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The Effects of Different Cross Section on Natural Frequency of the Advanced Composite Materials Road Structures

복합신소재 도로구조물의 변환단면이 고유진동수에 미치는 영향

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ABSTRACT

PURPOSES : This paper aims to give a guideline and the way to apply the advanced composite materials theory to the road structures with different cross sections to the practicing engineers.

METHODS : To simple but exact method of calculating natural frequencies corresponding to the modes of vibration of road structures with different cross sections and arbitrary boundary conditions. The effect of the D_{22} stiffness on the natural frequency is rigorously investigated.

RESULTS : Simple method of vibration analysis for calculating the natural frequency of the different cross sections is presented.

CONCLUSIONS : Simple method of vibration analysis for calculating the natural frequency of the different cross sections is presented. This method is a simple but exact method of calculating natural frequencies of the road structures with different cross sections. This method is extended to be applied to two dimensional problems including composite laminated road structures.

Keywords
advanced composite materials, natural frequencies, simple method of vibration analysis, different cross section

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1. INTRODUCTION

The advanced composite materials can be used economically and efficiently in broad civil engineering applications when standards and processes for analysis, design, fabrication, construction and quality control are established. The problem of deteriorating infrastructures is very serious in our country.

The advanced composite materials can be effectively used for repairing such structures. Because of the advantages of these materials, such repair job can fulfill two goals :

(1) Repairing existing damage caused by corrosion, impact, earthquake, and others.

(2) Reinforcing the structure against anticipated future situation which will require the increase of the load beyond the design parameters used for this structure.

Before making any decision on repair work, reliable non-destructive evaluation is necessary. One of the dependable methods is to evaluate the in-situ stiffness of the structure by means of obtaining the natural frequency. By comparing the in-situ stiffness with the one obtained at the design stage, the degree of damage can be estimated rather accurately.

The reinforced concrete slab can be assumed as a [0, 90, 0]r type specially orthotropic plate as a close approximation, assuming that the influence of B_{16}, B_{26}, D_{16} and D_{26} stiffness are negligible. Many of the bridge and building floor systems, including the girders and cross beams, also behave as similar specially orthotropic plates. Such plates are subject to the concentrated mass/masses in the form of traffic loads, or the test equipments such as the accelerator in addition to their own masses. Analysis of such problems is usually very difficult.

Most design engineers for construction have bachelors degree level of academic background. Theories for advanced composite structures are too difficult for such engineers, so simple but still accurate methods are needed.

Most of the civil structures are high in size and the numbers of laminae are high, even though the thickness to length ratios are small enough to neglect the transverse shear deformation effects in stress analysis. For such plates, the fiber orientations given above work as specially orthotropic plates and simple formulas developed by the reference [Kim 1995, Han & Kim, 2001, 2003, 2009] can be used.

Most of the bridge and building slabs on girders have high aspect ratios. For such cases, further simplification is possible by neglecting the effect of the longitudinal moment terms (M_x) on the relevant partial differential equations of equilibrium [Han & Kim, 2001]. This paper presents the result of the study on the subject problem. Even with such assumption, the specially orthotropic plate with boundary conditions other than Navier or Levy solution types, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical methods for eigenvalue problems are also very much involved for the solution [Ashton, Pagano, Whitney, 1970, Timoshenko, 1989].

2. METHOD OF ANALYSIS

2.1. Finite Diffence Method

The equilibrium equation for the specially orthotropic plate is

$$D_1 \frac{\partial^4 w}{\partial x^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4} = q(x, y) \qquad (1)$$

where $D_1 = D_{11},\ D_2 = D_{22},\ D_3 = D_{12} + 2D_{66}$

The assumptions needed for this equation are :

(1) The transverse shear deformation is neglected.

(2) Specially orthotropic layers are arranged so that no coupling terms exist, i.e. $B_{ij} = 0,\ (\ )_{16} = (\ )_{26} = 0$

(3) No temperature or hygrothermal terms exist.

The purpose of this paper is to demonstrate, to the practicing engineers, how to apply this equation to the slab systems made of plate girders and cross beams.

In the case of an orthotropic plate with boundary conditions other than Navier or Levy solution type, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical methods for eigenvalue problems are also very much involved for the solution. As one solution, finite difference method is used in this paper. The resulting linear algebraic equations can be used for any cases with minor modifications at the boundaries, and so on.

The problem of deteriorating infrastructures is very serious all over the world. Before making any decision on repair work, reliable non-destructive evaluation is necessary. One of the dependable methods is to evaluate the in-situ stiffness of the structure by means of obtaining the natural frequency. By comparing the in-situ stiffness with the one obtained at the design stage, the degree of damage can be estimated rather accurately.

