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The Effects of the Aspect Ratio on Natural Frequency of the Advanced Composite Materials Road Structures

복합신소재 도로구조물의 형상비가 고유진동수에 미치는 영향

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ABSTRACT

PURPOSES : Current theories for composite structures are too difficult for design engineers for construction. The purpose of this paper is to demonstrate to the practicing engineers, how to apply the advanced composite materials theory to the road structures.

METHODS : Some laminate orientations have decreasing values of D_{16} , B_{16} , D_{26} and B_{26} stiffnesses as the ply number increases. The plate aspect ratio considered is from 1 to 5. In order to study the effect of M_x on the equilibrium equations, two cases are considered. M_x term is considered or neglected.

RESULTS : Most of the road structures have high aspect ratios, for such cases further simplification is possible by neglecting the effect of the longitudinal moment terms.

CONCLUSIONS : Most of the road structures have plate aspect ratios higher than 2. It is concluded that, for all boundary conditions, neglecting the longitudinal moment(M_x) terms is acceptable if the aspect ratio (a/b) is equal to or higher than 2. This conclusion gives good guide line for design of the road structures.

Keywords
advanced composite materials, aspect ratio, natural frequencies, simple method of vibration analysis

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1. INTRODUCTION

The future of material industry will depend on when the conventional construction materials are replaced by advanced composite materials. If composite materials are used for construction, the quantity is huge : in tons, not in kilos or pounds. composite materials can be used economically and efficiently in broad applications in civil engineering when standards and processes for analysis, design, fabrication, construction and quality control are established. The problem of deteriorating infrastructures is very serious all over the world. The U.S. Civil Engineering Research Foundation (CERF) report, “High-Performance Construction Material and System : An Essential Program for America and its Infrastructure” - published in collaboration with several organizations such as U.S. Department of Transportation - figures as follows :

(1) The road bridge condition in U.S.A int the year 2009 : 149,654 of the Americans 603,259 bridges are structurally deficient or obsolete. (71,177 were

structurally deficient and 78,477 were functionally obsolete)

(2) 199,584 of these bridges were more than 50 years old and unsuitable for current or projected traffic.

(3) Traffic delays alone will cost $115 billion per year for lost work time and fuel by the year 2009.

Steel girders become rusty. The reinforcing bars embedded in concrete beams or slabs are subject to corrosion caused by electro-chemical action. Underground fuel tanks are under similar condition. In 1979, the U.S. Bureau of Standards (NIST) study showed that yearly loss caused by corrosion related damages mounted to 82 billion dollars which is about 4.9% of GNP. About 32 billion dollars could be saved if existing technologies were used to prevent such losses.

These figures show the condition of the United States of America where various federal, state, and other agencies are doing their best in maintaining such structures in good condition. The issue of deteriorating and damaged infrastructures and lifelines has become a critically important subject in the United States as well as Japan and Europe. The problem in developing nations, where degree of construction quality control and maintenance are in question, must be much more profound [Kim 1995, Han & Kim 2004].

The advanced composite materials can be effectively used for repairing such structures. Before making any decision on repair work, reliable non-destructive evaluation is necessary. One of the dependable methods is to evaluate the in-situ stiffness of the structure by means of obtaining the natural frequency. By comparing the in-situ stiffness with the one obtained at the design stage, the degree of damage can be estimated rather accurately.

The reinforced concrete slab can be assumed as a $[90^0, 0^0, 90^0]_r$ type specially orthotropic plate as a close approximation, assuming that the effect of B_{16}, B_{26}, D_{16} and D_{26} stiffness are negligible. Many of the bridge and building floor systems, including the girders and cross-beams, also behave as similar specially orthotropic plates. Such plates are subject to the concentrated mass/masses in the form of traffic loads, or the test equipments such as the accelerator in addition to their own masses. Analysis of such problems is usually very difficult.

Most of the design engineers for construction have bachelors degree level of academic background. Theories for advanced composite structures are too difficult for such engineers, so simple but still accurate methods is needed.

