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On ϕ-pseudo Symmetries of $(LCS)_n$-Manifolds

Shyamal Kumar Hui
Nikhil Banga Sikshan Mahavidyalaya, Bishnupur, Bankura – 722 122, West Bengal, India
e-mail : shyamal_hui@yahoo.co.in

Abstract. The present paper deals with a study of ϕ-pseudo symmetric and ϕ-pseudo Ricci symmetric $(LCS)_n$-manifolds. It is shown that every ϕ-pseudo symmetric $(LCS)_n$-manifold and ϕ-pseudo Ricci symmetric $(LCS)_n$-manifold are η-Einstein manifold.

1. Introduction

In 2003 Shaikh [24] introduced the notion of Lorentzian concircular structure manifolds (briefly, $(LCS)_n$-manifolds), with an example, which generalizes the notion of LP-Sasakian manifolds introduced by Matsumoto [17] and also by Mihai and Rosca [18]. Then Shaikh and Baishya ([26], [27]) investigated the applications of $(LCS)_n$-manifolds to the general theory of relativity and cosmology. The $(LCS)_n$-manifolds is also studied by Atceken [2], Hui and Atceken [16], Shaikh [25], Shaikh and Binh [31], Shaikh and Hui [32], Sreenivasa, Venkatesha and Bagewadi [33] and others.

The study of Riemann symmetric manifolds began with the work of Cartan [3]. A Riemannian manifold (M^n, g) is said to be locally symmetric due to Cartan [3] if its curvature tensor R satisfies the relation $\nabla R = 0$, where ∇ denotes the operator of covariant differentiation with respect to the metric tensor g.

During the last five decades the notion of locally symmetric manifolds has been weakened by many authors in several ways to a different extent such as recurrent manifold by Walker [39], semi symmetric manifold by Szabó [34], pseudo symmetric manifold in the sense of Deszcz [14], pseudo symmetric manifold in the sense of Chaki [4].

A non-flat Riemannian manifold (M^n, g) $(n > 2)$ is said to be pseudo symmetric in the sense of Chaki [4] if it satisfies the relation

$$(1.1)\quad (\nabla_W R)(X,Y,Z,U) = 2A(W)R(X,Y,Z,U) + A(X)R(W,Y,Z,U) + A(Y)R(X,W,Z,U) + A(Z)R(X,Y,W,U) + A(U)R(X,Y,Z,W),$$

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i.e.,

$$(1.2)\quad (\nabla_W R)(X,Y)Z = 2A(W)R(X,Y)Z + A(X)R(W,Y)Z + A(Y)R(X,W)Z + A(Z)R(X,Y)W + g(R(X,Y)Z,W)\rho$$

for any vector fields X, Y, Z, U and W on M, where R is the Riemannian curvature tensor of the manifold M, A is a non-zero 1-form such that $g(X,\rho) = A(X)$ for every vector field X. Such an n-dimensional manifold is denoted by $(PS)_n$.

Every recurrent manifold is pseudo symmetric in the sense of Chaki [4] but not conversely. Also the pseudo symmetry in the sense of Chaki is not equivalent to that in the sense of Deszcz [14]. However, pseudo symmetry by Chaki will be the pseudo symmetry by Deszcz if and only if the non-zero 1-form associated with $(PS)_n$, is closed. Pseudo symmetric manifolds in the sense of Chaki have been studied by Chaki and Chaki [6], Chaki and De [7], De [9], De and Biswas [10], De and Guha [11], De, Murathan and Özgür [13], Özen and Altay ([21], [22]), Tarafder ([36], [37]), Tarafder and De [38] and others.

A Riemannian manifold M is said to be Ricci symmetric if its Ricci tensor S of type (0,2) satisfies $\nabla S = 0$, where ∇ denotes the Riemannian connection on M. During the last five decades, the notion of Ricci symmetry has been weakened by many authors in several ways to a different extent such as Ricci recurrent manifold [23], Ricci semi symmetric manifold [34], pseudo Ricci symmetric manifold by Deszcz [15], pseudo Ricci symmetric manifold by Chaki [5].

