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A NOTE ON SCALAR CURVATURE FUNCTIONS OF ALMOST-KÄHLER METRICS

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Abstract. We present a 4-dimensional nil-manifold as the first example of a closed non-Kählerian symplectic manifold with the following property: a function is the scalar curvature of some almost Kähler metric iff it is negative somewhere. This is motivated by the Kazdan-Warner's work on classifying smooth closed manifolds according to the possible scalar curvature functions.

1. Introduction

Kazdan and Warner classified closed smooth manifolds of dimension>2 into three classes according to what the scalar curvature functions can be on a manifold [3, Th. 4.35]:

(a) Any smooth function is the scalar curvature of some Riemannian metric.
(b) A smooth function is the scalar curvature of some Riemannian metric iff it is either identically zero or somewhere negative.
(c) A function is the scalar curvature of some Riemannian metric iff it is negative somewhere.

This interesting theorem was proved based on the existence of many Riemannian metrics of constant scalar curvature, which is due to the resolution of Yamabe problem [2, 11].

Recently, there has been much interest on symplectic manifolds [5, 6, 10]. So we would like to study an analogous question on the scalar curvature functions of almost Kähler metrics on symplectic manifolds. An almost Kähler metric is a Riemannian metric compatible with a symplectic structure, see Subsection 2.1. We ask if any closed symplectic manifold of dimension>2 falls into one of the three classes:

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(a') Any smooth function is the scalar curvature of some almost Kähler metric.

(b') A smooth function is the scalar curvature of some almost Kähler metric iff it is either identically zero or somewhere negative.

(c') A function is the scalar curvature of some almost Kähler metric iff it is negative somewhere.

This classification may be difficult to resolve at this time, since we do not have a general existence theory of almost Kähler metrics of constant scalar curvature. In fact, we do not even have a Yamabe type problem defined. As little is known about this problem, we first try to get examples.

Many of the manifolds admitting Kähler metrics with zero scalar curvature may be examples in class (a'). According to the section 5 in [9], some symplectic tori are examples in the class (b'). For class (c'), we just suspect that many manifolds admitting Kähler metrics with negative constant scalar curvature may belong to this class. Usually, Kähler examples are easy to deal with. But the focus of the above classification problem is on non-Kählerian symplectic manifolds.

The main result in this article is to present the so-called Kodaira-Thurston symplectic manifold as the first non-Kählerian example in the class (c');

Theorem 1.1. *On a symplectic compact quotient M of the 4-dimensional Kodaira-Thurston nil-manifold, a smooth function is the scalar curvature of some almost-Kähler metrics if and only if it is somewhere negative.*

2. Preliminaries

2.1. Almost-Kähler metric For this subsection a good reference is [4]. An *almost-Kähler* metric on a smooth manifold M^{2n} of real dimension $2n$ is a Riemannian metric g compatible with a symplectic structure ω, i.e. $\omega(X,Y) = g(X,JY)$ for an almost complex structure J, where X, Y are tangent vectors at a point of the manifold. Note that given ω, g determines J and vice versa. We call a Riemannian metric g ω-almost Kähler if g is compatible with ω and denote by $\Omega_\omega := \Omega_\omega(M)$ the set of all C^∞ ω-almost Kähler metrics on M. An almost-Kähler metric (g, ω, J) is Kähler if and only if J is integrable.

An almost complex structure J gives rise to a type decomposition of symmetric (2,0)-tensors. For any symmetric (2,0)-tensor field h, we have the splitting $h = h^+ + h^-$, where

$$h^+(X,Y) = \frac{1}{2}\{h(X,Y) + h(JX,JY)\}$$

and

$$h^-(X,Y) = \frac{1}{2}\{h(X,Y) - h(JX,JY)\}.$$

A symmetric (2,0)-tensor field h is called J-invariant [or J-anti-invariant] if $h^- = 0$ [or $h^+ = 0$, respectively].