The author has reported that some laminate orientations such as $[\alpha, \beta]_r$, $[\alpha, \beta, \gamma]_r$, $[\alpha, \beta, \beta, \alpha, \alpha, \beta]_r$ and $[\alpha, \beta, \beta, \gamma, \alpha, \alpha, \beta]_r$ with , $\alpha = -\beta$ and $\gamma = 0^0$ or 90^0 and with increasing r, have decreasing values of B_{16}, B_{26}, D_{16} and D_{26} stiffness. Most of the civil and architectural structures are high in size and the numbers of laminae are big, even though the thickness to length ratios are small enough to neglect the transverse shear deformation effects in stress analysis. For such plates, the fiber orientations given above work as specially orthotropic plates and simple formulas developed by the reference [Han & Kim 2001, 2004] can be used. Most of the bridge and building slabs on girders have high aspect ratios. For such cases, further simplification is possible by neglecting the effect of the longitudinal moment terms (M_x) on the relevant partial differential equations of equilibrium [Han & Kim 2001]. This paper presents the result of the study on the subject problem.

Even with such assumption, the specially orthotropic plate with boundary conditions other than Navier or Levy solution types, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain.

The simple method of vibration analysis is the one developed by the author. This method is a simple but exact method of calculating the natural frequency of beam and tower structures with irregular cross-sections and attached mass/masses.

This method has been extended to be applied in two-dimensional problems with several types of given conditions and has been reported at several international conferences. This method uses the deflection influence surfaces. The finite difference method is used for this purpose in this paper.

2. METHOD OF ANALYSIS

The equilibrium equation for the specially orthotropic plate is:

$$D_1 \frac{\partial^4 w}{\partial x^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4} = q(x, y) \qquad (1)$$

where $D_1 = D_{11},\ D_2 = D_{22},\ D_3 = D_{12} + 2D_{66}$

The assumptions needed for this equation are :

(1) The transverse shear deformation is neglected.

(2) Specially orthotropic layers are arranged so that no coupling terms exist, i.e.,. $B_{ij} = 0,\ (\ )_{16} = (\ )_{26} = 0$

(3) No temperature or hydrothermal terms exist.

The purpose of this paper is to demonstrate, to the practicing engineers, how to apply this equation to the slab systems made of plate girders and cross-beams.

In the case of an orthotropic plate with boundary conditions other than Navier or Levy solution type or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical methods for eigenvalue problems are also very much involved for the solution [Ashton, Pagano, Whitney 1970, Timoshenko 1989]. As one solution, finite difference method (F.D.M) is used in this paper. The resulting linear algebraic equations can be used for any cases with minor modifications at the boundaries, and so on.

The problem of deteriorating infrastructures is very serious all over the world. Before making any decision on repair work, reliable non-destructive evaluation is necessary. One of the dependable methods is to evaluate the in-situ stiffness of the structure by means of obtaining the natural frequency. By comparing the in- situ stiffness with the one obtained at the design stage, the degree of damage can be estimated rather accurately.

The basic concept of the Rayleigh method - the most popular analytical method for vibration analysis of a single degree of freedom system - is the principle of conservation of energy : the energy in a free vibrating system must remain constant if no damping forces act to absorb it. In case of a beam which has an infinite number of degree of freedom, it is necessary to assume a shape function in order to reduce the beam to a single degree of freedom system. The frequency of vibration can be found by equating the maximum strain energy developed during the motion to the maximum kinetic energy. This method, however, yields the solution either equal to or higher than the real one. Recall Rayleigh's quotient ≥ 1. For a complex beam, assuming a correct shape function is not possible. In such cases, the solution obtained is higher than the real one.

Design engineers need to calculate the natural frequencies of such element, but obtaining exact solution to such problems is very much difficult. Pretlove reported a method of analyzing beams with attached masses, using the concept of effective mass. This method, however, is useful only for certain simple types of beams. Such problems can easily be solved by the method presented.

