# Joint Beamforming and Power Allocation for Multiple Primary Users and Secondary Users in Cognitive MIMO Systems via Game Theory

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## Abstract

We consider a system where a licensed radio spectrum is shared by multiple primary users (PUs) and secondary users (SUs). As the spectrum of interest is licensed to primary network, power and channel allocation must be carried out within the cognitive radio network so that no excessive interference is caused to PUs. For this system, we study the joint beamforming and power allocation problem via game theory in this paper. The problem is formulated as a non-cooperative beamforming and power allocation game, subject to the interference constraints of PUs as well as the peak transmission power constraints of SUs. We design a joint beamforming and power allocation algorithm for maximizing the total throughput of SUs, which is implemented by alternating iteration of minimum mean square error based decision feedback beamforming and a best response based iterative power allocation algorithm. Simulation results show that the algorithm has better performance than an existing algorithm and can converge to a locally optimal sum utility.

***Keywords*:** Cognitive radio, MIMO, beamforming, power allocation, game theory.

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## 1. Introduction

It is well-known that the exclusive use of the licensed radio spectrum is highly inefficient. As a novel approach to enhancing the efficiency of utilizing the scarce radio spectrum, cognitive radio (CR) [1] has attracted tremendous interests recently. On the other hand, multiple-input multiple-output (MIMO) technique, with its significantly increased channel capacity, has become a dominating technique in the future-generation wireless systems [2]. It is thus quite natural to combine these two techniques together to achieve overall spectral efficiency. This technological combination results in the so-called cognitive MIMO radio [3].

MIMO CR networks were recently studied in [4]-[8]. A semi-distributed algorithm was proposed in [4] to obtain a locally optimal solution to the SU beamforming problem. On the other hand, under the ideal assumption that the PUs can act as a scheduler for SUs transmissions, an opportunistic orthogonalization scheme was proposed in [5]. Assuming that the SU has full CSI and there is no interference from the PU to the SU, the authors studied the optimal secondary transmit spatial spectrum which can achieve the capacity of the secondary transmission for a single SU and provided better intuition in [6]. When the secondary transmitter has complete, partial, or no knowledge about the channels to the primary receivers, [7] studied the optimal secondary-link beamforming pattern that balances between the SU's throughput and the interference. Owing to the fact that the cognitive MIMO interference channel (IC) consists of multiple cognitive point-to-point MIMO links, [8] pointed out that the optimization of cognitive MIMO IC can be separated into several iterative optimizations of each secondary point-to-point link.

As an effective interference suppression technique, joint beamforming and power allocation had been widely used in communication systems with multi-antenna [9]-[11]. Different from the traditional communication systems, in CR network, the interference constraints of PUs is the first-line issue that SUs should consider, which calls for new algorithms. A robust power control scheme via link gain pricing with $H_\infty$ estimator for cognitive spectrum underlay network was proposed in [12]. The scheme guaranteed that the interference temperature of the PUs through operating in the network-centric manner, and kept the fairness between the SUs through link gain pricing. In [13], joint power control and beamforming in the downlink of the cognitive radio network was considered and two iterative algorithms were formulated considering two different scenarios. However, it only considered a single primary user. In [14], a joint beamforming and power allocation algorithm for total throughput maximization in CR network was proposed. We denote it by ZF-CML. The ZF-CML algorithm has the following two limitations: First, ZF-CML requires the number of active SUs does not exceed the number of antennae at the cognitive base station, which may bring deterioration in total throughput and fairness of CR network; second, the interference from PUs to SUs and the noise power were not considered. When the number of PUs is large or their transmission powers are high, the total throughput of SUs will fall rapidly.

As multiple non-cooperative SUs share the same frequency band licensed to PUs, game theory can be naturally applied to CR networks. The authors in [15] considered joint power and rate control using a game-theoretic approach, where the SUs were considered as active players in the game. The extension to the cognitive MIMO system was considered in [16, 17]. Therein, both theoretic analysis and algorithm were carefully investigated. However, the problem of joint beamforming and power allocation for cognitive MIMO systems is different from the traditional radio networks. To the best of our knowledge, a few studies have been performed to look at this problem in a cognitive MIMO radio environment via game theory. In [18], the authors considered the non-cooperative maximization of mutual information in the Gaussian interference channel in a fully distributed fashion via game theory. [19] formulated the design of the SU network as a non-cooperative game, where the SUs compete with each other over the resources made available by the PUs, by maximizing their own information rates subject to the transmit power and robust interference constraints.

Inspired by these considerations, in this paper, under a game-theoretic framework, we study the problem of joint beamforming and power allocation in a cognitive MIMO system wherein multiple primary users and secondary users are co-located. We design an algorithm based on the minimum mean square error based decision feedback (MMSE-DFE) beamforming algorithm and a best response based (BR) iterative power control algorithm [20]. The proposed algorithm can achieve better throughput performance considering the interference and noise power, and does not limit the number of active users. So it is more robust and practical. Since we consider the cognitive MIMO downlink, we focus on the game of the SUs. Specifically, we consider a strategic non-cooperative game, in which the Nash Equilibrium (NE) is considered as the solution of this game, and the pricing function is the cost of the spectrum. Even though the main objective of this non-cooperative game formulation is to maximize the profit of all SUs, based on the equilibrium adopted by all SUs, the revenue of the PUs can be maximized as well. In our earlier works [21, 22], the beamforming problem in cognitive MIMO systems was studied from the game-theoretic perspective. However, their problem formulations and algorithms were different from the ones in this paper.