The space Ω_ω is a smooth Fréchet manifold, and the tangent space $T_g\Omega_\omega$ at $g \in \Omega$ is exactly the set of J-anti-invariant symmetric (2,0)-tensor fields, where J is the almost complex structure corresponding to (g,ω). The space Ω_ω admits a natural parametrization by the exponential map; for $g \in \Omega_\omega$, define $\mathcal{E}_g : T_g\Omega_\omega \to \Omega_\omega$ by $\mathcal{E}_g(h) = g \cdot \exp^h$, with $g \cdot \exp^h(X,Y) = g(X, \exp^{\hat{h}}(Y))$, where X, Y are tangent vectors at a point of M and $\hat{h}$ is the $(1,1)$-tensor field lifted from h with respect to g so that $\exp^{\hat{h}}(Y) = Y + \sum_{k=1}^{\infty} \frac{1}{k!}\hat{h}^k(Y)$. Clearly we have;

$$\frac{d\{g \cdot \exp^{th}\}}{dt}|_{t=0} = h. \tag{2.1}$$

We denote by ∇, R and s the Levi-Civita connection, the Riemannian curvature tensor and the scalar curvature of a Riemannian manifold (M,g). For tangent vector fields X,Y,Z,W, the Riemannian curvature tensor R is defined by $R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$.

3. Scalar Curvature Functions of Almost Kähler metrics

The purpose of this section is mainly to recall the argument in [9, Section 5] which is relevant to our Kazdan-Warner type result, but we also modify the proof of Lemma 5 in [9] into a little more understandable argument.

3.1. Derivative of the scalar curvature functional We consider the scalar curvature map defined on the space $\mathcal{M}$ of Riemannian metrics on a manifold;

$$S(g) := \text{the scalar curvature of } g.$$

Recall that the differential at g, in the direction of a symmetric (2,0)-tensor h, of S is given by

$$\Delta_g(\mathrm{tr_g}h) + \delta_\mathrm{g}(\delta_\mathrm{g}h) - \mathrm{g}(\mathrm{r}, h), \tag{3.1}$$

where r is the Ricci curvature tensor of g, Δ_g is the Laplacian, $\mathrm{tr_g}(h)$ is the trace of h with respect to g, $\delta_g h$ is the divergence of h which can be written in local coordinates as $(\delta_g h)_\lambda = -\nabla^\nu h_{\nu\lambda}$ and finally $\delta_g(\cdot)$ for 1-forms is the formal adjoint of the exterior differential on functions, [3].

Now we restrict the scalar curvature map to the space Ω_ω;

$$S_\omega := S|_{\Omega_\omega}$$

As a J-anti-invariant symmetric (2,0)-tensor is of trace zero, from (3.1), we have;

$$D_g S_\omega(h) = \delta_g(\delta_g h) - g(r, h).$$

So $D_g S_\omega$ is an under-determined elliptic operator for any $g \in \Omega_\omega$. The formal adjoint operator $(D_g S_\omega)^* : C^\infty(M) \to T_g\Omega$ of $D_g S_\omega$ with respect to the L^2 inner product induced from g is then as follows:

$$(D_g S_\omega)^*(\psi) = \nabla^- d\psi - r^-\psi. \tag{3.2}$$

where $\nabla^- d\psi$ and r^- are the J-anti-invariant part of $\nabla d\psi$ and r, respectively, and J is the corresponding almost complex structure to (g, ω).

3.2. Scalar Curvature Map in L^p Setting We shall now consider the scalar curvature map in L^p setting and discuss the surjectivity of its derivative map as a sufficient condition for the local surjectivity of the scalar curvature map [3, Chap.4, Section E].

By standard argument, the scalar curvature map $S_\omega : \Omega_\omega \longrightarrow C^\infty(M)$ can be extended to a smooth map from the space of L^p_2 ω-almost-Kähler metrics, $L^p_2(\Omega_\omega)$, to the space of L^p functions, $L^p(M)$, if $p > \dim_{\mathbb{R}}(M)$, which will be assumed in this section. Note that $L^p_2(\Omega_\omega)$ is a Banach algebra.

