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# Probabilistic Approach on Railway Infrastructure Stability and Settlement Analysis

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**Abstract**

Railway construction needs vast soil investigation for its infrastructure foundation designs along the planned railway path to identify the design parameters for stability and serviceability checks. The soil investigation data are usually classified and grouped to decide design input parameters per each construction section and budget estimates. Deterministic design method which most civil engineer and practitioner are familiar with has a clear limitation in construction/maintenance budget control, and occasionally produced overdesigned or unsafe design problems. Instead of using a batch type analysis with predetermined input parameters, data population collected from site soil investigation and design load condition can be statistically estimated for the mean and variance to present the feature of data distribution and optimized with a best fitting probability function. Probabilistic approach using entire feature of design input data enables to predict the worst, best and most probable cases based on identified ranges of soil and load data, which will help railway designer select construction method to save the time and cost. This paper introduces two Monte Carlo simulations actually applied on estimation of retaining wall external stability and long term settlement of organic soil in soil investigation area for a recent high speed railway project.

***Keywords :*** *Probabilistic, Monte Carlo, Simulation, Wall, Stability, Settlement, Mean, Variance, Stochastic*

## 1. Nomenclature

$\hat{x}$ : Random variable, x
$\bar{x}$ : Mean value of random variable, x
s: Standard deviation of random variable, x
ρ: Correlation coefficient between random variables
ϕ: drained internal friction angle of site soil
PDF: probability density function
$f(x)$: Beta distribution PDF for x
FS: Factor of Safety for Wall External Stability
S1: Probabilistic primary settlement amount
S2: Probabilistic secondary settlement amount

## 2. Introduction

Deterministic approach, familiar to many civil engineers and predominant in most railway infrastructure design only handles representative or site averaged design parameters adequate for a single-batch type analysis. This deterministic factor of safety should not be applied monotonously to every construction situation, different from site to site. Some construction sites have a relatively uniform soil type and consistent soil parameters, in which a factor of safety value, generally accepted would produce overly designed foundation specifications for overall project site. Meanwhile, a factor of safety value evaluated from the averaged soil condition cannot guarantee stability and serviceability of entire infrastructure foundations in project site having a various soil profile and inconsistent test values. Based on Load and Resistance Factor Design (LRFD) newly adopted in Unite States, AASHTO bridge design manual (2008) deals these different project situations and soil conditions to allow cost-effective design for the project site having a consistent and uniform soil condition and to demand a more conservative design for the project site having a poor and variable soil conditions. However, current LRFD system, based on nationwide input database also has a limitation of applicability to local railway

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projects showing clear regional and temporal differences of soil parameters, requiring for a more site specific investigation.

Due to nature of elongated railway profile, extensive subsurface investigation and field/lab test programs are necessary to build durable infrastructure on variable foundation soils along the railway path. To group and analyze the vast field/lab data, statistical analysis should be adopted to estimate the mean and variance values of each data group. A probabilistic approach on the data distribution of input parameters has been successfully applied to many railway projects such as structural safety evaluation on old railway bridges (Casas and Wisniewski, 2013), decision making on resilient railway tunnel serviceability (Cedergren, 2012), stochastic approach on minimization of passenger travel time for single or partially double track railway (Yang *et al*, 2010), random set finite element analysis on tunnel excavation (Schweiger *et al*, 2007), stochastic analysis on dynamic interaction between train and railway turn-out (Kassa and Nielsen, 2008), and modeling and managing rockfall hazard *(*Straub and Schubert, 2007). However, probabilistic or stochastic analysis on foundation soil parameters (internal friction angle, cohesion, void ratio, compression indices and etc.) for railway infrastructure design has rarely been conducted to date.

Probabilistic approach was applied to a high speed railway foundation design between Madison and Milwaukee, Wisconsin in 2010 to evaluate the mean and variance of soil input parameters from the field and laboratory investigations. Accumulated regional/chronological data were prescreened and classified into soil database of entire railway project region in view of data reliability and confidence level. Feature of data distribution (log-normal, normal or Poisson distribution) was evaluated from statistical analyses on the accumulated soil investigation data. After distribution and relationships were analyzed for the soil input parameters, the probability density function (PDF) of factor of safety or settlement estimate could be evaluated based on Monte Carlo simulation or point estimation method. This research used a beta-distribution suggested by Harr (1996) to characterize the distributional features of soil parameters due to the flexibility handling various probability distributions and the capability of limiting input value ranges.

## 2. Research Methodology

Monte Carlo simulation enables to assess a probability distribution of output value using an established relationship between input parameters and output value with each input parameter assumed to follow a probability distribution function (PDF) of best fitting probability model. If no correlation is found between two input parameters, the input parameters can be called mutually independent to each other. In the case, the Monte Carlo simulation will randomly select a candidate value of input parameters with considering each PDF shape and value limits. If any positive or negative relationship is found between the input parameters from a precedent parametric study, the data set of input parameters can be called mutually associated. In the case, Monte Carlo method should select random input variables with considering correlation factors among the data sets.

Most probabilistic or stochastic software have been commercially invented and actively used in financial application areas such as stock market or insurance industry. However, the existing commercial products are not easy to run nor compatible to engineering spreadsheet programs most of civil engineers and practitioners are familiar with. Thus, add-in program capable of handling random input parameters is considered practically useful for pre-developed spreadsheet programs to solve the civil and railway problems with Monte Carlo simulation.

For this research, author used free source Excel add-in program titled "ran_var.xla" (publicly available in http://www.me.utexas.edu/~jensen/ORMM/computation/index.html) to apply to customized engineering spreadsheet programs to calculate external stabilities of retaining wall and primary and creep settlement of organic soil under preloading fill.

