LDGM와 LDPC code를 이용한 Raptor code의 비교

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A Comparison of Raptor Code Using LDGM and LDPC code

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요 약

본 논문에서는 LDPC 코드 대신 LDGM 코드를 이용한 랩터(Raptor) 코드 구성을 제안한다. 그리고 실험을 통하여 두 코드 성능을 비교한다. 시뮬레이션으로 두 코드 성능을 비교한 결과 LDGM 코드를 이용한 랩터 코드가 LDPC 코드를 이용한 랩터 코드보다 성능은 좋지 않았지만 인코딩 복잡도는 더 간단한 것으로 확인되었다.

ABSTRACT

In this paper, we propose the construction of Raptor code using LDGM code instead of LDPC code as precode. To see their merits, the simulations are done. The results show that the performance in case of using LDGM code is worse than that of using LDPC code as precode, and the complexity of encoding LDGM code is lower than that of LDPC.

키워드

Raptor code, LDPC code, LDGM code, LT code
Raptor 코드, LDPC 코드, LDGM 코드, LT 코드

Ⅰ. Introduction

Raptor code is a significant theoretical and practical improvement over LT codes, which were the first practical class of fountain codes. Raptor code consists of LT code and precode. LDPC code, which is a linear code with a parity-check matrix[3], was used as precode in 2006. The performance of LDPC codes has been determined to approach the Shannon limit by using probabilistic iterative decoding algorithms[1]. Because of the encoding complexity of Raptor code[2], LDGM codes are considered. Then LDGM code is regarded as a special type of LDPC code, and the encoding complexity of LDGM codes is much less than LDPC codes. And since the parity-check matrix is also sparse[4], LDGM codes can be decoded using the same techniques as those for LDPC codes. In spite of these advantages, the performance of LDGM codes is asymptotically bad since they have too many degree-1 columns. So in this paper, We proposed a design for construction of Raptor code which require LDGM code instead of LDPC code as precode. and showed their performances and complexities with simulation by Matlab. We could verify that in case of

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LDGM, the code performance is worse than that of LDPC code, and encoding complexity is lower[5][6][7].

Ⅱ. LDPC code

LDPC codes are linear codes obtained from sparse bipartite graphs. Suppose that G is a graph with n left nodes (called message nodes) and r right nodes (called check nodes). The graph gives rise to a linear code of block length n and dimension at least $n-r$ in the following way: The n coordinates of the codewords are associated with the n message nodes. The codewords are those vectors $(c_1,...,c_n)$ such that for all check nodes the sum of the neighboring positions among the message nodes is zero. Fig. 1 gives an example.

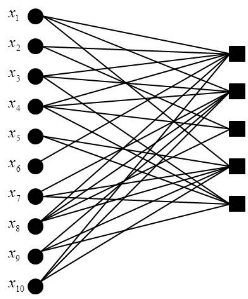

Fig. 1 Tanner graph of LDPC code

The graph representation is analogous to a matrix representation by looking at the adjacency matrix of the graph: let H be a binary $r\times n$ matrix in which the entry (i,j) is 1 if and only if the ith check node is connected to the jth message node in the graph. Then the LDPC code defined by the graph is the set of vectors $c=(c_1,...,c_n)$ such that $H\cdot c^{\top}=0$. The matrix H is called a parity check matrix for the code. Conversely, any binary $r\times n$ matrix gives rise to a bipartite graph between n message and r check nodes, and the code defined as the null space of H is precisely the code associated to this graph. Therefore, any linear code has a representation as a code associated to a bipartite graph. However, not every binary linear code has a representation by a sparse bipartite graph. If it does, then the code is called a low-density parity-check (LDPC) code. Fig. 1 can also express as equation below:

$$H=\begin{bmatrix}1111000000\\1000111000\\0100110110\\0010000101\\0001001011\end{bmatrix} \tag{1}$$

Equation (1) express LDPC code which row weight is 4 and column weight is 2.