Now at $g \in \Omega_\omega$ with J, consider the linearized map of S_ω, $DS_\omega|_g : L^p_2(T_g\Omega_\omega) \longrightarrow L^p(M)$. The space $L^p_2(T_g\Omega_\omega)$ consists of L^p_2 J-anti invariant symmetric 2-tensor fields h. As $DS_\omega|_g$ is an under-determined elliptic operator at any $g \in \Omega_\omega$, by the elliptic regularity theory [3, page 464], we have a decomposition:

$$L^p(M) = DS_\omega|_g(L^p_2(T_g\Omega_\omega)) \oplus \ker(DS_\omega|_g)^*.$$

and the kernel $\ker(DS_\omega|_g)^*$ of the formal adjoint map $(DS_\omega|_g)^*$ is finite dimensional and consists of C^∞ functions on M. Therefore in order to prove that $DS_\omega|_g$ is surjective, we need to show that $\ker(DS_\omega|_g)^*$ is zero.

Lemma 1 and Lemma 2 below are the keys to a Kazdan-Warner type result; refer to [8]. We adapt them to our map S_ω.

Lemma 1. *If $DS_\omega|_g$ is surjective at an almost-Kähler metric g, then the scalar curvature map S_ω is locally surjective at g, i.e. there exists $\epsilon > 0$ such that, if f is in $L^p(M)$ and $||f - S_\omega(g)||_{L^p} < \epsilon$, there is an L^p_2 almost-Kähler metric $\tilde{g}$ such that $f = S_\omega(\tilde{g})$. Furthermore if f is C^∞, so is $\tilde{g}$.*

Proof. The argument is a simple modification of the general Riemannian case. Define a map $A : L^p_4(M) \longrightarrow L^p(M)$, by $A(\psi) = S_\omega(g \cdot \exp^{(DS_\omega|_g)^*(\psi)})$. By standard argument one can see that A is a smooth differential operator and is elliptic (at least for ψ near 0). Using (2.1), one check that the derivative of A at 0, DA_0, is equal to $DS_\omega|_g \circ (DS_\omega|_g)^*$ and so is an isomorphism from the surjectivity of $DS_\omega|_g$. We apply the Inverse Function Theorem to conclude that S_ω is locally surjective at g. Elliptic regularity applied to A implies that for a C^∞ function f near 0 in L^p norm, one can choose a C^∞ almost Kähler metric $\tilde{g} = g \cdot \exp^{(DS_\omega|_g)^*(\tilde{\psi})}$ compatible with ω such that $f = S_\omega(\tilde{g})$. □

The following approximation lemma handles arbitrary smooth functions on M. Let $\mathcal{D}$ be the diffeomorphism group of M.

Lemma 2 ([8]). *If $\dim_{\mathbb{R}}(M) \geq 2$ and if $f \in C^0(M)$, then an L^p function f_1 belongs to the L^p closure of the set $\{f \circ \phi, \phi \in \mathcal{D}\}$ if and only if $\inf f \leq f_1(x) \leq \sup f$ almost everywhere.*

Now we can state [9];

Proposition 1. *Suppose that there exists an almost-Kähler metric (g, ω) with constant scalar curvature s_g and that $DS_\omega|_g$ is surjective at g, then any smooth function f with $\inf f \leq s_g \leq \sup f$ is the scalar curvature of an almost-Kähler metric $\tilde{g}$ for some symplectic form $\tilde{\omega}$ which has the same volume as ω.*

Proof. Given a smooth function f with $\inf f \leq s_g \leq \sup f$, by Lemma 1 and Lemma 2 there exist a diffeomorphism ϕ and a C^∞ ω-almost-Kähler metric $\tilde{g}$ such that the scalar curvature of pulled-back metric $\phi^*\tilde{g}$ equals f. Then the pulled-back pair $(\tilde{\omega} := \phi^*\omega,\ \phi^*\tilde{g})$ is an almost-Kähler structure with the scalar curvature f and the same volume as ω. □

4. Computations on the Kodaira-Thurston Nil-manifold

In order to use Proposition 1, we need to have almost-Kähler metrics (g, ω) with constant scalar curvature such that $DS_\omega|_g$ is surjective. The problem is that we do not have many examples of non-Kähler almost Kähler metrics with constant scalar curvature. Another problem is that it is difficult to verify the surjectivity of $DS_\omega|_g$ especially for non-Kähler almost Kähler metrics, in contrast to the general Riemannian case in [3, 4.37].

