} + \frac{c_2 N_0}{1-\zeta})}{P_i^2 \exp(\frac{c_1 N_0}{P_i \zeta} + \frac{c_2 N_0}{P_i (1-\zeta)})} = \lambda \end{cases} \tag{30}$$

Clearly, we should allocate equal power to each unit in order to make the sum of outage probability of all CUs minimum, i.e., $P_i = \frac{P_{total}}{K}$.

4. Numerical Results and Analysis

In this section, we give numerical results of the outage probability of a CU. First, we assume that each CU is allocated with equal power P. We assume that the target spectral efficiency R =1 bps/Hz , and there is a total of 4 CUs (K=4), $\lambda_{S_iR_i} > \lambda_{S_iR_j}$ ($i \neq j$), which means that the channel gain between the source node to its relay node in each CU is better than the interference channels, and $\lambda_{R_iD_i} > \lambda_{R_iD_j}$ ($i \neq j$), which means that the channel gain between the relay node to its destination node in each CU is better than the interference channels. Because each CU is allocated the same power and has the same structure, we can study CU 1 as the investigated CU to depict the performance of this wireless network.

Fig. 2 shows the outage probability of CU 1 versus P/N_0 for $\lambda_{S_iR_i} = \lambda_{R_iD_i} = 100$ ($i = 1, 2, \ldots, K$) and $\lambda_{S_iR_i} = \lambda_{R_iD_i} = 1000$ ($i = 1, 2, \ldots, K$) in two cases. We assume that $\lambda_{S_iR_j} = \lambda_{R_iD_j} = 1$ ($i \neq j$) and $\zeta_i = 0.5$ ($i = 1, 2, \ldots, K$). From the figure, we observe that although the outage probability decreases as the SNR increases, it remains unchanged when the SNR is sufficiently large. This observed result is consistent with the analysis in the above section. The interference at each CU is increased as the SNR increases due to the equal power allocation of each CU. From **Fig. 2**, we observe that the outage probability decreases by about 0.14 at 15dB in the case of $\lambda_{S_iR_i} = \lambda_{R_iD_i} = 1000$ compared to the case of $\lambda_{S_iR_i} = \lambda_{R_iD_i} = 100$. Therefore, if the channel gain between SN and RN or between RN and DN in a CU is high, then the outage probability of this CU will be smaller, as discussed in the above section. We also compare the outage probability obtained using a Monte-Carlo simulation with that computed from analytical expression (18). The results for both are in good agreement with each other, verifying the accuracy of our analysis for the outage probability.

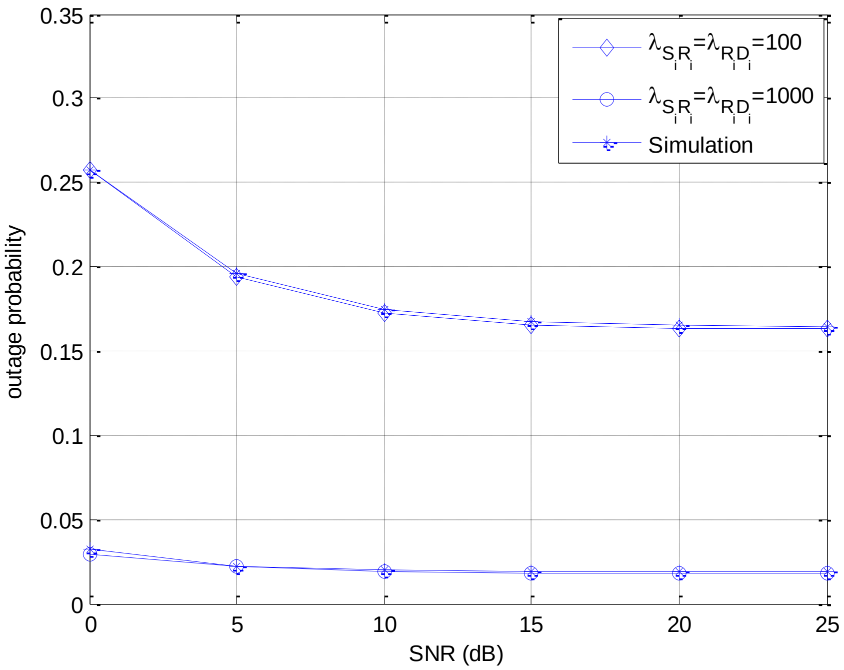

Fig. 2. Outage probability as a function of SNR (P/N_0) when the power factors are all equal to 0.5.