It is a simple but exact method of calculating the natural frequency corresponding to the first mode of vibration of beam and tower structures with irregular cross sections and attached mass/masses. This method consists of determining the deflected mode shape of the member due to the inertia force under resonance condition. Beginning with initially “guessed” mode shape, “exact” mode shape is obtained by the process similar to iteration. Recently, this method was extended to be applied to two dimensional problems including composite laminates, and has been applied to composite plates with various boundary conditions with/without shear deformation effects and reported.

This method is used for vibration analysis in this paper. A natural frequency of a structure is the frequency under which the deflected mode shape corresponding to this frequency begins to diverge under the resonance condition. From the deflection caused by the free vibration, the force required to make this deflection can be found, and from this force, resulting deflection can be obtained. If the mode shape as determined by the series of this process is sufficiently accurate, then the relative deflections (maximum) of both the converged and the previous one should remain unchanged under the inertia force related with this natural frequency. Vibration of a structure is a harmonic motion and the amplitude may contain a part expressed by a trigonometric function.

2.1. Simple Method of Vibration Analysis

In this paper, the simple method of vibration analysis is given in as follows.

The magnitudes of the maximum deflection at a certain number of points are arbitrarily given as

$$w(i, j)^{(1)} = W(i, j)^{(1)} \tag{2}$$

where $j)[\omega$ denotes the point under consideration. This is absolutely arbitrary but educated guess is good for accelerating convergence. The dynamic force corresponding to this (maximum) amplitude is

$$F(i,j)^{(1)} = m(i,j)[\omega(i,j)^{(1)}]^2 w(i,j)^{(1)} \tag{3}$$

The "new" deflection caused by this force is a function of f and can be expressed as

$$\begin{aligned} w(i,j)^{(2)} &= f\{m(i,j)[\omega(i,j)^{(1)}]^{(2)} w(i,j)^{(1)}\} \\ &= \sum^{k,l} \Delta(i,j,k,l)\{m(i,j)[\omega(i,j)^{(1)}]^2 w(k,l)^{(1)}\} \end{aligned} \tag{4}$$

where Δ is the deflection influence surface. The relative (maximum) deflections at each point under consideration of a structural member under resonance condition, $w(i,j)^{(1)}$ and $w(i,j)^{(2)}$, have to remain unchanged and the following condition has to be held :

$$w(i,j)^{(1)} / w(i,j)^{(2)} = 1 \tag{5}$$

From this equation, $w(i,j)^{(1)}$ at each point of (i,j) can be obtained. But they are not equal in most cases. Since the natural frequency of a structural member has to be equal at all points of the member, i.e. $w(i,j)$ should be equal for all , (i,j) this step is repeated until sufficient equal magnitude of $w(i,j)$ is obtained at all (i,j) points.

However, in most cases, the difference between the maximum and the minimum values of $w(i,j)$ obtained by the first cycle of calculation is sufficiently negligible for engineering purposes. The accuracy can be improved by simply taking the average of the maximum and the minimum, or by taking the value of $w(i,j)$ where the deflection is the maximum. For the second cycle, $w(i,j)^{(3)}$ is

$$w(i,j)^{(3)} = f\{m(i,j)[\omega(i,j)^{(2)}]^2 w(i,j)^{(2)}\} \tag{6}$$

the absolute numerics of $w(i,j)^{(2)}$ can be used for convenience.

2.2. Finite Difference Method

The method used in this paper requires the deflection influence surfaces. F.D.M is applied to the governing equation of the specially orthotropic plates,

$$\begin{aligned} &D_1\frac{\partial^4 w}{\partial x^4} + 2D_3\frac{\partial^4 w}{\partial x^2\partial y^2} + D_2\frac{\partial^4 w}{\partial y^4} \\ &= q(x,y) - kw + Nx\frac{\partial^2 w}{\partial x^2} + Ny\frac{\partial^2 w}{\partial y^2} + 2Nxy\frac{\partial^2 w}{\partial x\partial y} \end{aligned} \tag{7}$$

where, $D_1 = D_{11}$, $D_2 = D_{22}$, $D_3 = (D_{12} + 2D_{66})$

The number of the pivotal points required in the case of the order of error Δ^2, where Δ is the mesh size, is five for the central differences,