Here we consider a left-invariant almost Kähler metric g on a compact quotient M of the 4-dimensional Kodaira-Thurston nil-manifold [1, 13]. We shall check for the surjectivity of $DS_\omega|_g$.

On the universal cover they are described as follows: the metric can be written on $\mathbb{R}^4 = \{(x,y,z,t)|x,y,z,t \in \mathbb{R}\}$ as $g = dx^2 + dy^2 + (dz - xdy)^2 + dt^2$ and the left-invariant symplectic form is $\omega = dx \wedge dt + dz \wedge dy$. The almost complex structure J is then given by $J(e_4) = e_1$, $J(e_1) = -e_4$, $J(e_2) = e_3$, $J(e_3) = -e_2$, where $e_1 = \frac{\partial}{\partial x}$, $e_2 = \frac{\partial}{\partial y} + x\frac{\partial}{\partial z}$, $e_3 = \frac{\partial}{\partial z}$, $e_4 = \frac{\partial}{\partial t}$ which form an orthonormal frame for the metric. Consider the discrete subgroup Γ of the isometry group of g spanned by $\{\gamma_i, i = 1,2,3,4\}$ where $\gamma_1(x,y,z,t) = (x+1, y, y+z, t)$, $\gamma_2(x,y,z,t) = (x, y+1, z, t)$, $\gamma_3(x,y,z,t) = (x,y,z+1,t)$ and $\gamma_4(x,y,z,t) = (x,y,z,t+1)$. Then $\mathbb{R}^4/\Gamma$ is a compact quotient smooth manifold with the quotient metric which we still denote by g.

By routine computation one can find the components r_{ij} of Ricci curvature as follows; $r_{11} = -\frac{1}{2}, r_{22} = -\frac{1}{2}, r_{33} = \frac{1}{2}, r_{44} = 0$, and $r_{ij} = 0$ for $i \neq j$. Then the components $r^-_{ij} = r^-(e_i, e_j) = \frac{1}{2}\{r(e_i,e_j) - r(Je_i, Je_j)\}$ of the J-anti-invariant part of the Ricci tensor are as follows: $r^-_{11} = -\frac{1}{4}$, $r^-_{22} = -\frac{1}{2}$, $r^-_{33} = \frac{1}{2}$, $r^-_{44} = \frac{1}{4}$, and other components r^-_{ij} $(i \neq j)$ are identically zero.

Suppose that a smooth function ψ belongs to $\mathrm{Ker}(D_gS)^*$. Equivalently, it satisfies ${\nabla_g}^- d\psi - \psi r^-_g = 0$. Now one computes the J-anti-invariant part $\nabla^- d\psi$ of the Hessian of ψ.

For convenience we denote $\frac{\partial \psi}{\partial x}$ by ψ_x and $\frac{\partial^2 \psi}{\partial x \partial y}$ by ψ_{yx}, etc.. The Riemannian connection ∇ can be computed by the formula

$$2\langle \nabla_X Y, Z\rangle = X\langle Y, Z\rangle + Y\langle X, Z\rangle - Z\langle X, Y\rangle - \langle X, [Y,Z]\rangle - \langle Y, [X,Z]\rangle + \langle Z, [X,Y]\rangle;$$

$$\text{For } i = 1,2,3,4, \quad \nabla_{e_i} e_i = 0, \quad \nabla_{e_4} e_i = \nabla_{e_i} e_4 = 0.$$

$$\nabla_{e_1} e_2 = -\nabla_{e_2} e_1 = \frac{1}{2} e_3, \qquad \nabla_{e_1} e_3 = \nabla_{e_3} e_1 = -\frac{1}{2} e_2, \qquad \nabla_{e_2} e_3 = \nabla_{e_3} e_2 = \frac{1}{2} e_1.$$