A non-flat Riemannian manifold (M^n, g) is said to be pseudo Ricci symmetric [5] if its Ricci tensor S of type (0,2) is not identically zero and satisfies the condition

$$(1.3)\qquad (\nabla_X S)(Y,Z) = 2A(X)S(Y,Z) + A(Y)S(X,Z) + A(Z)S(Y,X)$$

for any vector fields X, Y, Z on M, where A is a nowhere vanishing 1-form and ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. Such an n-dimensional manifold is denoted by $(PRS)_n$. The pseudo Ricci symmetric manifolds have been also studied by Arslan et. al [1], Chaki and Saha [8], De and Mazumder [12], De, Murathan and Özgür [13], Özen [20] and many others.

The relation (1.3) can be written as

$$(1.4)\qquad (\nabla_X Q)(Y) = 2A(X)Q(Y) + A(Y)Q(X) + S(Y,X)\rho,$$

where ρ is the vector field associated to the 1-form A such that $A(X) = g(X,\rho)$ and Q is the Ricci operator, i.e., $g(QX,Y) = S(X,Y)$ for all X, Y on M.

As a weaker version of local symmetry, the notion of locally ϕ-symmetric Sasakian manifolds was introduced by Takahashi [35]. In the context of Lorentzian geometry, Shaikh and Baishya [28] introduced and studied the notion of local ϕ-symmetry on an LP-Sasakian manifold with several examples. Again Shaikh et. al. ([29], [30]) studied locally ϕ-symmetric and locally ϕ-recurrent $(LCS)_n$-manifolds.

The object of the paper is to study ϕ-pseudo symmetric and ϕ-pseudo Ricci symmetric $(LCS)_n$-manifolds. The paper is organized as follows. Section 2 is concerned with some preliminaries. Section 3 deals with a study of ϕ-pseudo symmetric $(LCS)_n$-manifold. It is shown that every ϕ-pseudo symmetric $(LCS)_n$-manifold is η-Einstein. In section 4, we have studied ϕ-pseudo Ricci symmetric $(LCS)_n$-manifolds and it is proved that every ϕ-pseudo Ricci symmetric $(LCS)_n$-manifold is also an η-Einstein manifold.

2. Preliminaries

An n-dimensional Lorentzian manifold M is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric g, that is, M admits a smooth symmetric tensor field g of type (0,2) such that for each point $p \in M$, the tensor $g_p : T_pM \times T_pM \to \mathbb{R}$ is a non-degenerate inner product of signature $(-, +, \cdots, +)$, where T_pM denotes the tangent vector space of M at p and $\mathbb{R}$ is the real number space. A non-zero vector $v \in T_pM$ is said to be timelike (resp., non-spacelike, null, spacelike) if it satisfies $g_p(v, v) < 0$ (resp, ≤ 0, $= 0$, > 0) [19].

Definition 2.1([24]). In a Lorentzian manifold (M, g) a vector field P defined by

$$g(X, P) = A(X),$$

for any X on M, is said to be a concircular vector field if

$$(\nabla_X A)(Y) = \alpha\{g(X, Y) + \omega(X)A(Y)\},$$

where α is a non-zero scalar and ω is a closed 1-form and ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g.

Let M be an n-dimensional Lorentzian manifold admitting a unit timelike concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

$$g(\xi, \xi) = -1. \tag{2.1}$$

Since ξ is a unit concircular vector field, it follows that there exists a non-zero 1-form η such that for

$$g(X, \xi) = \eta(X), \tag{2.2}$$

the equation of the following form holds

$$(\nabla_X \eta)(Y) = \alpha\{g(X, Y) + \eta(X)\eta(Y)\}, \quad \alpha \neq 0 \tag{2.3}$$

for any vector fields X, Y on M, where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g and α is a non-zero scalar function satisfies

$$\nabla_X \alpha = (X\alpha) = d\alpha(X) = \rho\eta(X), \tag{2.4}$$

ρ being a certain scalar function given by $\rho = -(\xi\alpha)$.