This makes the procedure at the boundaries complicated. In order to solve such a problem, the three simultaneous partial differential equations of equilibrium with three dependent variables, w, M_x and M_y, are used instead of Eq.(7) with $N_x = N_y = N_{xy} = 0$ [Han & Kim 2001].

$$D_1\frac{\partial^2 Mx}{\partial x^2} + 4D_{66}\frac{\partial^4 w}{\partial x^2\partial y^2} + \frac{\partial^2 My}{\partial y^4} = -q(x,y) + kw(x,y) \tag{8}$$

$$M_x = -D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} \tag{9}$$

$$M_y = -D_{12}\frac{\partial^2 w}{\partial x^2} - D_{22}\frac{\partial^2 w}{\partial y^2} \tag{10}$$

If F.D.M. is applied to these equations, the resulting matrix equation is very high in sizes, but the tridiagonal matrix calculation scheme is very efficient to solve such equations.

In order to confirm the accuracy of the F.D.M., $[A/B/A]r$ type laminate with aspect ratio of a/b=1m/1m=1 is considered.

For simplicity, it is assumed that A=0°, B=90° and r=1. Since one of the few efficient analytical solutions of the specially orthotropic plate is Navier solution, and this is good for the case of the four edges simple supported, F.D.M. is used to solve this problem and the result is compared with the Navier solution.

Calculation is carried out with different mesh sizes.

The error is less than 1%. This is smaller than the predicted theoretical errors :

If F.D.M is applied to these equations, the resulting matrix equation is very high in sizes, but the tridiagonal matrix calculation scheme used by Kim & Han is very efficient to solve such equations [Han & Kim 2001, 2004, 2010].

Since one of the few efficient analytical solutions of the specially orthotropic plate is Navier solution, and this is good for the case of the four edges simple supported, F.D.M is used to solve this problem. The result is satisfactory as expected.

By neglecting the Mx terms, the sizes of the matrices needed to solve the resulting linear equations are reduced to two thirds of the "non-modified" equations.

3. NUMERICAL EXAMINATION

3.1. Specially Orthotropic Plate

The structure under consideration is as shown in Fig.1.

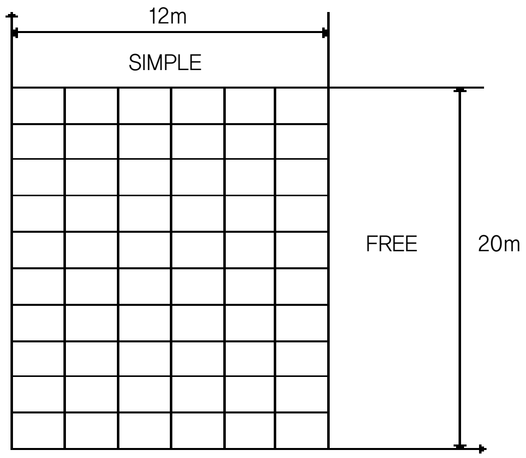

Fig. 1 Structure under Consideration

E_s = 200,000 MPa, ν = 0.3

The stiffnesses are given in Table 1. Type 1 is for the specially orthotropic plate and Type 2 is the case of a simple beam. In order to a study the effect of the cross-beam sizes, variable values of D_{22} are given, in Table 2.

Table 1. Stiffnesses

$D_{ij}(N \cdot m)$	Type 1	Type 2
D_{11} D_{22}	101,199,927 21,757,838	101,199,928 0.00

Table 2. Stiffnesses with Variable D_{22}

$D_{ij}(N \cdot m)$	Case 1	Case 2	Case 3
D_{11} D_{22}	101,199,928 21,757,838	101,199,928 41,618,360	101,199,928 61,478,883

Analysis is carried out and the result is given by Tables from 3 to 6. As Table 3 shows, the deflection of Type 2, based on beam theory, is 2.43 times that of specially orthotropic theory. Increase of the cross-beam sizes does not produce profound change of deflection, Table 4. Similar conclusion can be obtained from the frequency, Table 5 and 6. The specially orthotropic plate theory yields much stiffer structure than by beam theory.