We set $\nabla d\psi_{ij} = \nabla d\psi(e_i, e_j)$. From $\nabla d\psi(X,Y) = X(Y\psi) - (\nabla_X Y)\psi$;

$$\begin{aligned}
&\nabla d\psi_{11} = \psi_{xx}, && \nabla d\psi_{22} = \psi_{yy} + 2x\psi_{yz} + x^2\psi_{zz}, && \nabla d\psi_{33} = \psi_{zz},\\
&\nabla d\psi_{44} = \psi_{tt}, && \nabla d\psi_{12} = \psi_{xy} + x\psi_{xz} + \frac{1}{2}\psi_z, && \nabla d\psi_{13} = \psi_{xz} + \frac{1}{2}\psi_y + \frac{1}{2}x\psi_z,\\
&\nabla d\psi_{14} = \psi_{xt}, && \nabla d\psi_{23} = \psi_{yz} + x\psi_{zz} - \frac{1}{2}\psi_x, && \nabla d\psi_{24} = \psi_{yt} + x\psi_{zt},\\
&\nabla d\psi_{34} = \psi_{zt}.
\end{aligned}$$

From $\nabla^- d\psi(X,Y) = \frac{1}{2}\{\nabla d\psi(X,Y) - \nabla d\psi(JX, JY)\}$;

$$2\nabla^- d\psi_{11} = \psi_{xx} - \psi_{tt}, \qquad 2\nabla^- d\psi_{22} = \psi_{yy} + 2x\psi_{yz} + (x^2-1)\psi_{zz},$$
$$2\nabla^- d\psi_{12} = \psi_{xy} + x\psi_{xz} + \tfrac{1}{2}\psi_z + \psi_{zt}, \quad 2\nabla d^- \psi_{13} = \psi_{xz} + \frac{1}{2}\psi_y + \frac{1}{2}x\psi_z - \psi_{yt} - x\psi_{zt},$$
$$2\nabla^- d\psi_{23} = 2(\psi_{yz} + x\psi_{zz} - \tfrac{1}{2}\psi_x), \qquad 2\nabla^- d\psi_{14} = 2\psi_{xt}.$$

We deduce that the equation $\nabla_g{}^- d\psi - \psi r_g^- = 0$ is equivalent to the following system of six differential equations.

$$\langle 1\rangle\ \psi_{xx} - \psi_{tt} = -\tfrac{1}{2}\psi, \qquad \langle 2\rangle\ \psi_{yy} + 2x\psi_{yz} + (x^2-1)\psi_{zz} = -\psi,$$
$$\langle 3\rangle\ \psi_{xy} + x\psi_{xz} + \tfrac{1}{2}\psi_z + \psi_{zt} = 0, \qquad \langle 4\rangle\ \psi_{xz} + \frac{1}{2}\psi_y + \frac{1}{2}x\psi_z - \psi_{yt} - x\psi_{zt} = 0,$$
$$\langle 5\rangle\ \psi_{yz} + x\psi_{zz} - \tfrac{1}{2}\psi_x = 0, \qquad \langle 6\rangle\ \psi_{xt} = 0.$$

Now the surjectivity of D_gS follows by showing that ψ should be necessarily zero. The computation is elementary and it goes as follows.

From the equation $\langle 6\rangle$, $\psi(x,y,z,t)$ can be written as $a(x,y,z) + b(y,z,t)$. From $\langle 1\rangle$, $a_{xx} - b_{tt} = -\frac{1}{2}(a+b)$. So, $a_{xx} + \frac{1}{2}a = b_{tt} - \frac{1}{2}b$. LHS is a function of x,y,z whereas RHS is a function of y,z,t. So, both sides are functions of y and z only. By differentiating RHS, we have $b_{ttt} - \frac{1}{2}b_t = 0$. So $b_t = b_1(y,z)e^{\frac{1}{\sqrt{2}}t} + b_2(y,z)e^{-\frac{1}{\sqrt{2}}t}$. And $b = \sqrt{2}b_1(y,z)e^{\frac{1}{\sqrt{2}}t} - \sqrt{2}b_2(y,z)e^{-\frac{1}{\sqrt{2}}t} + b_3(y,z)$. $\psi(x,y,z,t) = a(x,y,z) + b(y,z,t)$ should be invariant under the repeated action of $\gamma_4(x,y,z,t) = (x,y,z,t+1)$. It forces $b_1 = b_2 = 0$ and $b = b(y,z)$. So, $\psi_t = b_t = 0$. Taking $\frac{\partial}{\partial x}$ to $\langle 3\rangle$, we get $\psi_{xxy} + x\psi_{xxz} + \frac{3}{2}\psi_{xz} = 0$. As $\psi_{xx} = -\frac{1}{2}\psi$ from $\langle 1\rangle$, we have $-\frac{1}{2}\psi_y - \frac{1}{2}x\psi_z + \frac{3}{2}\psi_{xz} = 0$. Comparing this with $\langle 4\rangle$, we get $\psi_y + x\psi_z = 0$. Take $\frac{\partial}{\partial z}$ to get $\psi_{yz} + x\psi_{zz} = 0$. With $\langle 5\rangle$, it follows that $\psi_x = 0$. From $\langle 1\rangle$, $\psi = 0$. So $\mathrm{Ker}(D_gS)^* = 0$.