The rest of the paper is organized as follows. In Section 2 we introduce the cognitive MIMO system model and formulate the throughput optimization problem under the interference constraints of PUs as well as the peak transmission power constraints of SUs. In Section 3, the optimization problem is formulated as a non-cooperative beamforming and power allocation game. We choose a proper utility function with pricing to characterize the data transmission for all SUs. In Section 4 we present the joint beamforming and power allocation algorithm. Numerical simulation results are given in Section 5. Finally, Section 6 concludes the paper.

The following notations are used in this paper. The capital boldface is used to denote matrices, and the lowercase in boldface denotes vectors. $(\cdot)^H$ and $(\cdot)^T$ denote the conjugate transpose operation and transpose operation, respectively.

## 2. System Model and Problem Formulation

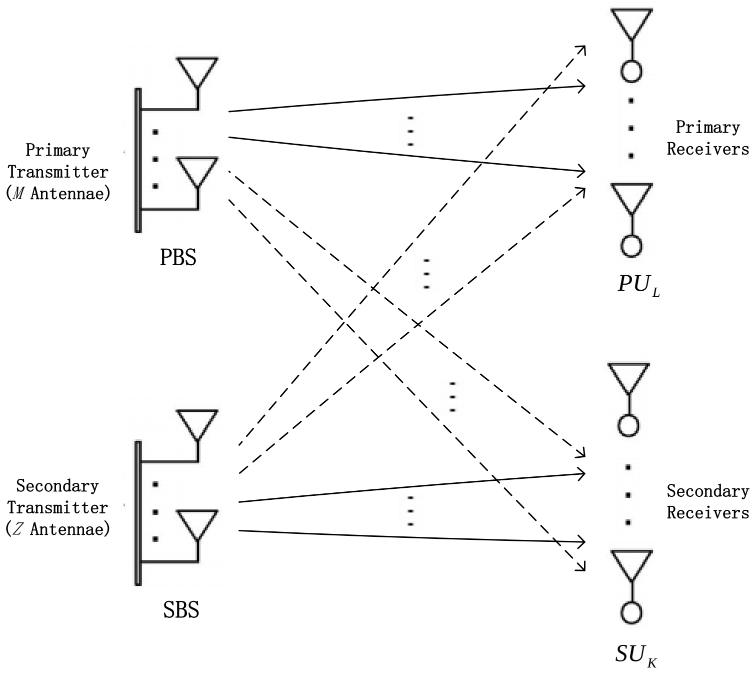


**Fig.1.** Cognitive MIMO system

We consider that the primary and secondary communication links share the same frequency band with bandwidth *B*, as shown in **Fig. 1**. All channels follow independent Rayleigh block fading. The channel coefficients of the primary and secondary links are independent and identically distributed complex Gaussian random variables with zero mean and unit variance. The primary network consists of a primary base station (PBS), equipped with *M* antennae, which transmits signals to *L* primary users ($PU_1,...,PU_L$). Each PU has a single antenna. The secondary network comprises a secondary multi-antenna transmitter (SBS) and *K* secondary single-antenna receivers ($SU_1,...,SU_K$). Due to the sharing of the same frequency band, the received signals at the PUs are interfered by the signals transmitted from the SBS. Similarly, the received signals at the SUs are interfered by the signals transmitted from the PBS.

The $Z \times 1$ received signal vector $\boldsymbol{y}$ can be represented as

$$\boldsymbol{y} = \sum_{k=1}^{K} \sqrt{p_k} \boldsymbol{h_k} x_k + \sum_{l=1}^{L} \sqrt{p_l} \boldsymbol{h_l} x_l + \boldsymbol{Q} \tag{1}$$

The $k$-th SU transmits signal $x_k$ with power $p_k$. The $l$-th PU transmitter sends signal $x_l$ with power $p_l$. $\boldsymbol{Q}$ is the Gaussian noise vector whose entries are independent Gaussian random variables with zero mean and unit variance.

For each SU, the output of the Z array elements at SBS is weighted and added by a beamformer. Let $\mathbf{w_k} = [w_k^{(1)}, ..., w_k^{(Z)}]^T, \forall k \in [1, K]$, be the Z-component complex weight vector for the $k$-th SU. Then, the signal-to-interference-plus-noise-ratio (SINR) of the $k$-th SU is calculated by

$$SINR_k = \frac{\left|\boldsymbol{w_k^H h_k}\right|^2 p_k}{\sum_{j=1\ j\neq k}^{K} \left|\boldsymbol{w_k^H h_j}\right|^2 p_j + \sum_{l=1}^{L} \left|\boldsymbol{w_k^H h_l}\right|^2 p_l + \left\|\boldsymbol{w_k}\right\|^2 \sigma_k^2} \tag{2}$$

where $\boldsymbol{h_k}$ and $\boldsymbol{h_l}$ denote the channel response vector from $k$-th SU and $l$-th PU to the SBS, $\sigma_k^2$ is the noise power. The beamforming weights are normalized such that $\left\|\mathbf{w_k}\right\|^2$ =1. A specified measurement point is set in the primary network to measure the interference caused by secondary network, where $g_k$ is the channel response from SU $k$ to the measurement point, $\boldsymbol{g}=\left[g_1, g_2, ...., g_k\right]^T$. To simplify the analysis, all the channel response vectors $\boldsymbol{h_k}$, $\boldsymbol{h_l}$ and $\boldsymbol{g}$ are assumed to be perfectly known at the SBS. The achievable rate of the $k$-th SU can be expressed as