Ⅲ. LDGM code

An LDPC code is a linear code with a parity-check matrix that contains a small number of ones. By using probabilistic iterative decoding algorithms, the performance of LDPC codes is known to approach the Shannon limit. The main advantage of LDPC codes over turbo codes is a fully parallelizable decoder which allows fast decoding. On the other hand, the encoding complexity is much larger than that of Turbo codes since the encoding an LDPC code is usually based on the matrix multiplication of large size. Therefore, it is essential to search for (a family of) LDPC codes with efficient encoding algorithms as well as efficient decoding algorithms. We consider LDGM codes which can be regarded as a special type of LDPC codes.

A binary linear code C of rate $R=\frac{m}{n}$,and can be represented by a generator matrix, say $G\in\{0,1\}^{n\times m}$. In this generator representation, each codeword $x\in C$ belongs to the range space of G, and so can be written in the form $x=Gz$, for a suitable sequence of information bits $z\in\{0,1\}^m$. Here all arithmetic (addition and multiplication) is performed in modulo-two. Presuming that G is full rank, the code C then consists of 2^m possible n-bit strings, and so has rate $R=\frac{m}{n}$.

The structure of a given generator matrix G can be captured by its factor graph, which is a bipartite graph in which circular nodes represent code bits x_i (or row of G), and square nodes represent the information bits(or columns of G). For instance, Fig. 2 shows a binary linear code of block length $n=12$ and $m=9$ information bits, represented in factor graph form by its generator matrix $G\in\{0,1\}^{12\times 9}$, with an overall rate of $R=3/4$. The degree of the check and variable nodes in the factor graph are $\Upsilon_c=3$ and $\Upsilon_v=4$, respectively, so that the associated generator matrix G has three "1"s in each row, and four "1"s in each column, when the generator matrix is sparse in this sense, we refer to the resulting code as a low-density generator matrix code(or LDGM code for short).

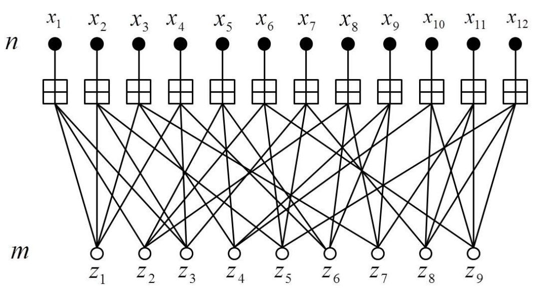

Fig. 2 Binary linear code

LDGM codes are linear codes with generator matrix[12], $G=[I\ P]$, where I is a $k\times k$ identity matrix and P is a $k\times(n-k)$ sparse matrix. Here, k denotes the number of information bits and n denotes the number of bits of a codeword. The parity check matrix of the codes is $H=[P^T\ I]$. Fig. 3 shows the bipartite graph representation of LDGM codes. There exist $(n-k)$ coded bit nodes of degree-1 and k bits nodes corresponding to the systematic bits. Since the messages propagated from the degree-1 coded bit nodes to their corresponding check nodes are always the same, LDGM codes have high error floors. But the number of errors for the codewords in error decays very fast, and the outputs obtained from the decoding of LDGM codes can be seen as a priori probability produced by an equivalent channel introducing a small amount of erasures at specific locations. So if we use these outputs to initialize the bit nodes of the outer LDGM decoder in the decoding process, the number of residual errors can be reduced.