Now the linearized map $DS_\omega|_g$ is surjective. The scalar curvature of $c^2g, c>0$ can be any negative constant and c^2g is an almost Kähler metric compatible with the symplectic structure $c^2\omega$. Clearly $DS_{c^2\omega}|_{c^2g}$ is also surjective. So from Proposition 1 we get the 'if' part of Theorem 1.1. According to [7, Theorem A], as M is an *enlargeable* manifold, it does not admit any Riemannian metric with nonnegative and not identically zero scalar curvature. It is well known that M does not admit a zero-scalar-curved metric. Indeed, if it does, the metric should be ricci-flat and then should be Kähler by Hitchin's theorem [12]. This proves Theorem 1.1.

Remark 1. We only computed for one 4-dimensional metric g on the Kodaira-Thurston manifold, but one can get similar results to Theorem 1.1 in higher dimensions. For example, one may try the product metric $g + g_T$ where g_T is the flat Kähler metric on the $2k$-dimensional torus, $k \geq 1$.

Remark 2. One may produce some more examples of similar kind in 4 dimension belonging to the class (c'). For instance, we have available some explicit almost Kähler metrics on solvmanifolds.

Remark 3. We hope Theorem 1.1 gives a motivation to pursue for the problem of characterizing the scalar curvature functions of almost-Kähler metrics for general symplectic manifolds.

REFERENCES

1. E. Abbena: An example of an almost Kahler manifold which is not Kahlerian. *Boll. Un. Mat. Ital.* A **3** (1984), no. 6, 383-392.
2. T. Aubin: *Some nonlinear problems in Riemannian geometry.* Springer Monographs in Mathematics, Springer-Verlag, 1998.
3. A.L. Besse: *Einstein manifolds.* Ergebnisse der Mathematik, 3 Folge, Band 10, Springer-Verlag, 1987.
4. D.E. Blair: On the set of metrics associated to a symplectic or contact form, *Bull. Institute of Math. Academia Sinica.* **11** (1983), 297-308.
5. R.E. Gompf & A.I. Stipsicz: *4-Manifolds and Kirby Calculus.* Graduate Studies in Math. v.20, American Math. Soc., 1999
6. M. Gromov: Pseudoholomorphic curves in symplectic manifolds. *Invent. Math.* **82** (1985), 307-347.
7. M. Gromov & H.B. Lawson: Spin and scalar curvature in the presence of a fundamental group. I. *Ann. of Math.* **111** (1980), no. 2, 209-230.
8. J.L. Kazdan & F.W. Warner: Existence and conformal deformation of metrics with prescribed gaussian and scalar curvatures. *Ann. of Math.* **101** (1975), no. 2, 317-331.
9. J. Kim & C. Sung: Deformation of almost Kähler metrics with constant scalar curvature on compact Kähler manifolds. *Ann. Global. Anal. and Geom.* **22** (2002), 49-73.
10. D. McDuff & D. Salamon: *Introduction to Symplectic Topology.* Oxford University Press, New York 1998.
11. R. Schoen: Conformal deformation of a Riemannian metric to constant scalar curvature. *J. Diff. Geom.* **20** (1984), 479-495.
12. N. Hitchin: On compact four-dimensional Einstein manifolds. *J. Diff. Geom.* **9** (1974), 435-442.
13. W. Thurston: Some simple examples of symplectic manifolds. *Proc. Amer. Math. Soc.* **55** (1976), 467-468.

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