Fig. 3 shows the outage probability of CU 1 versus P/N_0 when the power factor ζ of each CU is the same. The power factor: [0.1 0.1 0.1 0.1] shown in the legends represents the power factor of CU 1 $\zeta_1 = 0.1$, the power factor of CU 2 $\zeta_2 = 0.1$, the power factor of CU 3 $\zeta_3 = 0.1$, and the power factor of CU 4 $\zeta_4 = 0.1$. We assume that $\lambda_{S_i R_j} = \lambda_{R_i D_j} = 1 (i \neq j)$, $\lambda_{S_i R_i} = \lambda_{R_i D_i} = 100$, and the target spectral efficiency R =1 bps/Hz. It is observed from the figure that the outage probability of CU 1 is a minimum when each CU allocates the same power to its source and destination nodes, i.e., $\zeta = 0.5$. For the case of $\lambda_{S_i R_i} \neq \lambda_{R_i D_i}$ (e.g., $\lambda_{S_i R_i} = 100, \lambda_{R_i D_i} = 50$), the optimal power factor is $\dfrac{c_1 - \sqrt{c_1 c_2}}{c_1 - c_2} = 0.4142$, as shown in **Fig. 4.** These results agree with our analysis in Equation (24).

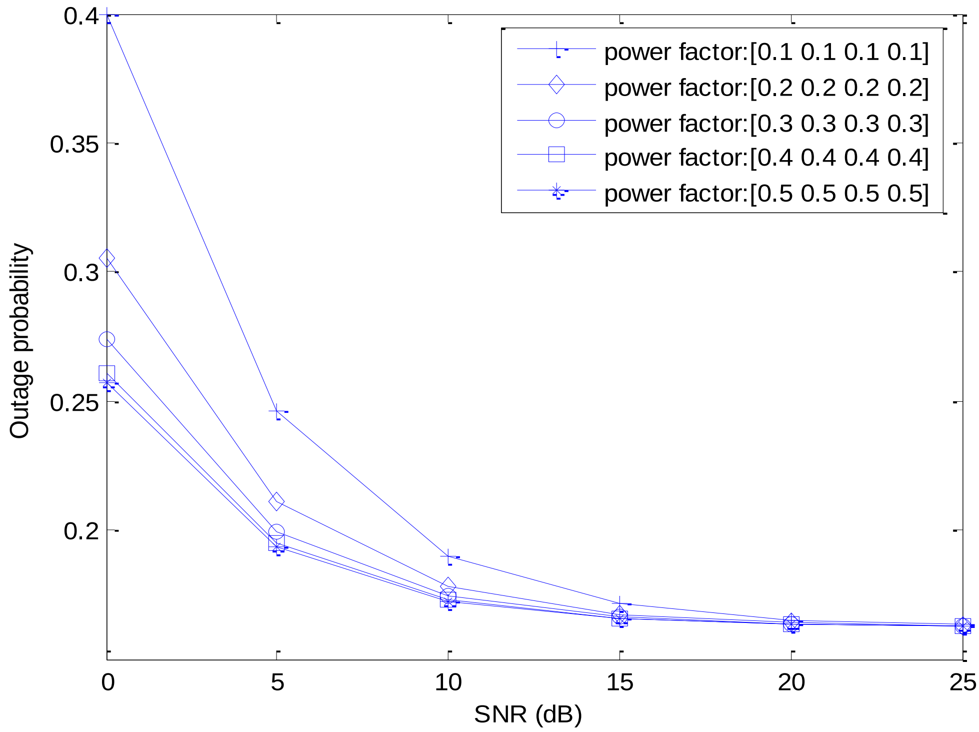

Fig. 3. Outage probability as a function of SNR when the power factor of CU 1 is equal to the power factors of the other CUs and $\lambda_{S_i R_i} = \lambda_{R_i D_i} = 100$.