$$R_k = \log_2(1 + SINR_k) \tag{3}$$

In order for a CR network to coexist with the PUs, the interference powers received by the $l$-th PU from the SBS should be below certain thresholds, which are usually dependent on the quality of service (QoS) of the $l$-th PU. It is therefore essential to control the transmission powers of the SUs. On the other hand, to ensure QoS of SUs, power allocation in a CR network should be appropriately determined to optimize the performance metrics of the SUs, which can be reflected through the parameters such as the sum-rate or SINRs.

Motivated by the considerations described above, we formulate the design of CR networks into an optimization problem. The problem is to maximize the sum-rate of the SUs subject to the individual peak transmission power constraint of each SU, as well as the interference power constraints of PUs. The interference power received by *l*-th PU from all SUs is characterized by $\boldsymbol{g}^T\boldsymbol{p}$, $\boldsymbol{p}=\left[p_1, p_2, ...., p_k\right]^T$. It can be formally stated as follows

$$\max_{p.w} \sum_{k=1}^{K} R_k$$

$$\text{subject to } \boldsymbol{g}^T\boldsymbol{p} \le I_{th} \tag{4}$$

$$0 \le p_k \le p_{k,\max}, \forall k \in [1, K]$$

The interference constraints of PUs is denoted by $I_{th}$, and the peak transmission power constraint of *k*-th SU is denoted by $p_{k,\max}$.

## 3. Non-cooperative Game

### 3.1 Game-theoretic Formulation

Game theory is an effective tool to analyze competitive optimization problem. Particularly, emergent potential games have a special property that the incentive of all users to change their strategies can be expressed in a global function. That is, users in a potential game can serve the greater good by furthering their own interests. These have inspired us to explore a game-theoretic approach for cognitive MIMO. Suppose that the secondary users in the cognitive MIMO are selfish and non-cooperative. Then each secondary user's transmission is a source of interference for the others. The strategies chosen by different SUs depend on each other. Based on the system model described above, a non-cooperative game can be formulated as follows [23]

$$Q = \left\{\Omega, \left\{\mathbf{w_k}, p_k\right\}_{k\in\Omega}, \left\{\mu_k\right\}_{k\in\Omega}\right\} \tag{5}$$

The players in this game are the SUs. The strategy of each player includes beamforming weights and transmit power (denoted by $\mathbf{w_k}$ and $p_k$ for the *k*-th SU, which is non-negative). The utility for each player is the profit (i.e., revenue minus cost, denoted by $\mu_k$ for the *k*-th SU) in sharing the spectrum with the PUs and the other SUs. Consequently, the utility function can be designed based on the achievable rate, i.e.

$$\mu_k = \log_2(1 + SINR_k) \tag{6}$$

Due to greediness, a payoff function based on (6) leads to an inefficient outcome, i.e., each player focuses on the forming of its own beam without nulling the interference to the PUs. To prevent this selfish circumstance, pricing has been used as an effective tool to give distributed players incentives to cooperate in resource usages. Therefore, the payoff function should consist of revenue and cost, the new utility function of the k-th $SU_K$ with pricing is rewritten as follows

$$\mu_k = \log_2(1+SINR_k) - \lambda p_k \sum_{l=1}^{L} \left| \boldsymbol{h_l} \mathbf{w}_{\boldsymbol{k}}^{\boldsymbol{H}} \right|^2 \tag{7}$$

where $\lambda$ is a positive constant and has an effect to reflect the potential interference to the PUs. The non-cooperative game is formulated as

$$\max \sum_{k=1}^{K} \mu_k$$

$$\text{subject to} \quad \boldsymbol{g}^{\boldsymbol{T}} \boldsymbol{p} \leq I_{th} \tag{8}$$

$$0 \leq p_k \leq p_{k,\max}, \forall k \in [1, K]$$

Here, each SU competes against the others by choosing its beamforming vector $\mathbf{w_k}$ and transmission power $p_k$ to maximize its own utility function.

### 3.2 Existence of Nash Equilibrium

To analyze the outcome of the game, the existence of a NE is a well-known optimality criterion. At the NE point, no user has any incentive to change its strategy with its own action. According to the fundamental game theory result [23], the existence conditions of Nash Equilibrium are given by

(i) The feasible set $B_k = \left\{ \mathbf{w}_{\boldsymbol{k}}^{\boldsymbol{H}}, p_k \right\}$ is a nonempty compact convex subset of a Euclidean space.

(ii) The utility function $\mu_k(\cdot)$ is continuous and quasi-concave on $B_k = \left\{ \mathbf{w}_{\boldsymbol{k}}^{\boldsymbol{H}}, p_k \right\}$.

