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Some Common Fixed Points for Type(β) Compatible Maps in an Intuitionistic Fuzzy Metric Space

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Abstract

Previously, Park et al. (2005) defined an intuitionistic fuzzy metric space and studied several fixed-point theories in this space. This paper provides definitions and describe the properties of type(β) compatible mappings, and prove some common fixed points for four self-mappings that are compatible with type(β) in an intuitionistic fuzzy metric space. This paper also presents an example of a common fixed point that satisfies the conditions of Theorem 4.1 in an intuitionistic fuzzy metric space.

Keywords: Compatible map, Type(β) compatible map, Fixed point

1. Introduction

Grabiec [1] demonstrated the Banach contraction theorem in the fuzzy metric spaces introduced by Kramosil and Michalek [2]. Park [3–5], Park and Kim [6] also proved a fixed-point theorem in a fuzzy metric space.

Recently, Park et al. [7] defined an intuitionistic fuzzy metric space while Park et al. [8] proved a fixed-point Banach theorem for the contractive mapping of a complete intuitionistic fuzzy metric space. Park et al. [9] defined a type(α) compatible map and obtained results for five mappings using a type(α) compatibility map in intuitionistic fuzzy metric spaces. Furthermore, Park [10] introduced a type(β) compatible mapping and proved some of the properties of the type(β) compatibility mapping in an intuitionistic fuzzy metric space.

This paper proves some common fixed points for four self-mappings that satisfy type(β) compatibility mapping in intuitionistic fuzzy metric space, while it also provides an example in the given conditions for an intuitionistic fuzzy metric space.

2. Preliminaries

First, some definitions and properties of the intuitionistic fuzzy metric space X are provided, as follows.

Let us recall ([11]) that a continuous $t-$norm is a binary operation $* : [0,1]\times[0,1] \to [0,1]$, which satisfies the following conditions: (a) $*$ is commutative and associative; (b) $*$ is continuous; (c) $a * 1 = a$ for all $a \in [0,1]$; (d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$.

Similarly, a continuous $t-$conorm is a binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$, which satisfies the following conditions: (a) $\diamond$ is commutative and associative; (b) $\diamond$ is continuous;

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(c) $a \diamond 0 = a$ for all $a \in [0, 1]$; (d) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0, 1])$.

Definition 2.1. [12] The $5-$tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous $t-$norm, $\diamond$ is a continuous $t-$conorm, and M, N are fuzzy sets in $X^2 \times (0, \infty)$, which satisfy the following conditions: for all $x, y, z \in X$, such that

(a) $M(x, y, t) > 0$,

(b) $M(x, y, t) = 1 \Longleftrightarrow x = y$,

(c) $M(x, y, t) = M(y, x, t)$,

(d) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

(e) $M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous,

(f) $N(x, y, t) > 0$,

(g) $N(x, y, t) = 0 \Longleftrightarrow x = y$,

(h) $N(x, y, t) = N(y, x, t)$,

(i) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,

(j) $N(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous.

Note that (M, N) is referred to as an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of proximity and the degree of non-proximity between x and y with respect to t, respectively.

Example 2.2. [13] Let (X, d) be a metric space. Denote $a * b = ab$ and $a \diamond b = \min\{1, a + b\}$ for all $a, b \in [0, 1]$ and let M_d, N_d be the fuzzy sets on $X^2 \times (0, \infty)$, which are defined as follows :

$$M_d(x, y, t) = \frac{kt^n}{kt^n + md(x, y)},$$
$$N_d(x, y, t) = \frac{d(x, y)}{kt^n + md(x, y)}$$

for $k, m, n \in R^+ (m \geq 1)$. Thus, $(X, M_d, N_d, *, \diamond)$ is an intuitionistic fuzzy metric space, i.e., the intuitionistic fuzzy metric space induced by the metric d.

Definition 2.3. [13] Let X be an intuitionistic fuzzy metric space.

(a) $\{x_n\}$ is said to be convergent to a point $x \in X$ by $\lim_{n\to\infty} x_n = x$ if

$$\lim_{n\to\infty} M(x_n, x, t) = 1,$$
$$\lim_{n\to\infty} N(x_n, x, t) = 0$$

for all $t > 0$.

(b) $\{x_n\}$ is a Cauchy sequence if

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1,$$
$$\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$$

for all $t > 0$ and $p > 0$.

(c) X is complete if every Cauchy sequence converges on X.