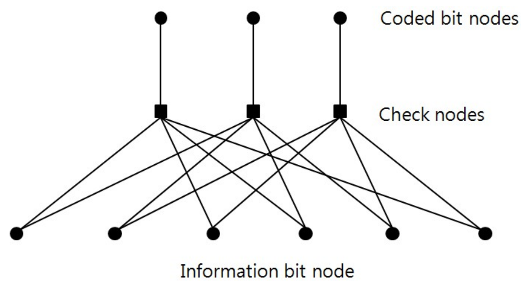

Fig. 3 Tanner graph of LDGM code

Fig. 3 can also express as equation (2)

$$H=\begin{bmatrix}101101 & 100\\ 110110 & 010\\ 011011 & 001\end{bmatrix} \tag{2}$$

LDGM code present a complexity advantage over standard LDPC code. Since the parity check matrix of systematic LDGM code is also sparse, they are in fact a subset of LDPC code and can be decoded in the same manner and with the same complexity as standard LDPC code. The LDPC versus LDGM distinction is important and it impacts:

- the encoding speed: encoding is faster with LDGM than with LDPC.
- the error correction capabilities: LDPC has a major advantage over LDGM from this point of view, because parity packets are better protected. An LDPC parity packet can be recovered (because it shares edges with several check nodes) which is not possible with LDGM.

Ⅳ. Construction of Raptor code

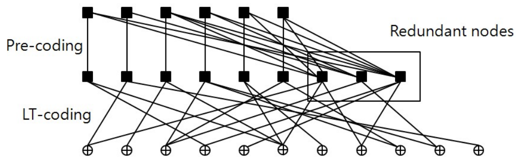

Fig. 4 Raptor code

Raptor code is consist of traditional erasure correcting code and LT code as Fig. 4. We encode the input symbols using a traditional erasure correcting code, and then apply an appropriate LT-code to the new set of symbols in a way that the traditional code is capable of recovering all the input symbols even in face of a fixed fraction of erasure. The traditional erasure correcting code usually use the LDPC code.

Fig. 3 shows that the input symbols are appended by redundant symbols(black squares)in the case of a systematic precode. An appropriate LT-code is used to generate output symbols from the preceded input symbols.

The data is produced by sourcefile and obtain the symbols which have fixed code length through the LDPC / LDGM encoding. Then produce the code symbols by LT encoder and transmit by AWGN channel, as shown in Fig. 5.

The receiver receives these code symbols then try to decode these code symbols. If fails, receivers could receive more code symbols and try to decode. Receivers will stop this process when decode successfully and sent a simple signal to sender. LT encoder stop encoding this frame date and sourcefile then sent next frame data to receivers.

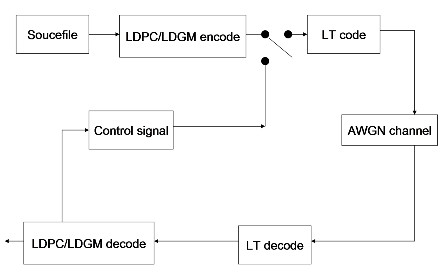

Fig. 5 The process of Raptor code encode/decode

Raptor code encode/decode process as below :

① Sourcefile produce the original data $s=(s_0,s_1,...s_{k-1})$.

② Using LDPC / LDGM code to pre-encode the original data s, obtain the among symbol $u=(u_0,u_1,...u_{n-1})$.

③ Using LT code to encode among symbol u, obtain the encode symbol $t=(t_0,t_1,...t_{n-1})$ and generator matrix $G_{n\times n}$.

④ Modulate encode symbol and transmit to receivers by AWGN channels, information of generator matrix is transmitted to receivers too, receivers receive the receiving sequence $r=(r_0,r_1...r_{n-1})$.

Consider $i=0$, circulation begins:

⑤ LT encoder encode the among symbol u and obtain the encode symbol increment $\triangle t=(\triangle t_0,\triangle t_1,...\triangle t_{\triangle n-1})$ and generator matrix increment $g_{\triangle n\times n}$, i++.

⑥ The same as ③ , transmit $\triangle t$ and generator matrix to receivers, receivers receive the receiving sequence $r=(r_0,r_1...r_{n-1+i\cdot\triangle n})$, and generator matrix is $G_{n\times(n+i\cdot\triangle n)}$.