By taking the first derivative of $\mu_k(\cdot)$ with respect to $p_k$ and $\left| \boldsymbol{w}_{\boldsymbol{k}}^{\boldsymbol{H}} \right|^{\boldsymbol{2}}$, respectively, we have

$$\frac{\partial \mu_k}{\partial p_k} = \frac{1}{\ln 2} \frac{\left|\mathbf{w}_k^H \boldsymbol{h}_k\right|^2}{\sum_{j=1, j\neq k}^{K} \left|\mathbf{w}_k^H \boldsymbol{h}_j\right|^2 p_j + \sum_{l=1}^{L} \left|\mathbf{w}_k^H p_l\right| + \sigma_k^2 + p_k \left|\mathbf{w}_k^H \boldsymbol{h}_k\right|^2} - \lambda \sum_{l=1}^{L} \left|\boldsymbol{h}_l \mathbf{w}_k^H\right|^2 \tag{9}$$

$$\frac{\partial \mu_k}{\partial \left|\mathbf{w}_k^H\right|^2} = \frac{1}{\ln 2} \frac{p_k \left|\boldsymbol{h}_k\right|^2}{\sum_{j=1, j\neq k}^{K} \left|\mathbf{w}_k^H \boldsymbol{h}_j\right|^2 p_j + \sum_{l=1}^{L} \left|\mathbf{w}_k^H p_l\right| + \sigma_k^2 + p_k \left|\mathbf{w}_k^H \boldsymbol{h}_k\right|^2} - \lambda p_k \sum_{l=1}^{L} \left|\boldsymbol{h}_l\right|^2 \tag{10}$$

Moreover, by finding the second derivative of $\mu_k(\cdot)$ with respect to $p_k$ and $\left|\mathbf{w}_k^H\right|^2$, respectively, we get

$$\frac{\partial {\mu_k}^2}{\partial^2 p_k} = -\frac{1}{\ln 2} \frac{\left|\mathbf{w}_k^H \boldsymbol{h}_k\right|^4}{\left[\sum_{j=1, j\neq k}^{K} \left|\mathbf{w}_k^H \boldsymbol{h}_j\right|^2 p_j + \sum_{l=1}^{L} \left|\mathbf{w}_k^H p_l\right| + \sigma_k^2 + p_k \left|\mathbf{w}_k^H \boldsymbol{h}_k\right|^2\right]^2} \tag{11}$$

$$\frac{\partial {\mu_k}^2}{\partial \left|\mathbf{w}_k^H\right|^2} = -\frac{1}{\ln 2} \frac{{p_k}^2 \left|\boldsymbol{h}_k\right|^4}{\left[\sum_{j=1, j\neq k}^{K} \left|\mathbf{w}_k^H \boldsymbol{h}_j\right|^2 p_j + \sum_{l=1}^{L} \left|\mathbf{w}_k^H p_l\right| + \sigma_k^2 + p_k \left|\mathbf{w}_k^H \boldsymbol{h}_k\right|^2\right]^2} \tag{12}$$

As $\left|\mathbf{w}_k^H \boldsymbol{h}_k\right|^4 \geq 0$ and ${p_k}^2 \left|\boldsymbol{h}_k\right|^4 \geq 0$, it is easy to check that $\frac{\partial {\mu_k}^2}{\partial^2 p_k} \leq 0$ and $\frac{\partial {\mu_k}^2}{\partial \left|\mathbf{w}_k^H\right|^2} \leq 0$. Consequently, the utility functions of $SU_K$ satisfy all the required conditions (i) (ii) for the existence of at least one NE.

## 4. Joint Beamforming and Power Allocation Algorithm

The alternating iteration is a low-complexity method for multi-variable optimization problem and used widely. We employ an iterative algorithm that repeats two sets of optimization variables: beamforming matrix **w** and transmission power vector ***p*** until convergence to solve the optimization problem [20].

During beamforming matrix optimization, the beamforming vector for each SU is identified for a given transmission power vector $\boldsymbol{p}^0$. The beamforming vector of different SUs can be equivalent to the following

$$\max_{w_k} SINR_k\left(\mathbf{w_k}, \boldsymbol{p}^0\right) \tag{13}$$

It is well-known that the optimal solutions to (4) can be achieved by the classical MMSE beamforming algorithm, so the optimal beamforming vector of $k$-th SU is

$$\hat{\mathbf{w}}_{\mathbf{k}} = \alpha \mathbf{\Gamma}_{\mathbf{k}} \boldsymbol{h}_k \tag{14}$$

where $\alpha$ is a constant used to make $\left\| \hat{\mathbf{w}}_{\mathbf{k}} \right\| = 1$, and $\mathbf{\Gamma}_{\mathbf{k}} = \sum_{j=k+1}^{K} p_j \boldsymbol{h}_j \boldsymbol{h}_j^{\boldsymbol{H}} + \sum_{l=1}^{L} p_l \boldsymbol{h}_l \boldsymbol{h}_l^{\boldsymbol{H}} + \sigma^2$.