In this paper, X is considered to be the intuitionistic fuzzy metric space with the following condition:

$$\lim_{t\to\infty} M(x, y, t) = 1, \qquad (1)$$
$$\lim_{t\to\infty} N(x, y, t) = 0$$

for all $x, y \in X$ and $t > 0$.

Lemma 2.4. [6] Let $\{x_n\}$ be a sequence in an intuitionistic fuzzy metric space X with the condition (1). If there exists a number $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(x_{n+2}, x_{n+1}, kt) \geq M(x_{n+1}, x_n, t),$$
$$N(x_{n+2}, x_{n+1}, kt) \leq N(x_{n+1}, x_n, t) \qquad (2)$$

for all $t > 0$ and $n = 1, 2, 3 \cdots$, then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 2.5. [14] Let X be an intuitionistic fuzzy metric space. If there exists a number $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(x, y, kt) \geq M(x, y, t),$$
$$N(x, y, kt) \leq N(x, y, t),$$

then $x = y$.

3. Properties of type(β) compatible mappings and an example

This section introduces type(α) and type(β) compatible maps in an intuitionistic fuzzy metric space, and it also presents an example of the relations of type(β) compatible maps.

Definition 3.1. [14] Let A, B be mappings from the intuitionistic fuzzy metric space X into itself. These mappings are said to be compatible if

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1,$$
$$\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$$

for all $t > 0$, whenever $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Definition 3.2. ([10]) Let A, B be mappings from the intuitionistic fuzzy metric space X into itself. The mappings are said to be type(β) compatible if

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1,$$
$$\lim_{n\to\infty} N(AAx_n, BBx_n, t) = 0$$

for all $t > 0$, whenever $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Proposition 3.3. [10] Let X be an intuitionistic fuzzy metric space and A, B be the continuous mappings from X into itself. Thus, A and B are compatible if they are type(β) compatible.

Proposition 3.4. [10] Let X be an intuitionistic fuzzy metric space and A, B be mappings from X into itself. If A, B are type(β) compatible and $Ax = Bx$ for some $x \in X$, then $ABx = BBx = BAx = AAx$.

Proposition 3.5. [10] Let X be an intuitionistic fuzzy metric space and A, B be type(β) compatible mappings from X into itself. Let $\{x_n\} \subset X$ so $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$, then

(a)$\lim_{n\to\infty} BBx_n = Ax$ if A is continuous at $x \in X$,

(b)$\lim_{n\to\infty} AAx_n = Bx$ if B is continuous at $x \in X$,

(c)$ABx = BAx$ and $Ax = Bx$ if A and B are continuous at $x \in X$.

Example 3.6. Let $X = [0, \infty)$ with the metric d defined by $d(x, y) = |x - y|$ and for each $t > 0$, let M_d, N_d be fuzzy sets on $X^2 \times [0, \infty)$, which are defined as follows

$$M_d(x, y, t) = \frac{t}{t + d(x, y)},$$
$$N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

for all $x, y \in X$. Clearly, $(X, M_d, N_d, *, \diamond)$ is an intuitionistic fuzzy metric space where $*, \diamond$ are defined by $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$ for all $a, b \in [0, 1]$. Let us define $A, B : X \to X$ as

$$Ax = \begin{cases} 1 & \text{if } x \in [0, 1], \\ 1 + x & \text{if } x \in (1, \infty), \end{cases}$$
$$Bx = \begin{cases} 1 + x & \text{if } x \in [0, 1], \\ 1 & \text{if } x \in (1, \infty). \end{cases}$$

Thus, A, B are discontinuous at $x = 1$. Let $\{x_n\} \subset X$ be defined by $x_n = \frac{1}{n}, n = 1, 2, 3, \cdots$. Next, we have $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = 1$.

Furthermore,

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) \neq 1,$$
$$\lim_{n\to\infty} N(ABx_n, BAx_n, t) \neq 0$$

and

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1,$$
$$\lim_{n\to\infty} N(AAx_n, BBx_n, t) = 0.$$

Therefore, A, B are type(β) compatible but they are not compatible.

4. Main Results and Example

This section proves the main theorem and presents an example using the given conditions in an intuitionistic fuzzy metric space.