Let us take

$$\phi X = \frac{1}{\alpha}\nabla_X \xi, \tag{2.5}$$

then from (2.3) and (2.5) we have

$$\phi X = X + \eta(X)\xi, \tag{2.6}$$

from which it follows that ϕ is a symmetric (1,1) tensor and called the structure tensor of the manifold. Thus the Lorentzian manifold M together with the unit timelike concircular vector field ξ, its associated 1-form η and an (1,1) tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly, $(LCS)_n$-manifold) [24]. Especially, if we take $\alpha = 1$, then we can obtain the LP-Sasakian structure of Matsumoto [17]. In a $(LCS)_n$-manifold ($n > 2$), the following relations hold [24]:

$$\eta(\xi) = -1, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \quad g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.7}$$

$$\phi^2 X = X + \eta(X)\xi, \tag{2.8}$$

$$S(X,\xi) = (n-1)(\alpha^2 - \rho)\eta(X), \tag{2.9}$$

$$R(X,Y)\xi = (\alpha^2 - \rho)[\eta(Y)X - \eta(X)Y], \tag{2.10}$$

$$R(\xi,Y)Z = (\alpha^2 - \rho)[g(Y,Z)\xi - \eta(Z)Y], \tag{2.11}$$

$$(\nabla_X \phi)(Y) = \alpha\{g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\}, \tag{2.12}$$

$$(X\rho) = d\rho(X) = \beta\eta(X), \tag{2.13}$$

$$R(X,Y)Z = \phi R(X,Y)Z + (\alpha^2 - \rho)\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}\xi, \tag{2.14}$$

$$\begin{aligned}(\nabla_W R)(X,Y)Z &= (2\alpha\rho - \beta)\{\eta(Y)X - \eta(X)Y\}\eta(W) \\ &+ \alpha(\alpha^2 - \rho)\{g(Y,W)X - g(X,W)Y\} \\ &- \alpha R(X,Y)W,\end{aligned} \tag{2.15}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)(\alpha^2 - \rho)\eta(X)\eta(Y) \tag{2.16}$$

for any vector fields X, Y, Z on M and $\beta = -(\xi\rho)$ is a scalar function, where R is the curvature tensor and S is the Ricci tensor of the manifold.

Definition 2.2. A $(LCS)_n$-manifold M is said to be η-Einstein if its Ricci tensor S of type (0,2) is of the form

$$S = pg + q\eta \otimes \eta, \tag{2.17}$$

where p and q are smooth functions on M.

3. ϕ-pseudo symmetric $(LCS)_n$-manifolds

Definition 3.1. A $(LCS)_n$-manifold (M^n, g) is said to be *ϕ-pseudo symmetric* if the curvature tensor R satisfies

$$\begin{aligned} \phi^2((\nabla_W R)(X,Y)Z) &= 2A(W)R(X,Y)Z + A(X)R(W,Y)Z \\ &+ A(Y)R(X,W)Z + A(Z)R(X,Y)W \\ &+ g(R(X,Y)Z,W)\rho \end{aligned} \tag{3.1}$$

for any vector fields X, Y, Z and W on M, where A is a non-zero 1-form. In particular, if $A = 0$ then the manifold is said to be ϕ-symmetric [29].