Fortunately, due to the relationship $\mathbf{\Gamma}_{\mathbf{k-1}} = \mathbf{\Gamma}_{\mathbf{k}} + p_k \boldsymbol{h}_k \boldsymbol{h}_k^{\boldsymbol{H}}$, with the help of the famous Sherman-Morrison equation [24], we could solve the matrix inversion problem in (14) recursively as

$$\mathbf{\Gamma}_{\mathbf{k-1}}^{\mathbf{-1}} = (\mathbf{\Gamma}_{\mathbf{k}} + p_k \boldsymbol{h}_k \boldsymbol{h}_k^{\boldsymbol{H}})^{-1} = \mathbf{\Gamma}_{\mathbf{k}}^{\mathbf{-1}} - \frac{\mathbf{\Gamma}_{\mathbf{k}}^{\mathbf{-1}} p_k \boldsymbol{h}_k \boldsymbol{h}_k^{\boldsymbol{H}} \mathbf{\Gamma}_{\mathbf{k}}^{\mathbf{-1}}}{1 + p_k \boldsymbol{h}_k^{\boldsymbol{H}} \mathbf{\Gamma}_{\mathbf{k}}^{\mathbf{-1}} \boldsymbol{h}_k} \tag{15}$$

During the transmission power vector optimization, with the updated beamforming matrix $\hat{\mathbf{w}}$, the optimal $\hat{\boldsymbol{p}}$ is identified. We consider the optimization problem of $\boldsymbol{p}$. Obviously, the problem can be solved through the Lagrange algorithm. With the same apagoge as Theorem 1 in [14], we could prove that the optimal solution to the optimization of $\boldsymbol{p}$ must satisfy the following equivalent KKT conditions

$$p_k = \left[ \left( \sum_{c=k+1}^{K} \frac{G_{k,c}}{\sum_{j=1}^{c-1} p_j G_{j,c} + I_c} + \gamma \boldsymbol{g}_{\mathbf{k}} \right)^{-1} \right]_0^{p_{k,\max}}, \gamma \geq 0 \; \forall k \in [1,K], \tag{16}$$

$$\gamma \left( I_{th} - \sum_{k=1}^{K} p_k \boldsymbol{g}_{\mathbf{k}} \right) = 0 \tag{17}$$

$$\sum_{k=1}^{K} p_k \mathbf{g}_{\mathbf{k}} \leq I_{th} \tag{18}$$

where $G_{k,c} = \left| \hat{\mathbf{w}}_{\mathbf{c}} \boldsymbol{h}_k \right|^2$, $G_{j,c} = \left| \hat{\mathbf{w}}_{\mathbf{c}}^{\mathbf{H}} \boldsymbol{h}_j \right|^2$, $I_c = \sum_{l=1}^{L} \hat{p}_l \left| \hat{\mathbf{w}}_{\mathbf{c}}^{\mathbf{H}} \hat{\boldsymbol{h}}_l \right|^2 + \left\| \hat{\mathbf{w}}_{\mathbf{c}} \right\|^2 \sigma_c^2$, $\gamma$ is parameter. First, we fix $\gamma$ as any nonnegative value to solve equation (16), then we discuss how to identify the value of $\gamma$.

For any fixed $\gamma \geq 0$, the algorithm similar to sequential iterative water-filling [25] can be implemented to obtain the fixed point of equation (16). We assume the order of power adjustment is from SU 1 to SU *k*, then algorithm A helps us to solve equation (16) for any fixed $\gamma \geq 0$. The process of algorithm A could be regarded as a power game with a coordinate utility function. As the power adjustment strategy of each SU is the best response of the utility function, so the convergence and optimality of algorithm A can be guaranteed.

**Table 1.** Algorithm A

initialize: *n*=0, $p^{(0)} = \left[ p_1^{(0.)},..., p_k^{(0)},..., p_K^{(0)} \right], \gamma, \xi$

repeat: *n*=*n*+1

for k=1: K

initialize: *m*=0, $p_k^{(n),(0)}$, $\zeta$

repeat: *m*=*m*+1

$$p_k^{(n),(m)} = \left[ \left( \sum_{c=k+1}^{K} \frac{G_{k,c}}{p_k^{(n),(m-1)} G_{k,c} + \sum_{j=1}^{k-1} p_j^{(n)} G_{j,c} + \sum_{j \neq k+1}^{c-1} p_j^{(n-1)} G_{j,c} + I_c} + \gamma \boldsymbol{g_k} \right)^{-1} \right]_0^{p_{k,\max}}$$

until $\left| p_k^{(n),(m)} - p_k^{(n),(m-1)} \right| / p_k^{(n),(m-1)} < \zeta$

end

until $\left\| p_k^{(n)} - p_k^{(n-1)} \right\| / \left\| p_k^{(n-1)} \right\| < \xi$

In algorithm A, the iteration indexed by *n* is called the outer iteration, while the iteration indexed by *m* is called the inner iteration. In the *n*th outer iteration, when SU adjusts its transmission power, the power strategy vector of other SUs is denoted as $p_{-k}^{(n)} = [p_1^{(n)},..., p_{k-1}^{(n)}, p_{k+1}^{(n-1)},..., p_K^{(n-1)}]$. In the inner iteration, the effect caused by other SUs to SU *k* in the *m*-th outer iteration is denoted as $R_c(p_{-k}^{(n)}) = \sum_{j=1}^{k-1} p_j^{(n)} G_{j.c} + \sum_{j=k+1}^{c-1} p_j^{(n-1)} G_{j.c} + I_c$. Besides, $\xi$ and $\zeta$ are the stopping criterions of the outer and inter iteration respectively.