The basic concept of the Rayleigh method - the most popular analytical method for vibration analysis of a single degree of freedom system - is the principle of conservation of energy : the energy in a free vibrating system must remain constant if no damping forces act to absorb it. In case of a beam which has an infinite number of degree of freedom, it is necessary to assume a shape function in order to reduce the beam to a single degree of freedom system [Clough 1995]. The frequency of vibration can be found by equating the maximum strain energy developed during the motion to the maximum kinetic energy. This method, however, yields the solution either equal to or higher than the real one. Recall Rayleigh's quotient ≥ 1 [Kim, 1995]. For a complex beam, assuming a correct shape function is not possible. In such cases, the solution obtained is higher than the real one.

2.2. Simple Method of Vibration Analysis

Structural engineers need to calculate the natural frequencies of such element, but obtaining exact solution to such problems is very much difficult. Pretlove reported a method of analysis of beams with attached masses using the concept of effective mass. This method, however, is useful only for certain simple types of beams. I argue that the simple method of vibration analysis for calculating the natural frequency could be a possible solution. It is a simple but exact method of calculating the natural frequency corresponding to the first mode of vibration of beam and tower structures with different cross sections and attached mass/masses. This method consists of determining the deflected mode shape of the member due to the inertia force under resonance condition. Beginning with initially "guessed" mode shape, "exact" mode shape is obtained by the process similar to iteration. Recently, this method was extended to two dimensional problems including composite laminates, and has been applied to composite plates with various boundary conditions with/without shear deformation effects. This method is used for vibration analysis in this paper.

A natural frequency of a structure is the frequency under which the deflected mode shape corresponding to this frequency begins to diverge under the resonance condition. From the deflection caused by free vibration, the force required to make this deflection can be found, and from this force, resulting deflection can be obtained. If the mode shape as determined by the series of this process is sufficiently accurate, then the relative deflections (maximum) of both the converged and the previous one should remain unchanged under the inertia force related with this natural frequency. Vibration of a structure is a harmonic motion and the amplitude may contain a part expressed by a trigonometric function. Considering only the first mode as a start, the deflection shape of a structural member can be expressed as

$$w = W(x,y)\,F(t) = W(x,y)\sin\omega t \tag{2}$$

where

W : maximum amplitude

ω : circular frequency of vibration

t : time

By Newton's second law, the dynamic force of the vibrating mass, m, is

$$F = m\frac{\partial^2 w}{\partial t^2} \tag{3}$$

Substituting (2) into this,

$$F = -m(\omega)^2 W(x,y)\sin\omega t \tag{4}$$

In this expression, ω and W are unknowns. In order to obtain the natural circular frequency ω, the following process is taken.

The magnitudes of the maximum deflection at a certain number of points are arbitrarily given as

$$w(i,j)^{(1)} = W(i,j)^{(1)} \tag{5}$$

where (i,j) denotes the point under consideration. This is absolutely arbitrary but educated guess is good for accelerating convergence. The dynamic force corresponding to this (maximum) amplitude is

$$F(i,j)^{(1)} = m(i,j)[\omega(i,j)^{(1)}]^2\, w(i,j)^{(1)} \tag{6}$$

The "new" deflection caused by this force is a function of and can be expressed as

$$\begin{aligned} w(i,j)^{(2)} &= f[m(k,l)[\omega(i,j)^{(1)}]^2\, w(k,l)^{(1)}] \\ &= \sum^{k,l} \Delta(i,j,k,l)m(k,l)[\omega(i,j)^{(1)}]^2\, w(k,l)^{(1)} \end{aligned} \tag{7}$$

where Δ is the deflection influence surface. The relative (maximum) deflections at each point under consideration of a structural member under resonance condition, $w(i,j)^{(1)}$ and $w(i,j)^{(2)}$, have to remain unchanged and the following condition has to be held :

$$w(i,j)^{(1)} / w(i,j)^{(2)} = 1 \tag{8}$$

From this equation, $w(i,j)^{(1)}$ at each point of (i,j) can be obtained, but they are not equal in most cases. Since the natural frequency of a structural member has to be equal at all points of the member, i.e., $w(i,j)$ should be equal for all (i,j), this step $w(i,j)$ is repeated until sufficient equal magnitude is obtained at all (i,j) points.