Theorem 4.1. Let X be a complete intuitionistic fuzzy metric space where $t * t \geq t$, $t \diamond t \leq t$ for all $t \in [0, 1]$. Let A, B, S and T be mappings from X into itself so:

(a) $AT(X) \cup BS(X) \subset ST(X)$;

(b) there exists $k \in (0, 1)$ so for all $x, y \in X$ and $t > 0$,

$$\begin{aligned}
&M^2(Ax, By, kt) * [M(Sx, Ax, kt)M(Ty, By, kt)] \\
&*M^2(Ty, By, kt) + aM(Ty, By, kt)M(Sx, By, 2kt) \\
&\geq [pM(Sx, Ax, t) + qM(Sx, Ty, t)]M(Sx, By, 2kt), \\
&N^2(Ax, By, kt) \diamond [N(Sx, Ax, kt)N(Ty, By, kt)] \\
&\diamond N^2(Ty, By, kt) + aM(Ty, By, kt)N(Sx, By, 2kt) \\
&\leq [pN(Sx, Ax, t) + qN(Sx, Ty, t)]N(Sx, By, 2kt),
\end{aligned}$$

where $0 < p, q < 1$, $0 \leq a < 1$ such that $p + q - a = 1$;

(c) S and T are continuous and $ST = TS$;

(d) the pairs (A, S) and (B, T) are type(β) compatible.

Thus, A, B, S and T have a unique common fixed point in X.

Proof. Let x_0 be an arbitrary point of X. Using (a), we can construct an $\{x_n\} \subset X$ as follows:

$$ATx_{2n} = STx_{2n+1},\ BSx_{2n+1} = STx_{2n+2},\ n = 0, 1, 2, \cdots .$$

Next, let $z_n = STx_n$. Using (b), we obtain

$$
\begin{aligned}
&M^2(ATx_{2n}, BSx_{2n+1}, kt) * [M(STx_{2n}, ATx_{2n}, kt)\\
&\times M(TSx_{2n+1}, BSx_{2n+1}, kt)] * M^2(TSx_{2n+1},\\
&BSx_{2n+1}, kt) + aM(TSx_{2n+1}, BSx_{2n+1}, kt)\\
&\times M(STx_{2n}, BSx_{2n+1}, 2kt)\\
&\geq [pM(STx_{2n+1}, ATx_{2n}, t)\\
&\qquad +qM(STx_{2n}, TSx_{2n+1}, t)]\\
&\qquad \times M(STx_{2n}, BSx_{2n+1}, 2kt),\\
&N^2(ATx_{2n}, BSx_{2n+1}, kt) \diamond [N(STx_{2n}, ATx_{2n}, kt)\\
&\times N(TSx_{2n+1}, BSx_{2n+1}, kt)] \diamond N^2(TSx_{2n+1},\\
&BSx_{2n+1}, kt) + aN(TSx_{2n+1}, BSx_{2n+1}, kt)\\
&\times N(STx_{2n}, BSx_{2n+1}, 2kt)\\
&\leq [pN(STx_{2n+1}, ATx_{2n}, t)\\
&\qquad +qN(STx_{2n}, TSx_{2n+1}, t)]\\
&\qquad \times N(STx_{2n}, BSx_{2n+1}, 2kt)
\end{aligned}
$$

and

$$
\begin{aligned}
&M^2(STx_{2n+1}, STx_{2n+2}, kt) * [M(z_{2n}, STx_{2n+1}, kt)\\
&\times M(z_{2n+1}, STx_{2n+2}, kt)] * M^2(z_{2n+1}, STx_{2n+2}, kt)\\
&+aM(z_{2n+1}, STx_{2n+2}, kt)M(z_{2n}, STx_{2n+2}, 2kt)\\
&\geq [pM(z_{2n}, STx_{2n+1}, t) + qM(z_{2n}, z_{2n+1}, t)]\\
&\times M(z_{2n}, STx_{2n+2}, 2kt),\\
&N^2(STx_{2n+1}, STx_{2n+2}, kt) \diamond [N(z_{2n}, STx_{2n+1}, kt)\\
&\times N(z_{2n+1}, STx_{2n+2}, kt)] \diamond N^2(z_{2n+1}, STx_{2n+2}, kt)\\
&+aN(z_{2n+1}, STx_{2n+2}, kt)N(z_{2n}, STx_{2n+2}, 2kt)\\
&\leq [pN(z_{2n}, STx_{2n+1}, t) + qN(z_{2n}, z_{2n+1}, t)]\\
&\times N(z_{2n}, STx_{2n+2}, 2kt).
\end{aligned}
$$