Next we discuss how to ascertain the value of $\gamma$. According to the KKT condition (16), the potential value of $\gamma$ can be sorted as $\gamma = 0$ and $\gamma > 0$, corresponding to the following two sub-problems (SP)

SP1: $\gamma = 0$

$$p_k^{(n+1)} = \left[ \left( \sum_{c=k+1}^{K} \frac{G_{k,c}}{\sum_{j=1}^{c-1} p_j^{(n)} G_{j,c} + I_c} \right)^{-1} \right]_0^{p_{k,\max}} \tag{19}$$

$$\sum_{k=1}^{K} p_k \boldsymbol{g_k} \le I_{th}$$

SP2: $\gamma > 0$

$$p_k^{(n+1)} = \left[ \left( \sum_{c=k+1}^{K} \frac{G_{k,c}}{\sum_{j=1}^{c-1} p_j^{(n)} G_{j,c} + I_c} + \gamma \boldsymbol{g_k} \right)^{-1} \right]_0^{p_{k,\max}}, \sum_{k=1}^{K} p_k \boldsymbol{g_k} = I_{th} \tag{20}$$

$$\sum_{k=1}^{K} p_k \mathbf{g_k} = I_{th}$$

We assume that the optimal solution to (16) is $\hat{p}^*$, the solution to SP1 and SP2 are $\hat{p}(0)$ and $\hat{p}(\gamma^*)$, respectively. Then, $\hat{p}^*$ must be $\hat{p}(0)$ or $\hat{p}(\gamma^*)$. The following Theorem gives the relationships among $\hat{p}(0)$, $\hat{p}(\gamma^*)$, and $\hat{p}^*$.

Theorem: If $\hat{p}(0)$ satisfies the interference constraints of PUs, i.e., $\sum_{k=1}^{K} \hat{p}_k(0) \boldsymbol{g_k} \le I_{th}$, then $\hat{p}^* = \hat{p}(0)$; else if $\sum_{k=1}^{K} \hat{p}_k(0) \boldsymbol{g_k} > I_{th}$, then $\hat{p}^* = \hat{p}(\gamma^*)$.

The following Lemma 1 is presented to prove the Theorem.

Lemma 1: if $\gamma_1 < \gamma_2$, then $\hat{p}(\gamma_1)$ and $\hat{p}(\gamma_2)$, which are the corresponding solutions of

KKT condition (16) obtained by algorithm A, which must satisfy $\hat{p}(\gamma_1) > \hat{p}(\gamma_2)$ regardless of the initial values. Here $\hat{p}(\gamma_1) > \hat{p}(\gamma_2)$ means that every element in $\hat{p}(\gamma_1)$ is larger than the corresponding element in $\hat{p}(\gamma_2)$.

Proof of Lemma 1: Without loss of generality, suppose that when $\gamma_1 < \gamma_2$, $\xi = \max_j(\hat{p}_j(\gamma_2) / \hat{p}_j(\gamma_1)) \geq 1$, then

$$\begin{aligned} \hat{p}_j(\gamma_2) &= \left[ \left( \sum_{c=k+1}^{K} \frac{G_{k,c}}{\sum_{i=1}^{c-1} \hat{p}_i(\gamma_2) G_{i,c} + I_c} + \gamma_2 \mathbf{g_k} \right)^{-1} \right]_0^{P_{i.\max}} \\ &< \left[ \left( \sum_{c=k+1}^{K} \frac{G_{k,c}}{\sum_{i=1}^{c-1} \xi \hat{p}_i(\gamma_1) G_{i,c} + \xi I_c} + \frac{1}{\xi} \gamma_1 \mathbf{g_k} \right)^{-1} \right]_0^{P_{i.\max}} = \xi \hat{p}_j(\gamma_1) \end{aligned} \tag{21}$$

This is contradictive to the hypothesis, so when $\gamma_1 < \gamma_2$, $\xi = \max_j(\hat{p}_j(\gamma_2) / \hat{p}_j(\gamma_1)) < 1$, i.e., $\hat{p}(\gamma_1) > \hat{p}(\gamma_2)$.

Proof of Theorem: When $\gamma = 0$, if the solution $\hat{p}(0)$ to SP1 satisfies $\sum_{k=1}^{K} \hat{p}_k(0) \boldsymbol{g_k} \leq I_{th}$, then based on Lemma 1, for any $\gamma > 0$, the solution $\hat{p}(\gamma)$ obtained from algorithm A must satisfy $\hat{p}(\gamma) < \hat{p}(0)$, so $\hat{p}(\gamma)$ satisfies $\sum_{k=1}^{K} \hat{p}_k(0) \boldsymbol{g_k} \leq I_{th}$. This means that if $\hat{p}(0)$ satisfies $\sum_{k=1}^{K} \hat{p}_k(0) \boldsymbol{g_k} \leq I_{th}$, then for any $\gamma > 0$, the necessary condition (20) of SP2 cannot be satisfied, so $\hat{p}(0)$ is the optimal solution to (16). If $\hat{p}(0)$ destroys the interference constraints of PUs, it is necessary to solve SP2, and the solution to

SP2 $\hat{p}(\gamma^*)$ is the optimal solution to (16).

By all appearances, SP1 can be solved directly by algorithm A, but SP2 is dependent on the value of $\gamma$, so how to fix on the value of $\gamma$ is the key problem in solving SP2.

Lemma 2: let us define a function $f(\gamma)=\sum_{k=1}^{K}\hat{p}_k(\gamma)\mathbf{g_k}-I_{th}$, then $f(\gamma)$ is strictly monotonically decreasing in $\gamma$ (lemma 2 can be directly obtained from lemma 1).