However, in most cases, the difference between the maximum and the minimum values of $w(i,j)$ obtained by the first cycle of calculation is sufficiently negligible for engineering purposes. The accuracy can be improved by simply taking the average of the maximum and the minimum, or by taking the value of $w(i,j)^{(2)}$ where the deflection is the maximum. For the second cycle, $w(i,j)^{(3)}$ in

$$w(i,j)^{(3)}=f\,[m(i,j)[\omega(i,j)^{(2)}]^2\, w(i,j)^{(1)}] \tag{9}$$

the absolute numerics of $w(i,j)^{(2)}$ can be used for convenience. In case of a structural member with irregular section including composite one, and non-uniformly distributed load, regardless of the boundary conditions, it is convenient to consider the member as divided by finite number of elements. The accuracy of the result is proportional to the accuracy of the deflection calculation.

For practical design purposes, it is desirable to simplify the vibration analysis procedure. One of the methods is to neglect the weight of the structural element. The effect of neglecting the weight (thus mass) of the plate is studied as follows. If a weightless plate is acted upon by a concentrated load, $P=N\cdot q\cdot a\cdot b$ the critical circular frequency of this plate is

$$w_n=\sqrt{\frac{g}{\delta_{st}}} \tag{10}$$

where δ_{st} is the static deflection.

Similar result can be obtained by the use of Eqs. (7) and (8).

$$[w(i,j)]^2=\frac{1}{[\Delta(i,j,i,j)\cdot\dfrac{P(i,j)}{g}]} \tag{11}$$

where,

$$p(i,j)=N\cdot q\cdot a\cdot b \tag{12}$$

In case of the plate with more than one concentrated loads,

$$[w(i,j)]^2=\frac{1}{[\sum^{k,l}\Delta(i,j,k,l)\cdot\dfrac{P(k,l)}{g}]} \tag{13}$$

If we consider the mass of the plate as well as the concentrated loads,

$$\begin{aligned} w(i,j)^{(1)} = w(i,j)^{(2)} = \{&\sum^{k,l}\Delta(i,j,k,l)\cdot m(k,l)\cdot w(k,l)^{(1)} \\ &+\sum^{m,n}\Delta(i,j,m,n)\cdot\frac{P(m,n)}{g}\cdot w(m,n)^{(1)}\} \\ &\times[[\omega(i,j)^{(1)}]^2] \end{aligned} \tag{14}$$

where (m,n) is the location of the concentrated loads. The effect of neglecting the weight of the plate can be found by simply comparing Eqs. (13) and (14).

Since no reliable analytical method is available for the subject problem, F.D.M. is applied to the governing equation of the special orthotropic plates.

The number of the pivotal points required in the case of the order of error Δ^2 where Δ is the mesh size, is five for the central differences of the fourth order single derivative terms. This makes the procedure at the boundaries complicated. In order to solve such a problem, the three simultaneous partial differential equations of equilibrium with three dependent variables, w, M_x, and M_y, are used instead of Eq.(1) for the bending of the specially orthotropic plate.

$$\begin{aligned} &D_{11}\frac{\partial^2 Mx}{\partial x^2}+4D_{66}\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^2 My}{\partial y^2} \\ &=-q(x,y)+kw(x,y) \end{aligned} \tag{15}$$

$$M_x=-D_{11}\frac{\partial^2 w}{\partial x^2}-D_{12}\frac{\partial^2 w}{\partial y^2} \tag{16}$$

$$M_y=-D_{12}\frac{\partial^2 w}{\partial x^2}-D_{22}\frac{\partial^2 w}{\partial y^2} \tag{17}$$

If F.D.M. is applied to these equations, the resulting matrix equation is very high in sizes, but the tridiagonal matrix calculation scheme used by Kim [Kim, 1965, 1967] is very efficient to solve such equations.

In order to confirm the accuracy of the Simple method of vibration analysis, [*A* /*B* /*A*]r type laminate with aspect ratio of *a*/*b*=1m/1m=1 is considered. The material properties are :

$$E_1=67.36GPa,\ E_2=8.12GPa,$$
$$G_{12}=3.0217GPa,$$
$$\nu_{12}=0.272,\ \nu_{21}=0.0328$$

The thickness of a ply is 0.005m. As the r increases, B_{16}, B_{26}, D_{16} and D_{26} decrease and the equations for special orthotropic plates can be used. For simplicity, it is assumed that , A =0°, B =90° and r =1. Then D_{22} =18492 N-m.

Since one of the few efficient analytical solutions of the special orthotropic plate is Navier solution, and this is good for the case of the four edges simple supported, Simple method of vibration analysis is used to solve this problem and the result is compared with the Navier solution.

The mesh size is $\hat{x}=a/10$=0.1m, $\hat{y}=b/10$=0.1m. The deflection at (x, y), under the uniform load of 100N/m^2, the origin of the coordinates being at the corner of the plate, is obtained, and the ratio of the Navier solution to the Simple method of vibration analysis solution is 1.005~1.00028.