Table 3. Deflection at the Center(m) Loading 100kN at the Center

Type	1	2	Type 2 / Type 1
$\delta(m)$	0.6765E-01	0.1646E+00	2.43

Table 4. Deflection at the Center(m) Loading : 100kN at the Center

Case	1	2	3	4	5
$\delta(m)$	0.6765 E-01	0.6262 E-01	0.6061 E-01	0.5951 E-01	0.5881 E-01
Case i/ Case 1	1.0	1.0803	1.1162	1.1368	1.1503

Table 5. Natural Freqeuncy(rad/sec). Loading : 100kN at the Center

Type	1	2	Type 2 / Type 1
$w(rad/\sec)$	0.7313E+01	0.5133E+01	0.7019

Table 6. Natural Freqeuncy(rad/sec). Loading : 100kN at the Center

Case	1	2	3	4	5
$w(rad/\sec)$	0.7313 E+01	0.7471 E+01	0.7539 E+01	0.7577 E+01	0.7603 E+01
Case i/ Case 1	1.0	1.0216	1.0309	1.0361	1.0397

3.2. Effects of the Aspect Ratio

The plate are considered as [*a*/*b*/*a*]r composite laminated plate. The material properties are : E_1=67.36GPa, E_2=8.12GPa, ν_{12}=0.272, ν_{21}=0.0328, r=1.

In order to study the effect of Mx on the equilibrium equations, two cases are considered :

Case A : w, M_x and M_y are considered.

Case B : w and M_y are considered, M_x is neglected.

F.D.M. is used to obtain w, M_x, M_y and the natural frequency.

Plates with all edges simple supported (*SS*), the aspect ratio and the natural frequencies at the center of the uniformly loaded plate are as shown in Table 7.

Table 7. Effect of Aspect Ratio(*SS* case)

Aspect Ratio (a/b)	1	2	3	4	5
ω_A/ω_B	1.0960	1.0303	1.0189	1.0137	1.0107

Plates with one side simple and the other side free supported (*SF*), the aspect ratio and the natural frequencies at the center of the uniformly loaded plate are as shown in Table 8.

Table 8. Effect of Aspect Ratio(*SF* case)

Aspect Ratio (a/b)	1	2	3	4	5
ω_A/ω_B	0.9957	0.9977	0.9985	0.9988	0.9991

It is concluded that, for all boundary conditions, neglecting Mx terms is acceptable if the aspect ratio (*a*/*b*) is equal to or higher than 2.

4. CONCLUSION

Theories for composite structures are too difficult for such design engineers for construction, therefore simple but accurate enough methods are necessary. The simply supported laminated plates are analyzed by the specially orthotropic laminates theory. In this paper, the effects of the aspect ratio on the advanced composite materials road structures are studied. It is concluded that the method used is sufficiently accurate for engineering purposes.

The result of numerical examination is quite promising. When plates with all edges are simple supported (*SS*), the ratios of the natural frequencies (ω_A/ω_B) at the center of the uniformly loaded plate range from 1.0960 to 1.0107 according to its aspect ratio (*a*/*b*) from 1 to 5. In case Plates with one side simple and the other side free supported (*SF*), the ratios of the natural frequencies (ω_A/ω_B) at the center of the uniformly loaded plate range from 0.9957 to 0.9991 according to its aspect ratio (*a*/*b*) from 1 to 5.

Most of the road structures have plate aspect ratios higher than 2. It is concluded that, for all boundary conditions, neglecting the longitudinal moment (M_x) terms is acceptable if the aspect ratio (*a*/*b*) is equal to or higher than 2. This conclusion gives good guideline for design of the road structures.

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