As $f(\gamma)$ monotonically decreases in $\gamma$, there is only one point $\gamma^*$ which makes $f(\gamma)=\sum_{k=1}^{K}\hat{p}_k(\gamma)\mathbf{g_k}-I_{th}$. This means that the solution to SP2 is unique. Due to the monotonicity of $f(\gamma)$, the well-known bisection search algorithm can be used to find $\gamma^*$. How to set the two initial values $\gamma_-^{(0)}$ and $\gamma_+^{(0)}$, which make $f(\gamma_-^0)<0$ and $f(\gamma_+^0)>0$, respectively is the key problem of the bisection algorithm. Obviously, from the Theorem, if it is necessary to solve SP2, this means that $\hat{p}(0)$ makes $f(0)=\sum_{k=1}^{K}\hat{p}_k(0)\boldsymbol{g_k}-I_{th}>0$, so $\gamma_+^{(0)}$ can be set to 0; on the other hand, it is shown in (20) that, if $\gamma_-^{(0)}$ can make $f(\gamma_-^0)=0$ when $p_k=p_{k,\max}(\forall k\in[1,K])$, then $\gamma_-^{(0)}$ must satisfies $f(\gamma_-^0)\le 0$ when $p_k\le p_{k,\max}(\forall k\in[1,K])$, so the initial value $\gamma_-^{(0)}$ can be obtained by

$$\sum_{k=1}^{K}\left(\sum_{c=k+!}^{K}\frac{G_{k,c}}{\sum_{j=1}^{c-1}p_{j,\max}+I_c}+\gamma_-^{(0)}\boldsymbol{g_k}\right)^{-1}=I_{th} \tag{22}$$

Based on the above discussion, we summarize the process as the following algorithm B.

**Table 2.** Algorithm B

| |
|---|
| $\gamma=0$, solve SP1 by algorithm A, obtain the solution to SP1 $\hat{p}(0)$ |
| if $\sum_{k=1}^{K}\hat{p}_k(0)g_k\le I_{th}, \hat{p}^*=\hat{p}(0)$ |

else initialize: m=0, $\gamma_+^{(0)} = 0$, $\gamma_-^{(0)}$, $\varsigma$

repeat: m=m+1

$\gamma^{(m)} = (\gamma_-^{(m+1)} + \gamma_+^{(m-1)})/2$

compute $\hat{p}(\gamma^{(m)})$ by algorithm A

if $f(\gamma^{(m)}) < 0$ $\gamma_-^{(m)} = \gamma^{(m)}$; $\gamma_+^{(m)} = \gamma_+^{(m)}$

else if $f(\gamma^{(m)}) > 0$ $\gamma_+^{(m)} = \gamma^{(m)}$; $\gamma_-^{(m)} = \gamma_-^{(m)}$

else $\gamma^* = \gamma^{(m)}$; $\hat{p}^* = \hat{p}(\gamma^*)$; exit

end

until $\left|\gamma^{(m)} - \gamma^{(m-1)}\right| / \left|\gamma^{(m-1)}\right| \le \varsigma$

$\gamma^* = \gamma^{(m)}$, $\hat{p}^* = \hat{p}(\gamma^*)$

End

Our joint beamforming and power allocation algorithm is implemented by the alternating iteration of beamforming and power allocation, until the total throughput of CR network converges to a stable value. In the *n*-th iteration, two steps are involved: in the first step, the power vector is fixed as $\hat{\boldsymbol{p}}^{*(n-1)}$, which is the optimal solution to (16) in the (*n*-1)th iteration, then the optimal beamforming matrix $\hat{\mathbf{w}}^{(n)}$; in the second step, with the updated beamforming matrix $\hat{\mathbf{w}}^{(n)}$, we find the optimal transmission power vector $\hat{\boldsymbol{p}}^{*(n)}$ by algorithm B.

## 5. Numerical Simulations

In this section, simulations are conducted to examine the performance of the proposed algorithm. We consider such a simulation condition that the PUs are randomly distributed, the transmission powers for all SUs are identical, and all the SUs are uniformly distributed in an area with radius 200m. The channel fading coefficients are modeled as independent zero-mean complex Gaussian random variables with variance 1, and path loss exponent is set to 4. It is assumed that the SBS has perfect CSI about the fading channel coefficients from the SBS to both PUs and SUs.