3. NUMERICAL EXAMPLES

3.1. Simple Method of Vibration Analysis

As a calculations is the simple method of vibration analysis, a simply supported beam with uniform flexural rigidity, EI, is considered as shown in Fig. 1.

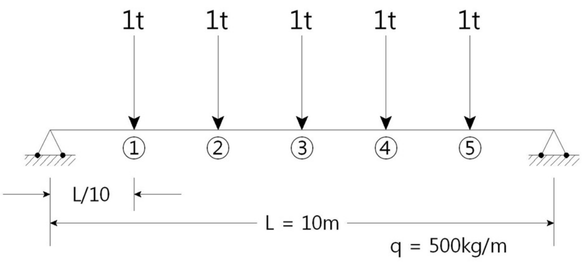

Fig.1 Simply Supported Beam with Uniform Flexural Rigidity

The length of the beam is 10m. The weight of the beam is assumed as 500kg/m. The weight acts as the mass when the beam vibrates and is treated as concentrated loads at five equally spaced points. Since a beam is one-dimensional, one subscript, i, is used.

The set of influence coefficients, $\Delta_{i,j}$, where i is the point under consideration and j is the loading point (unit load), is given in Table 1.

Table 1. Influence Coefficients, $EI\Delta_{i,j}$, for Cantilever Beam

i \ j	1	2	3	4	5
1	0.33	1.33	2.33	3.33	4.33
2	1.33	9.00	18.00	27.00	36.00
3	2.33	18.00	41.33	66.67	91.67
4	3.33	27.00	66.67	114.33	163.33
5	4.33	36.00	91.67	163.33	243.00

The initially guessed maximum amplitude, $W(i)^{(1)}$, can be arbitrary and the following values are given.

$$W(1)^{(1)}=W(5)^{(1)}=40$$
$$W(2)^{(1)}=W(4)^{(1)}=80$$
$$W(3)^{(1)}=100$$

These values are substituted into equations 4.77 and 4.78, and from equation 4.79, the following result is obtained:

$$w(1)^{(2)}=1616\text{m}(1)[\omega(1)^{(1)}]^2/EI$$
$$w(2)^{(2)}=4222\text{m}(2)[\omega(2)^{(1)}]^2/EI$$
$$w(3)^{(2)}=5216\text{m}(3)[\omega(3)^{(1)}]^2/EI$$

Letting , $w(i)^{(1)}/w(i)^{(2)}=1$, we get

$$w(1)^{(1)}=0.1573A(1)$$
$$w(2)^{(1)}=0.1376A(1)$$
$$w(3)^{(1)}=0.1385A(1)$$

where

$$A(i)=\sqrt{\frac{EI}{m(i)}}$$

Since all $w(i)^{(1)}s$ should be equal at all i points, this process has to be repeated. For the second cycle, only the relative magnitude of the amplitude is necessary, and $w(i)^{(2)}s$ are assigned as follows:

$$W(1)^{(2)}=W(5)^{(2)}=16.2$$
$$W(2)^{(2)}=W(4)^{(2)}=42.2$$
$$W(3)^{(2)}=52.2$$

The same influence coefficient for the first cycle is repeatedly used, and the “new” amplitude, $w(i)^{(3)}$, is obtained as

$$w(1)^{(3)}=827.76[\omega(1)^{(2)}]^2/[A(1)]^2$$
$$w(2)^{(3)}=2167.08[\omega(2)^{(2)}]^2/[A(2)]^2$$
$$w(3)^{(3)}=2678.64[\omega(3)^{(2)}]^2/[A(3)]^2$$

From , $w(i)^{(2)}/w(i)^{(3)}=1$,

$$w(1)^{(3)}=0.1397A(1)$$
$$w(2)^{(3)}=0.1396A(2)$$
$$w(3)^{(3)}=0.1395A(3)$$

One more process is executed in order to obtain better result as follows.

$$w(1)^{(3)}=0.139575A(1)$$
$$w(2)^{(3)}=0.139575A(2)$$
$$w(3)^{(3)}=0.139575A(3)$$

Note that all A are the same, and

$$w=0.139575A$$

The result obtained by the "exact" theory is $w=0.139575A$.

It is noted that the result of the first cycle is good enough for engineering purposes. If w at the point of the maximum deflection, $w(3)^{(1)}$, is considered, it is only 0.77% away from the "exact" result.

In the case of a variable cross section, including materials, $A(i)=E(i)(I_i)/m(i)$ should be used. Influence coefficients can be found with relative ease in any case.