We now consider a $(LCS)_n$-manifold (M^n, g), which is ϕ-pseudo symmetric. Then by virtue of (2.8), it follows from (3.1) that

$$\begin{aligned} &(\nabla_W R)(X,Y)Z + \eta((\nabla_W R)(X,Y)Z)\xi \\ =\ & 2A(W)R(X,Y)Z + A(X)R(W,Y)Z + A(Y)R(X,W)Z \\ +\ & A(Z)R(X,Y)W + g(R(X,Y)Z,W)\rho \end{aligned} \tag{3.2}$$

from which it follows that

$$\begin{aligned} &g((\nabla_W R)(X,Y)Z,U) + \eta((\nabla_W R)(X,Y)Z)\eta(U) \\ =\ & 2A(W)g(R(X,Y)Z,U) + A(X)g(R(W,Y)Z,U) + A(Y)g(R(X,W)Z,U) \\ +\ & A(Z)g(R(X,Y)W,U) + g(R(X,Y)Z,W)A(U). \end{aligned} \tag{3.3}$$

Taking an orthonormal frame field and then contracting (3.3) over X and U and then using (2.2), we get

$$\begin{aligned} &(\nabla_W S)(Y,Z) + g((\nabla_W R)(\xi,Y)Z,\xi) \\ =\ & 2A(W)S(Y,Z) + A(Y)S(W,Z) + A(Z)S(Y,W) \\ +\ & A(R(W,Y)Z) + A(R(W,Z)Y). \end{aligned} \tag{3.4}$$

Using (2.11), (2.15) and the relation $g((\nabla_W R)(X,Y)Z,U) = -g((\nabla_W R)(X,Y)U,Z)$, we have

$$g((\nabla_W R)(\xi,Y)Z,\xi) = -(2\alpha\rho - \beta)\{g(Y,Z) + \eta(Y)\eta(Z)\}\eta(W). \tag{3.5}$$

By virtue of (3.5) it follows from (3.4) that

$$(3.6)\quad \begin{aligned}(\nabla_W S)(Y,Z) &= 2A(W)S(Y,Z) + A(Y)S(W,Z) + A(Z)S(Y,W)\\ &+ A(R(W,Y)Z) + A(R(W,Z)Y)\\ &+ (2\alpha\rho - \beta)\{g(Y,Z) + \eta(Y)\eta(Z)\}\eta(W).\end{aligned}$$

This leads to the following:

Theorem 3.1 *A ϕ-pseudo symmetric $(LCS)_n$-manifold is pseudo Ricci symmetric if and only if*

$$\begin{aligned}&A(R(W,Y)Z) + A(R(W,Z)Y)\\ +\ &(2\alpha\rho - \beta)\{g(Y,Z) + \eta(Y)\eta(Z)\}\eta(W) = 0.\end{aligned}$$

Setting $Z = \xi$ in (3.2) and using (2.10) and (2.15), we get

$$(3.7)\quad \begin{aligned}&[\alpha + A(\xi)]R(X,Y)W\\ =\ &\alpha(\alpha^2 - \rho)\{g(Y,W)X - g(X,W)Y\}\\ +\ &(2\alpha\rho - \beta)\{\eta(Y)X - \eta(X)Y\}\eta(W)\\ +\ &(\alpha^2 - \rho)\big[2A(W)\{\eta(Y)X - \eta(X)Y\}\\ +\ &A(X)\{\eta(Y)W - \eta(W)Y\} + A(Y)\{\eta(W)X - \eta(X)W\}\\ +\ &\{\eta(Y)g(X,W) - \eta(X)g(Y,W)\}\rho\big].\end{aligned}$$

This leads to the following:

Theorem 3.2. *In a ϕ-pseudo symmetric $(LCS)_n$-manifold, the curvature tensor satisfies the relation* (3.7).

From (3.7), we get

$$(3.8)\quad \begin{aligned}[\alpha + A(\xi)]S(Y,W) &= [(n-1)\alpha - A(\xi)](\alpha^2 - \rho)g(Y,W)\\ &+ (n-1)(2\alpha\rho - \beta)\eta(Y)\eta(W)\\ &+ (\alpha^2 - \rho)\big[2nA(W)\eta(Y) + (n-2)A(Y)\eta(W)\big].\end{aligned}$$

Replacing Y by ϕY and W by ϕW in (3.8), we get

$$(3.9)\quad \{\alpha + A(\xi)\}S(\phi Y,\phi W) = [(n-1)\alpha - A(\xi)](\alpha^2 - \rho)g(\phi Y,\phi W).$$

By virtue of (2.7) and (2.16), we have from (3.9) that

$$(3.10)\quad S(Y,W) = ag(Y,W) + b\eta(Y)\eta(W),$$

where $a = \frac{[(n-1)\alpha - A(\xi)](\alpha^2-\rho)}{\alpha + A(\xi)}$ and $b = -\frac{n(\alpha^2-\rho)A(\xi)}{\alpha + A(\xi)}$, provided $\alpha + A(\xi) \neq 0$.