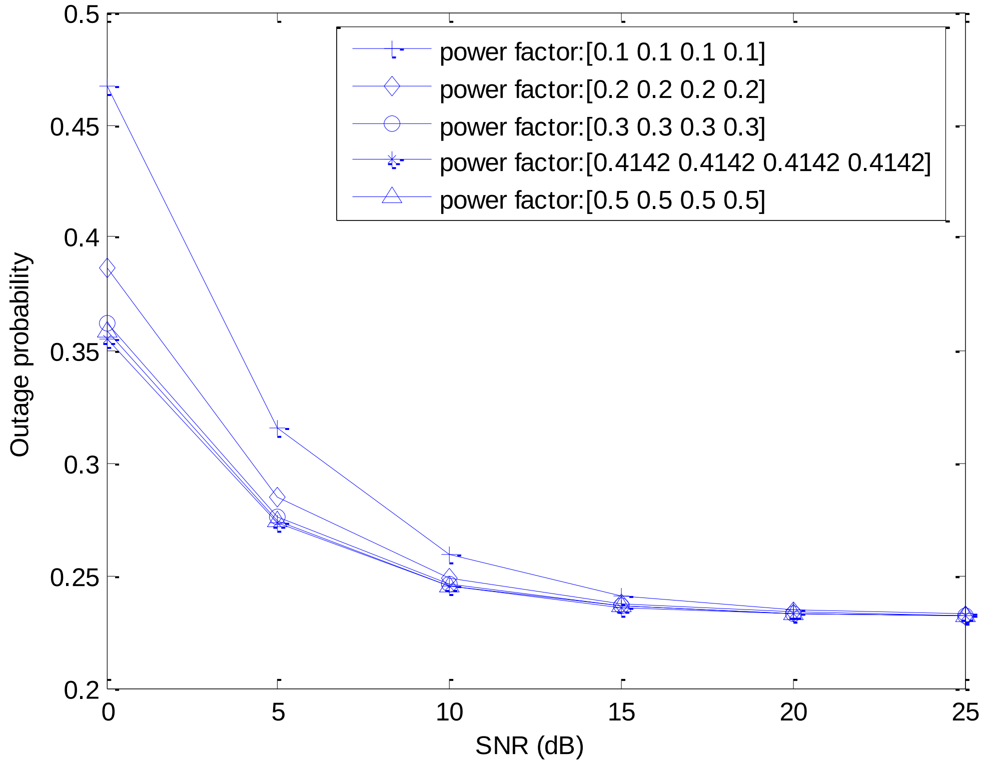

Fig. 4. Outage probability as a function of SNR when the power factor of CU 1 is equal to the power factors of the other CUs and $\lambda_{S_iR_i} = 100, \lambda_{R_iD_i} = 50$.

From our analysis in the above section, we note that the presence of more CUs in wireless networks will cause more interference at CU 1, **Fig. 5** shows that the outage probability of CU 1 is decreasing when the number of CUs in wireless networks becomes large.