⑦ Receivers try to decode. If decode successfully, the process is end; If decode fail, the process return to ⑤.

Ⅴ. Simulations

Fig. 6 shows the comparison of encoding complexity between LDGM code and LDPC code. In this simulation we encoded information bits with a (3, 57) outer LDGM code and (7,7) inner LDGM code. From the Figure we can see that at $time=0.5\times10^4ms$, the number of information

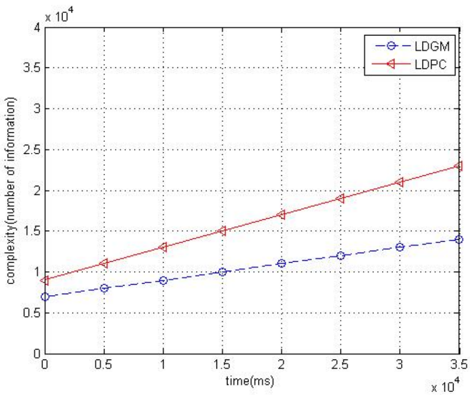

Fig. 6 Encoding Complexity of LDPC and LDGM code

which need to encode LDGM code is 0.7×10^4, the number of information which need to encode LDPC code is 0.9×10^4. But at $time = 3.5 \times 10^4 ms$, the number of information which need to encode LDGM code is 1.5×10^4, but the number of information which need to encode LDPC code is 2.3×10^4, so in the process of encoding, the number of information for LDGM code is less than the number of information for LDPC code so that encoding complexity of LDGM is much simple than encoding complexity of LDPC code. Because of the sparse generator matrix of LDGM code.

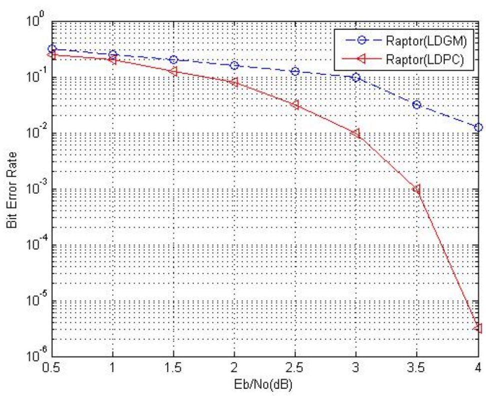

Fig. 7 Bit error rate of LDPC and LDGM code as precode of Raptor code

Fig. 7 shows the comparison of bit error rates between Raptor code using LDGM code and LDPC code($n = 1056, rate = 2/3$) as precode of Raptor code and a weakened LT code as inner code. About encoding of LDGM code, first, we encoded information bits with a $(3, 57)$ outer LDGM code to produce a total of 10000 bits. These bits were encoded again by a $(7,7)$ inner LDGM code. According to the comparison between Raptor code which is using LDPC code and LDGM code as precode, we can see that Raptor code which use LDPC code as precode can obtain the better performance. At the condition of $Eb/No = 4dB$, the bit error rate drop to 10^{-6}but the bit error rate of Raptor code which use LDGM code as precode only drop to 10^{-2}, so the performance of construction which use LDGM code as precode is worse than the construction which use LDPC code as precode. But the encoding complexity of Raptor code which is using LDGM code as precode is lower. Compare to LDPC code, the construction of LDGM code′s generator matrix is more simple.

Ⅵ. Conclusions

We introduce the construction of Raptor code using LDPC code and LDGM code and analysis the difference between them. Then a Raptor codes realization scheme is presented: a LDPC code or LDGM code as outer code and a weakened LT code as inner code. Finally, the performance of Raptor codes using LDPC codes as precode and Raptor code using LDGM codes as precode are compared. The simulation results show that the Raptor codes using LDGM codes as precode has worse performance than the Rpator codes using LDPC codes as precode. But in aspect of the complexity, the latter is better.

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