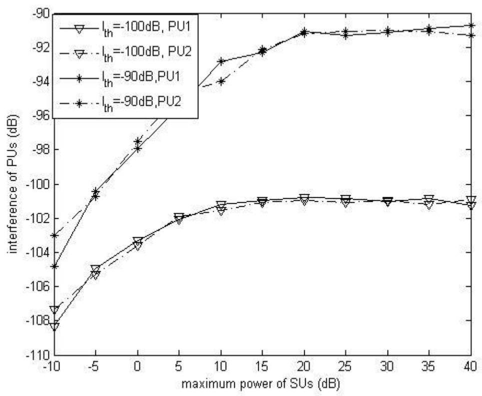


**Fig. 2.** The interference of PUs versus the maximum power of SUs

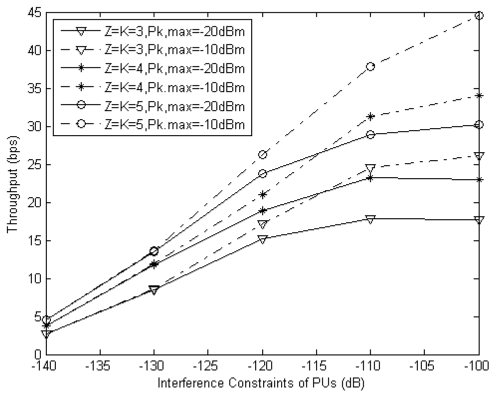


**Fig. 3.** Total throughput of SUs under various interference constraints of PUs (fixed *L*)

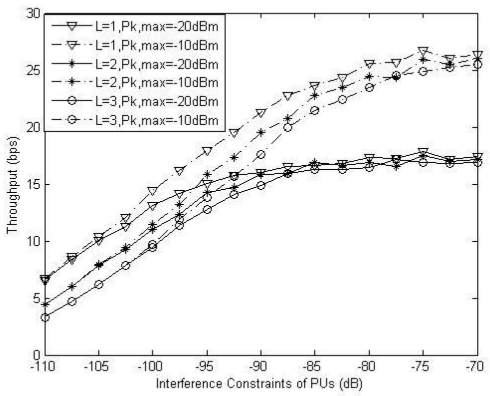


**Fig. 4.** Total throughput of SUs under various interference constraints of PUs (fixed *K* and *Z*)

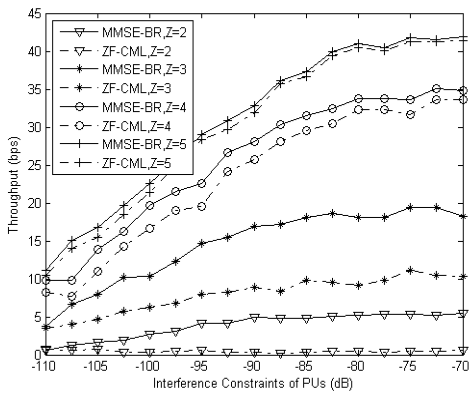


**Fig. 5.** Comparison of total throughput of SUs between MMSE-BR and ZF-CML

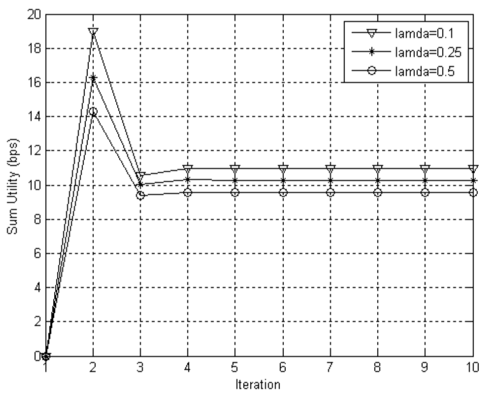


**Fig. 6.** Sum utility of SUs for different $\lambda$

First, in **Fig. 2** we show the interference of two primary users (PU1 and PU2) versus the maximum power of SUs under two different interference constraints of PUs. Other simulation parameters are $Z$=$K$=3, $\sigma^2 = -140$ dB. As shown in **Fig. 2**, when the interference of PUs is much lower than the interference constraints of PUs, it increases with the maximum power of SUs. But when the value of the interference of PUs is close to the interference constraints of PUs, no matter what the maximum power of SUs are, the interference of PUs do not exceed the interference constraints of PUs.

For a fixed $L$=4 in **Fig. 3** and a fixed $K$=$Z$=3 in **Fig. 4**, we investigate how the total throughput of SUs change with various interference constraints of PUs, and when $L$, $Z$ and $K$ change, how the total throughput of SUs change with the maximum power of SUs. Other simulation parameters are $\sigma^2 = -140$ dB, $p_l = -30$ dB. Clearly, the total throughput of SUs increase with the maximum power of SUs. When the interference constraints of PUs and the maximum power of SUs are same, the total throughput of SUs increase with the value of $L$, $Z$ and $K$ . But when the interference constraints of PUs achieves some certain value, the total throughput of SUs keeps constant.

In **Fig. 5**, for given $K$ and $Z$, we investigate how the throughput under the MMSE-BR algorithm and the ZF-CML algorithm change with the interference constraints of PUs, when $\sigma^2 = -140$ dB, $p_l = -30$ dB. The result shows no matter what $Z$ is, the throughput of MMSE-BR and ZF-CML increase with $I_{th}$ when $I_{th}$ is small. But when $I_{th}$ achieves a certain value, the throughput keepa constant. This is because when $I_{th}$ goes up to some value, the interference constraint can be satisfied even all the SUs use their peak transmission power. Then $I_{th}$ will not have any effect on the throughput of SUs. Moreover, it is easy to see that the MMSE-BR algorithm has better performance than the ZF-CML algorithm.

**Fig. 6** plots the sum utility of SUs versus the pricing factor $\lambda$ when the interference to PUs caused by SUs is restricted, with $\sigma^2 = -100$ dB, $p_{pu} = -40$ dB. There are two SUs and two PUs. As observed from **Fig. 6**, the sum utility of SUs decreases as $\lambda$ increases and can converge to a locally optimal value.

## 6. Conclusion

In this paper, the joint beamforming and power allocation problem in a cognitive MIMO network has been studied via game theory. Subject to the peak transmission power constraints of SUs as well as the interference constraints of PUs, a proper utility function with pricing was chosen to characterize the data transmission for SUs, and a joint beamforming and power allocation algorithm was designed to maximize the total throughput of SUs. Simulation results show the effectiveness of the proposed algorithm compared with the existing ZF-CML algorithm and the convergence property of the sum utility of SUs.

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