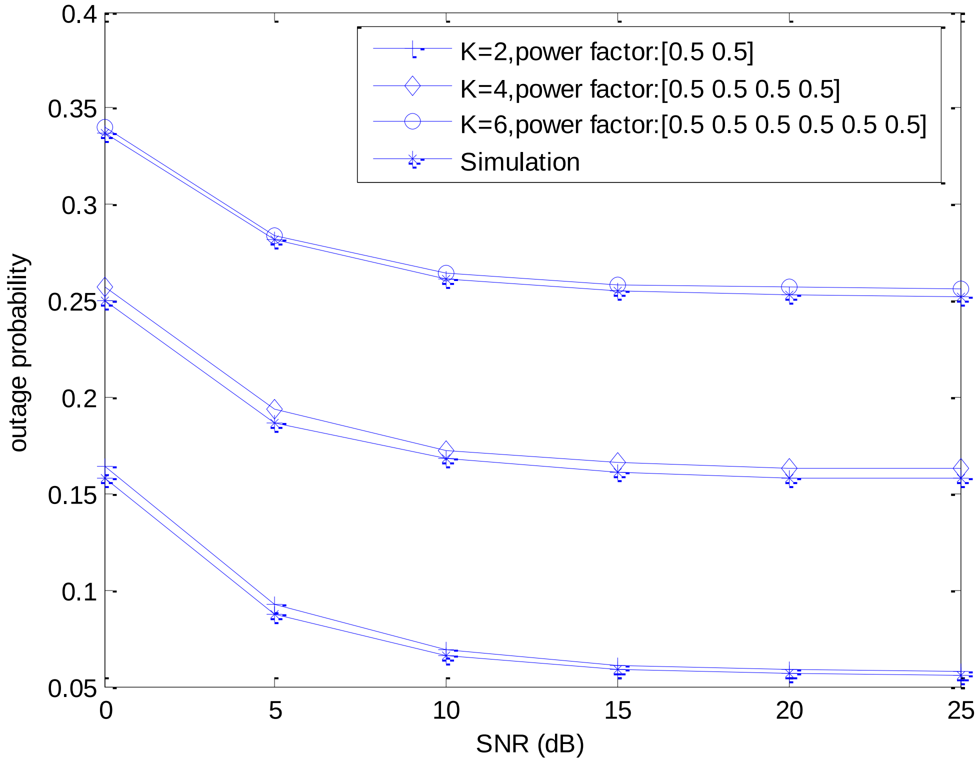

Fig. 5. Outage probability as a function of SNR as the number of CUs in wireless networks increases.

Fig. 6 shows the outage probability of CU 1 versus P / N_0 when the power factor of CU 1 ζ_1 is set to 0.8. It can be seen from the figure that if the power factor of the CU 1 is fixed, regardless of the allocation of power factors 0.3, 0.5, and 0.7 in the other CUs, the impact on the outage probability of CU 1 remains the same. The reason for this is that the interference at the RN in CU 1 is equal to the interference at the DN in CU 1.

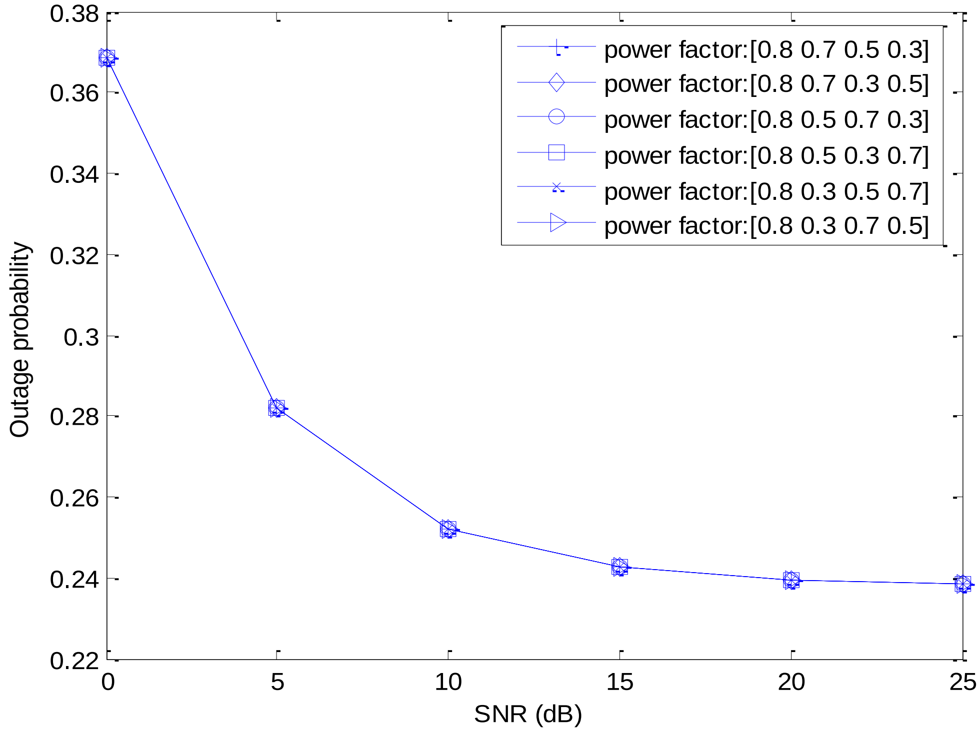

Fig. 6. Outage probability as a function of SNR when the power factor of CU 1 is fixed to 0.8 and the interference at the RN in CU 1 is equal to the interference at the DN in the CU 1.