Then,

$$
\begin{aligned}
&M^2(z_{2n+1}, z_{2n+2}, kt)\\
&\qquad *[M(z_{2n}, z_{2n+1}, kt)M(z_{2n+1}, z_{2n+2}, kt)]\\
&\qquad +aM(z_{2n+1}, z_{2n+2}, kt)M(z_{2n}, z_{2n+2}, 2kt)\\
&\geq [p+q]M(z_{2n}, z_{2n+1}, t)M(z_{2n}, z_{2n+2}, 2kt),\\
&N^2(z_{2n+1}, z_{2n+2}, kt)\\
&\qquad \diamond[N(z_{2n}, z_{2n+1}, kt)N(z_{2n+1}, z_{2n+2}, kt)]\\
&\qquad +aN(z_{2n+1}, z_{2n+2}, kt)N(z_{2n}, z_{2n+2}, 2kt)\\
&\leq [p+q]N(z_{2n}, z_{2n+1}, t)N(z_{2n}, z_{2n+2}, 2kt),
\end{aligned}
$$

and

$$
\begin{aligned}
&M^2(z_{2n+1}, z_{2n+2}, kt)M(z_{2n+1}, z_{2n+2}, kt)]\\
&\qquad +aM(z_{2n+1}, z_{2n+2}, kt)M(z_{2n}, z_{2n+2}, 2kt)\\
&\geq [p+q]M(z_{2n}, z_{2n+1}, t)M(z_{2n}, z_{2n+2}, 2kt),\\
&N^2(z_{2n+1}, z_{2n+2}, kt)N(z_{2n+1}, z_{2n+2}, kt)]\\
&\qquad +aN(z_{2n+1}, z_{2n+2}, kt)N(z_{2n}, z_{2n+2}, 2kt)\\
&\leq [p+q]N(z_{2n}, z_{2n+1}, t)N(z_{2n}, z_{2n+2}, 2kt).
\end{aligned}
$$

Therefore, it follows that

$$
\begin{aligned}
&M(z_{2n+1}, z_{2n+2}, kt) \geq M(z_{2n}, z_{2n+1}, t),\\
&N(z_{2n+1}, z_{2n+2}, kt) \leq N(z_{2n}, z_{2n+1}, t)
\end{aligned}
$$

for all $t > 0$ and $k \in (0, 1)$. In general, for $m = 1, 2, \cdots$, we have

$$
\begin{aligned}
&M(z_{m+1}, z_{m+2}, kt) \geq M(z_m, z_{m+1}, t),\\
&N(z_{m+1}, z_{m+2}, kt) \leq N(z_m, z_{m+1}, t)
\end{aligned}
$$

Thus, $\{z_n\}$ is a Cauchy sequence in X and, because X is complete, $\{z_n\}$ converges to a point $z \in X$. Since $\{ATx_{2n}\}$, $\{BSx_{2n+1}\}$ are subsequences of $\{z_n\}$, $\lim_{n\to\infty} ATx_{2n} = z = \lim_{n\to\infty} BSx_{2n+1}$.

Let $y_n = Tx_n$, $u_n = Sx_n$ for $n = 1, 2, \cdots$. Thus, we have $Ay_{2n} \to z$, $Sy_{2n} \to z$, $Tu_{2n+1} \to z$ and $Bu_{2n+1} \to z$. Furthermore,

$$
\begin{aligned}
&M(AAy_{2n}, SSy_{2n}, t) \to 1,\\
&M(BBu_{2n+1}, TT_{2n+1}, t) \to 1,\\
&N(AAy_{2n}, SSy_{2n}, t) \to 0,\\
&N(BBu_{2n+1}, TT_{2n+1}, t) \to 0
\end{aligned}
$$

as $n \to \infty$. Based on the continuity of T and Proposition 3.4, we obtain $TBu_{2n+1} \to Tz$, $BBu_{2n+1} \to Tz$.