This leads to the following:

Theorem 3.3. *A ϕ-pseudo symmetric $(LCS)_n$-manifold is an η-Einstein manifold.*

Corollary 3.1([29]). *A ϕ-symmetric $(LCS)_n$-manifold is an Einstein manifold.*

4. ϕ-pseudo Ricci symmetric $(LCS)_n$-manifolds

Definition 4.1. A $(LCS)_n$-manifold (M^n, g) is said to be *ϕ-pseudo Ricci symmetric* if the Ricci operator Q satisfies

$$\phi^2((\nabla_X Q)(Y)) = 2A(X)QY + A(Y)QX + S(Y,X)\rho \tag{4.1}$$

for any vector fields X, Y on M, where A is a non-zero 1-form.
In particular, if $A = 0$, then (4.1) turns into the notion of ϕ-Ricci symmetric $(LCS)_n$-manifold.

Let us take a $(LCS)_n$-manifold (M^n, g), which is ϕ-pseudo Ricci symmetric. Then by virtue of (2.8) it follows from (4.1) that

$$(\nabla_X Q)(Y) + \eta((\nabla_X Q)(Y))\xi = 2A(X)QY + A(Y)QX + S(Y,X)\rho$$

from which it follows that

$$\begin{aligned} & g(\nabla_X Q(Y), Z) - S(\nabla_X Y, Z) + \eta((\nabla_X Q)(Y))\eta(Z) \\ = \; & 2A(X)S(Y,Z) + A(Y)S(X,Z) + S(Y,X)A(Z). \end{aligned} \tag{4.2}$$

Putting $Y = \xi$ in (4.2) and using (2.5) and (2.9), we get

$$\begin{aligned} & A(\xi)S(X,Z) + \alpha S(\phi X, Z) \\ = \; & (n-1)(\alpha^2 - \rho)\big[\alpha g(\phi X, Z) - 2A(X)\eta(Z) - A(Z)\eta(X)\big]. \end{aligned} \tag{4.3}$$

Using (2.6) in (4.3), we obtain

$$\begin{aligned} [\alpha + A(\xi)]S(X,Z) \; = \; & (n-1)\alpha(\alpha^2 - \rho)g(X,Z) \\ - \; & (n-1)(\alpha^2 - \rho)[2A(X)\eta(Z) + A(Z)\eta(X)]. \end{aligned} \tag{4.4}$$

Replacing X by ϕX and Z by ϕZ in (4.4), we get

$$[\alpha + A(\xi)]S(\phi X, \phi Z) = (n-1)\alpha(\alpha^2 - \rho)g(\phi X, \phi Z). \tag{4.5}$$

In view of (2.7) and (2.16), (4.5) yields

$$S(X,Z) = cg(X,Z) + d\eta(X)\eta(Z), \tag{4.6}$$

where $c = \frac{(n-1)\alpha(\alpha^2-\rho)}{\alpha+A(\xi)}$ and $d = \frac{(n-1)(\alpha^2-\rho)A(\xi)}{\alpha+A(\xi)}$, provided $\alpha + A(\xi) \neq 0$, which implies that the manifold under consideration is η-Einstein.

Thus we can state the following:

Theorem 4.1. *Every ϕ-pseudo Ricci symmetric $(LCS)_n$-manifold is an η-Einstein manifold.*

Corollary 4.1. *A ϕ-Ricci symmetric $(LCS)_n$-manifold is an Einstein manifold.*

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