Fig. 7 shows that when the power factor of the CU 1 is set to 0.2, different outage results will be obtained by setting all of the other power factors to either low or high values (which means having low or high interference on the SN-RN link or the RN-DN link in CU 1). It is observed that the performance of CU 1 is better when the interference at the RN of CU 1 is less than that at the DN compared to the case when the interference at the RN is more than that at the DN. For example, when the power factor setting is [0.2 0.1 0.1 0.1], the interference at the relay node of CU 1 is smallest among the four cases and the performance of CU 1 is best. The result demonstrates that the interference at the RN of CU 1 causes more deterioration in the performance of CU 1 than does the interference at the DN of CU 1 when the power factor of CU 1 is low. From **Fig. 7**, we also observed that strong interference at the weak link of CU 1 causes the worst impairment to the performance of CU 1. For example, the power factor: [0.2 0.8 0.8 0.8] shown in legends in **Fig. 7** indicates that the weak link in CU 1 is the SN-RN link due to the lower source transmitting power in CU 1; however, this link suffers from strong interference from other CUs.

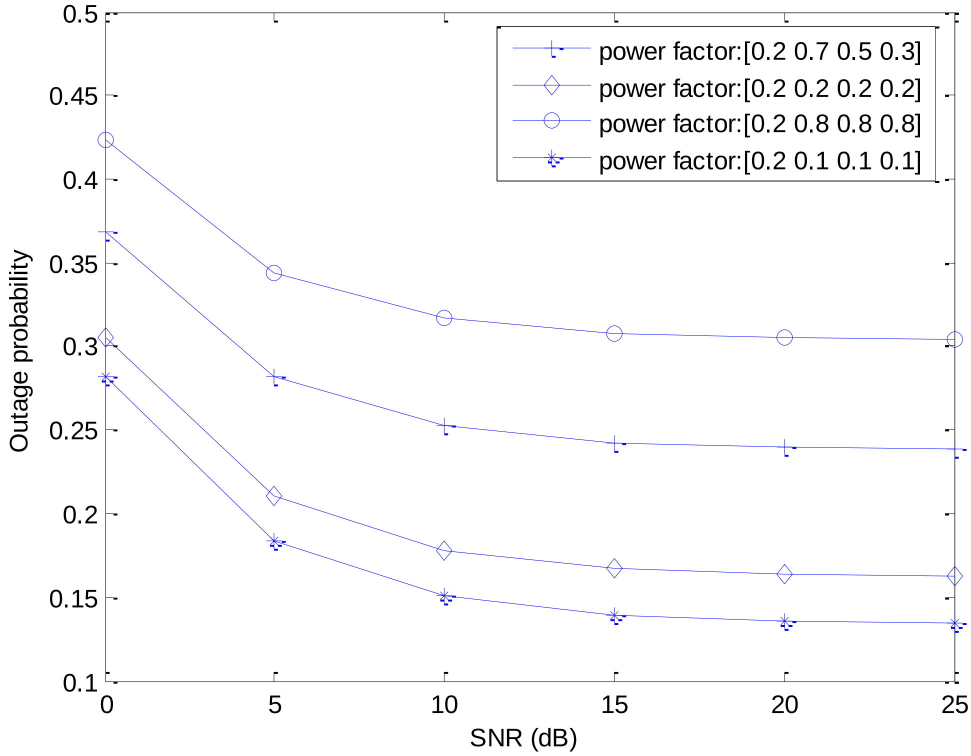

Fig. 7. Outage probability as a function of SNR when the power factor of the CU1 is fixed to 0.2 and the interference at the relay node in the CU 1 is not equal to the interference at the destination node in the CU 1.