Next, by taking $x = y_{2n}$, $y = Bu_{2n+1}$ in (b), for $n \to \infty$ we obtain,

$$
\begin{aligned}
&M^2(z, Tz, kt) * [M(z, z, kt)M(Tz, Tz, kt)]\\
&*M^2(Tz, Tz, kt) + aM(Tz, Tz, kt)M(z, Tz, 2kt)\\
&\geq [pM(z, z, t) + qM(z, Tz, t)]M(z, Tz, 2kt),\\
&N^2(z, Tz, kt) \diamond [N(z, z, kt)N(Tz, Tz, kt)]\\
&\diamond N^2(Tz, Tz, kt) + aN(Tz, Tz, kt)N(z, Tz, 2kt)\\
&\leq [pN(z, z, t) + qN(z, Tz, t)]N(z, Tz, 2kt),
\end{aligned}
$$

then

$$\begin{aligned}&M^2(z,Tz,kt)+aM(z,Tz,2kt)\\&\quad\geq[p+qM(z,Tz,t)]M(z,Tz,2kt),\\&N^2(z,Tz,kt)\leq qN(z,Tz,t)N(z,Tz,2kt).\end{aligned}$$

Since $M(x,y,\cdot)$ is nondecreasing and $N(x,y,\cdot)$ is nonincreasing for all $x,y\in X$, we obtain

$$\begin{aligned}&M(z,Tz,kt)+a\geq p+qM(z,Tz,t),\\&N(z,Tz,kt)\leq qN(z,Tz,t)\end{aligned}$$

and

$$\begin{aligned}&M(z,Tz,kt)\geq\frac{p-a}{1-q}=1,\\&N(z,Tz,kt)\leq\frac{0}{1-q}.\end{aligned}$$

Thus, $z=Tz$. Similarly, we have $z=Sz$.

Next, by taking $x=y_{2n}$ and $y=z$ in condition (b), for $n\to\infty$ we obtain

$$\begin{aligned}&M(z,Bz,kt)*M(z,Bz,kt)\\&\qquad+aM(z,Bz,kt)M(z,Bz,2kt)\\&\geq(p+q)M(z,Bz,2kt),\\&N(z,Bz,kt)\diamond N(z,Bz,kt)\\&\qquad+aN(z,Bz,kt)N(z,Bz,2kt)\leq 0.\end{aligned}$$

Thus,

$$\begin{aligned}&M(z,Bz,kt)+aM(z,Bz,kt)\geq p+q,\\&N(z,Bz,kt)+aN(z,Bz,kt)\leq 0.\end{aligned}$$

Therefore,

$$\begin{aligned}&M(z,Bz,kt)\geq 1,\\&N(z,Bz,kt)\leq 0\end{aligned}$$

for all $t>0$ and $k\in(0,1)$. Thus, $z=Bz$. Similarly, we obtain $z=Az$. Therefore, z is a common fixed point of A,B,S and T.

Let w be another common fixed point of A,B,S and T. Using condition (b), we have

$$\begin{aligned}&M^2(z,w,kt)*[M(z,z,kt)M(w,w,kt)]\\&*M^2(w,w,kt)+aM(w,w,kt)M(z,w,2kt)\\&\geq[pM(z,z,t)+qM(z,w,t)]M(z,w,2kt),\\&N^2(z,w,kt)\diamond[N(z,z,kt)N(w,w,kt)]\\&\diamond N^2(w,w,kt)+aN(w,w,kt)N(z,w,2kt)\\&\leq[pN(z,z,t)+qN(z,w,t)]M(z,w,2kt).\end{aligned}$$

Thus,

$$\begin{aligned}&M^2(z,w,kt)+M(z,w,2kt)\\&\quad\geq(p+qM(z,w,t))M(z,w,2kt),\\&N^2(z,w,kt)\leq qM(z,w,t)M(z,w,2kt).\end{aligned}$$

Therefore,

$$\begin{aligned}&M(z,w,kt)\leq M(z,w,2kt),\\&N(z,w,kt)\geq N(z,w,2kt),\end{aligned}$$

so

$$\begin{aligned}&M(z,w,kt)\geq\frac{p-a}{1-q}=1,\\&N(z,w,kt)\leq\frac{0}{1-q}.\end{aligned}$$

Thus, $z=w$. This means that A,B,S and T have a unique common fixed point. □

Corollary 4.2. Let X be a complete intuitionistic fuzzy metric space where $t*t\geq t, t\diamond t\leq t$ for all $t\in[0,1]$ and let A,B be mappings from X into itself such that:

(e) $A(X)\subset S(X)$,

(f) there exists $k\in(0,1)$ so for all $x,y\in X$ and $t>0$,

$$\begin{aligned}&M^2(Ax,Ay,kt)*[M(Sx,Ax,kt)M(Sy,Ay,kt)]\\&M^2(Sy,Ay,kt)+aM(Sy,Ay,kt)M(Sx,Ay,2kt)\\&\geq[pM(Sx,Ax,t)+qM(Sx,Sy,t)]M(Sx,Ay,2kt),\\&N^2(Ax,Ay,kt)\diamond[N(Sx,Ax,kt)N(Sy,Ay,kt)]\\&\diamond N^2(Sy,Ay,kt)+aM(Sy,Ay,kt)N(Sx,Ay,2kt)\\&\leq[pN(Sx,Ax,t)+qN(Sx,Sy,t)]N(Sx,Ay,2kt),\end{aligned}$$

where $0<p,q<1$, $0\leq a<1$ such that $p+q-a=1$,

(g) S is continuous,

(h) A and S are type(β) compatible.