Simple method of vibration analysis can be applied to any structural element with variable stiffnesses and loadings. And with any boundary conditions, including deep beams and thick plates for which an analytical solution is difficult to obtain. The accuracy of the result is proportional to that of deflection calculation. Calculation of the deflection influence surface is the fundamental first step in any structural analysis and design. Attention should be given to the fact that this method utilizes the deflection influence surfaces which are used at the beginning of the analysis and design.

3.2. The Effects of Different Cross Section on Natural Frequencies

Simple method of vibration analysis is used to study the effects of different cross section and moment of inertia on natural frequencies of simply supported beams, fixed beams.

For simply beams with different cross section as shown in Fig. 2.

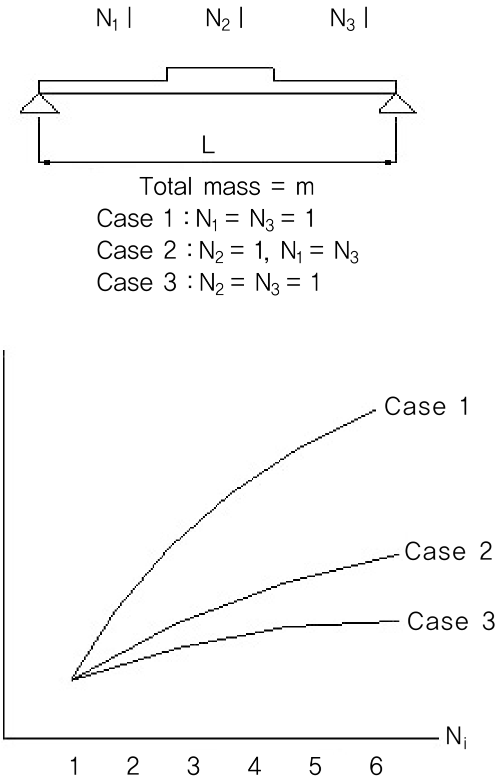

Fig. 2 Uniformly Loaded Simply Beams with Different Cross Section

For a fixed beam with different cross section as shown in Fig 3.

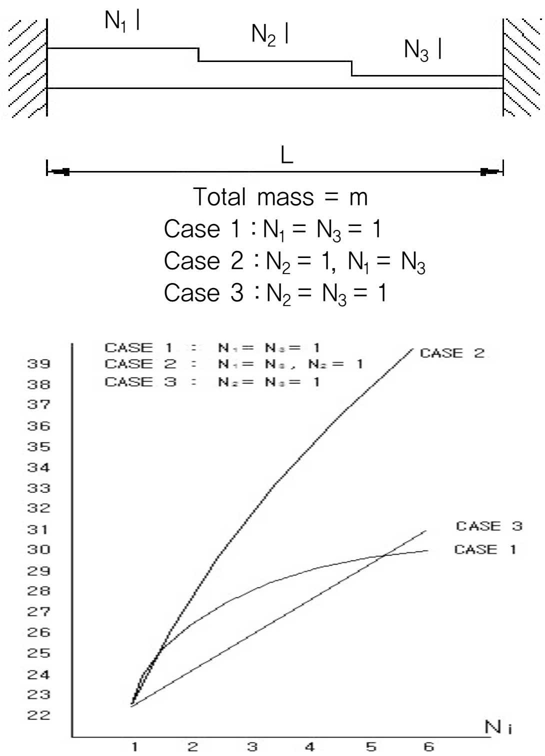

Fig. 3 Uniformly Loaded Fixed Beams with Different Cross Section

In this method, approximation of different cross section and the number of points under consideration do "not" affect much the vibration characteristics of beams. It is the influence coefficients which generate most differences. When "correct" influence coefficients are used, the results of six segmented (five pointed) beams and that of four segmented (three pointed) beams have exactly the same values to the accuracy of the computer truncating errors.

4. CONCLUSION

A natural frequency of a structure is the frequency under which the deflected mode shape corresponding to this frequency begins to diverge under the resonance condition. From the deflection caused by the free vibration, the force required to make this deflection can be found and from this force, the resulting deflection can be obtained. For practical design purposes, it is desirable to simplify the vibration analysis procedure. One of the methods is to neglect the weight of the beam.

Many practicing engineers are confused by such relations. This paper aims to give a guideline to apply the advanced composite materials theory to the road structures to the practicing engineers.

In this paper, the relation between the different cross section and the natural frequency of vibration of some structural elements is presented. This method utilizes the deflection influence surfaces which are used at the beginning of the analysis and design. This method is a simple but exact method of calculating natural frequencies of the road structures with different cross sections. This method can be extended to be applied to two dimensional problems including composite laminated road structures.

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