Second, we assume that there are total 2 CUs, each CU is allocated with different power, and the sum power constraint P_{total} is 10dB. We assume that the power factor ζ of each CU is the same, $\lambda_{S_iR_j} = \lambda_{R_iD_j} = 1(i \neq j)$, $\lambda_{S_iR_i} = \lambda_{R_iD_i} = 100$, and the target spectral efficiency R =1 bps/Hz. **Fig. 8** shows the sum outage probability versus the power of CU 1 when the power factor ζ of each CU is set to 0.5. It is observed from the figure that the sum of outage probability of all CUs is minimum when each CU is allocated the same power, i.e., $P_1 = P_2 = 5$ dB. This result tests our analysis in Equation (30).

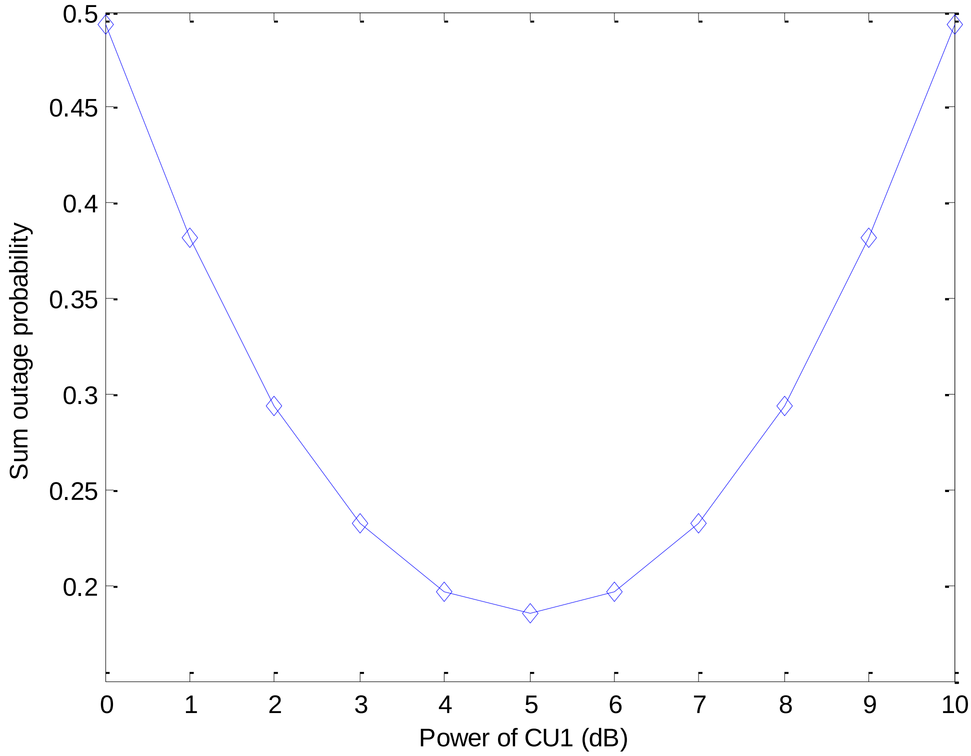

Fig. 8. Sum outage probability versus the power of CU1 when the power factor ζ of each CU is set to 0.5 and the sum power constraint is set to 10dB

5. Conclusions and future works

We have deduced the end-to-end outage probability performance in a CU over Rayleigh fading channels with relative interference for DF relaying strategy. Numerical results have shown that if the power factor of the investigated CU is fixed and the interference at the relay node is the same as that at the destination node of the investigated CU, the outage probability of the investigated CU is unrelated to the allocation of power in other CUs. If the interference at the relay node is not equal to that at the destination node, the link (i.e., the SN-RN link or RN-DN link) that suffers strong interference should be allocated a greater transmit power. Moreover, among various equal power allocation methods in the investigated CU, we determine that the optimal solution is to evenly allocate the total power between the source node and the relay node in each CU. When each CU is allocated different power and under the constrait of sum power of CUs, we determine that allocating the same power to each CU can make the sum of outage probability of all CUs minimum.

Future works can be conducted in the following aspect.In this work we assume that each node in a CU is equipped with single antenna. In practice, if nodes have enough power and advanced signal processing capability, we can install multiple antenna in each node in a CU. Moreover, in this work we consider a narrowband channel. We can expand our work to the case of broadband channel by incorperating orthogonal frequency division multiplexing (OFDM) technology which can turn a frequency-selective channel into a parallel collection of frequency flat sub-channels to mitigate the effects of intersymbol interference (ISI). Clearly, it is more challenging to incorperate multiple-input multiple-output (MIMO) and OFDM into the scheme.