Thus, A and S have a unique common fixed point in X.

Proof. Therefore, if we enter $A = B$ and $S = T$ into Theorem 4.1, all of the conditions of Theorem 4.1 are satisfied. Thus, the proof of this corollary follows from Theorem 4.1. □

Example 4.3. Let $X = \{\frac{1}{n}|n \in \mathbf{N}\} \cup \{0\}$ with the metric d defined by $d(x, y) = |x - y|$ and for each $t > 0$, let M_d, N_d be fuzzy sets on $X^2 \times [0, \infty)$, which are defined as follows

$$M_d(x, y, t) = \frac{t}{t + d(x, y)},$$
$$N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

for all $x, y \in X$. Clearly, $(X, M_d, N_d, *, \diamond)$ is a complete intuitionistic fuzzy metric space where $*, \diamond$ are defined by $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$ for all $a, b \in [0, 1]$. Let A, B, S and T be maps from X into itself, which are defined by

$$Ax = \frac{x}{6}, \quad Bx = 0, \quad Sx = \frac{x}{3}, \quad Tx = x$$

for all $x \in X$. Then,

$$AT(X) \cup BS(X) = \{\frac{1}{6n}|n \in \mathbf{N}\} \cup \{0\}$$
$$\subset \{\frac{1}{3n}|n \in \mathbf{N}\} \cup \{0\} = ST(X).$$

Furthermore, $ST = TS$ and S, T are continuous. If we take $k = \frac{1}{2}$ and $t = 1$, the condition (b) of Theorem 4.1 is satisfied. Moreover, A, S are type(β) compatible if $\lim_{n\to\infty} x_n = 0$ where $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 0$ for some $0 \in X$.

Similarly, B, T are type(β) compatible. Thus,

$$M(0, B0, kt) + aM(0, B0, kt) \geq p + q,$$
$$N(0, B0, kt) + aN(0, B0, kt) \leq 0.$$

Therefore, $M(0, B0, kt) \geq 1$ and $N(0, B0, kt) \leq 0$ for all $t > 0$ and $k \in (0, 1)$. Thus, $0 = B0$. Similarly, we obtain $0 = A0$. Therefore, 0 is a common fixed point of A, B, S and T.

Let w be another common fixed point of A, B, S and T. Then,

$$M^2(0, w, kt) + M(0, w, 2kt)$$
$$\geq (p + qM(0, w, t))M(0, w, 2kt),$$
$$N^2(0, w, kt) \leq qM(0, w, t)M(0, w, 2kt).$$

Therefore, because

$$M(0, w, kt) \leq M(0, w, 2kt),$$
$$N(0, w, kt) \geq N(0, w, 2kt),$$

Thus,

$$M(0, w, kt) \geq \frac{p - a}{1 - q} = 1,$$
$$N(0, w, kt) \leq \frac{0}{1 - q}.$$

Therefore, $0 = w$. Thus, A, B, S and T have a unique common fixed point 0.

5. Conclusion

Park et al. [7] defined an intuitionistic fuzzy metric space and Park et al. [8] proved a fixed-point Banach theorem for the contractive mapping of a complete intuitionistic fuzzy metric space. Park et al. [9] defined a type(α) compatible mapping and obtained results for five mappings using type(α) compatibility in intuitionistic fuzzy metric spaces. Furthermore, Park [10] introduced type(β) compatible mapping and proved some properties of type(β) compatibility in an intuitionistic fuzzy metric space. In this paper, we proved some common fixed points for four self-mappings that satisfy type(β) compatibility and we provided an example in the given conditions for an intuitionistic fuzzy metric space.

This paper attempted to develop a method to provide a proof based on the fundamental concepts and properties defined in the new space. I think that the results of this paper will be extended to the intuitionistic M-fuzzy metric space and other spaces. Further research should be conducted to determine how to combine the collaborative learning algorithm with our proof method in the future.

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