Appendix Ⅰ

Proof Of Theorem 1

Because U_1 is an exponential random variable with $\mathrm{E}\{U_1\}=\lambda_1$, the probability density function (p.d.f.) and the c.d.f. of U_1 can be given by

$$f_{U_1}(x)=\frac{1}{\lambda_1}\exp(-\frac{x}{\lambda_1}). \tag{31}$$

$$F_{U_1}(x)=1-\exp(-\frac{x}{\lambda_1}). \tag{32}$$

Because Y_j is an exponential random variable with $\mathrm{E}\{Y_j\}=\mu_j$, the p.d.f. of U_2 can be given by [20]

$$f_{U_2}(x)=\sum_{i=1}^{\eta(\mathbf{Q})}\sum_{j=1}^{\tau_i(\mathbf{Q})}\chi_{i,j}(\mathbf{Q})\frac{\mu_{<i>}^{-j}}{(j-1)!}x^{j-1}e^{-\frac{x}{\mu_{<i>}}}. \tag{33}$$

where $\mathbf{Q}=diag(\mu_1,\mu_2,...,\mu_N)$, $\eta(\mathbf{Q})$ is the number of distinct diagonal elements of the diagonal matrix $\mathbf{Q}$, $\mu_{<1>}>\mu_{<2>}>...>\mu_{<\eta(\mathbf{Q})>}$ are the distinct diagonal elements in decreasing order, $\tau_i(\mathbf{Q})$ is the multiplicity of $\mu_{<i>}$, and $\chi_{i,j}(\mathbf{Q})$ is the (*i*, *j*)th characteristic coefficient of $\mathbf{Q}$.

From Equation (31)-(33), we obtain the c.d.f. of U as

$$\begin{aligned}
&F_U(x)\\
&=\Pr(\frac{U_1}{1+U_2}\le x)\\
&=\mathrm{E}_{U_2}(1-\exp(-\frac{x(1+U_2)}{\lambda_1}))\\
&=1-\exp(-\frac{x}{\lambda_1})\int_0^{\infty}\sum_{i=1}^{\eta(\mathbf{Q})}\sum_{j=1}^{\tau_i(\mathbf{Q})}\chi_{i,j}(\mathbf{Q})\frac{\mu_{<i>}^{-j}}{(j-1)!}y^{j-1}\exp(-\frac{y}{\mu_{<i>}})\exp(-\frac{xy}{\lambda_1})dy\\
&=1-\exp(-\frac{x}{\lambda_1})\sum_{i=1}^{\eta(\mathbf{Q})}\sum_{j=1}^{\tau_i(\mathbf{Q})}\chi_{i,j}(\mathbf{Q})(1+\frac{x\mu_{<i>}}{\lambda_1})^{-j}
\end{aligned} \tag{34}$$

where the calculation of the integration in the final step is completed by using the identity [23,eq.(3.351.3)].

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Peisheng Pan received the B.S. degree in computer science and technology from Inner Mongolia University (IMU), Hohhot, China, in July 1988, the M.S. degree in computer science and technology from Nanjing University of Aeronautics and Astronautics (NUAA), Nanjing, China, in April 1994, and the Ph.D. degree in signal and information processing from Nanjing University of Posts and Telecommunications (NUPT), Nanjing, China, in January 2011. From August 1988 to August 1991, he was with the Department of Mathematics Science, Inner Mongolia Normal University (IMNU), Hohhot, China, as an assistant. Since 1994, he has worked with NUPT. Currently he is an associate professor with the college of communication and Information engineering of NUPT. His current research interests include multiuser MIMO, cognitive radio, and cooperative communication.

Baoyu Zheng received the B.S. and M.S.degrees from the Department of Circuit and Signal System, Nanjing University of Posts and Telecommunications (NUPT), Nanjing, China, in 1969 and 1981, respectively. Since then, he has been engaged in teaching and researching with the Signal and Information Processing. He is a full professor and doctoral advisor at NUPT. His research interests span the broad area of the intelligent signal processing, wireless network and signal processing for modern communication,and the quantum signal processing.