Random variables of input parameters used in the external stability check and long term settlement estimate were internal friction angle, cohesion, primary compression index, secondary (creep) compression index, initial void ratio, void ratio after primary consolidation, elapsed time of primary consolidation process. All input parameters were analyzed for the mean and variance were optimized to find a best fitting PDF. If any correlation is found between the input parameters, coefficient of correlation was evaluated between two data sets, which were applied for multivariate analysis.

Beta distribution was used in the Monte Carlo simulation to present soil input parameters having non-negative value and bounded value range since it allows setting both upper and lower limits, mean and deviation of random variables at the same time. PDF of Beta distribution for random variable x can be presented as following equations (Harr, 1996).

$$f(x) = C(x-a)^{\alpha}(b-x)^{\beta}$$

$$C = \frac{(\alpha+\beta+1)}{\alpha!\beta!(b-a)^{\alpha+\beta+1}}$$

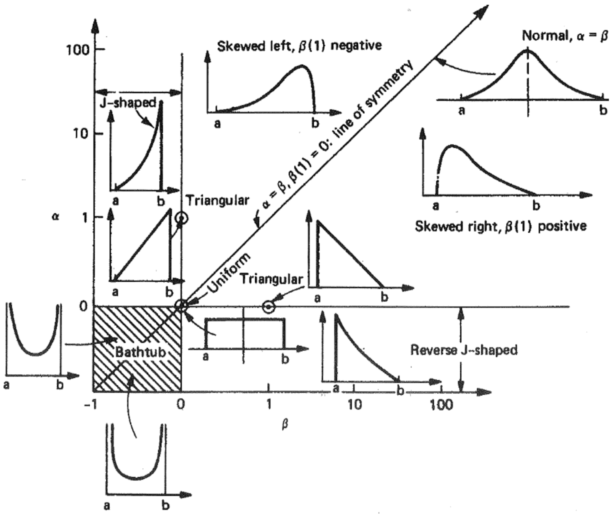


**Fig. 1** Various probability density function and optimized beta distribution parameters (Harr, 1996)

$$\alpha = \frac{X^2}{Y^2}(1-X)-(1+X) \quad \beta = \frac{\alpha+1}{X}-(\alpha+2)$$

$$X = \frac{(\bar{x}-a)}{(b-a)} \quad Y = \frac{\sigma}{(b-a)}$$

where a is the lower limit of random variable; b is the upper limit of random variable; $\bar{x}$ is the mean value of random variable ; σ is standard deviation of random variable.

First step to estimate beta distribution parameters was decision of mean and standard deviation values from the random variables. Then, X and Y could be determined based on the upper and lower limits of random soil variables. Alpha, beta and coefficient values could be determined consequently from the determined X and Y. Fig. 1 demonstrates how to model existing probability density functions with beta distribution parameters.

Point estimation method is useful to estimate the mean and variance of random outputs when each random input variable does not share a common distribution shape. This method can predict approximate mean and variance of new output values by addition or subtraction of two random variables to accommodate a correlation between the two random variables. Fig. 2 explains how to relate the correlation factors with weight factors to be multiplied with each output value at two third percentile (68% within one standard deviation) in quarter point estimate method. Multi point estimation approach greater than 4 points need to be referenced to original paper describing generalized point estimate method (Rosenblueth, 1975). However, the accuracy of point estimated values is known to deteriorate if data PDF shape is differentiated from Gaussian distribution.

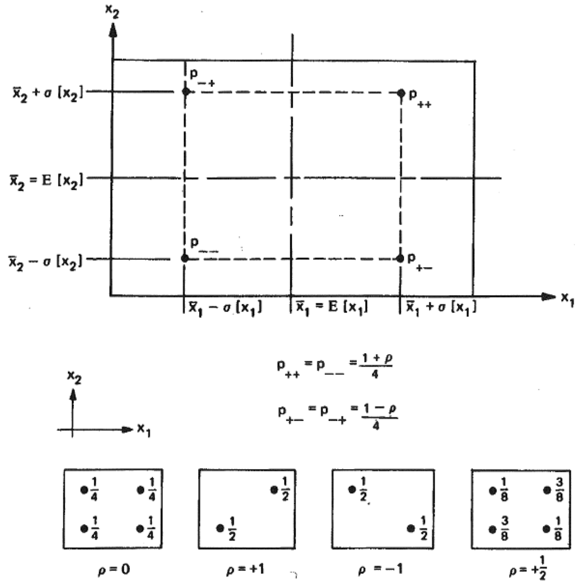


**Fig. 2** Influence of correlation factor on weight factors applied to two third (68%) percentile values of quarter point estimate method (Harr, 1996)

## 3. Example Calculations

### 3.1 Probabilistic evaluation of retaining wall external stability

In 2010, over 300 soil borings with standard penetration test (SPT) or cone penetration test (CPT) were performed for new high speed railway alignment between Madison and Milwaukee in Wisconsin. CU tri-axial tests and one-dimensional consolidation tests were also conducted to investigate shear strength and compressibility of clayey foundation soils predominant on project site. Synthetic gypsum produced from near power plant known as a cost effective alternative material for retaining wall backfill in terms of light unit weight and a large internal/reposed friction angle (up to 40 degree) along with a good drainage performance was proposed for selected wall construction sections recommended for cut and fill. Cantilever reverse T-shape wall was planned to retain the gypsum material and four external stability checks were conducted to verify feasibility of design back slope of gypsum backfill as shown in Fig. 3. Among the external stability check items, factor of safety against sliding dictated the final design of the wall dimensions due to a low friction angle of drained soil parameters for the clayey foundation soils whereas Wisconsin DOT manual (2010) specified a factor of safety against sliding should meet a minimum 1.5.

Precast T-wall footing dimension needs to accommodate lateral loads from retained gypsum material with back-slope

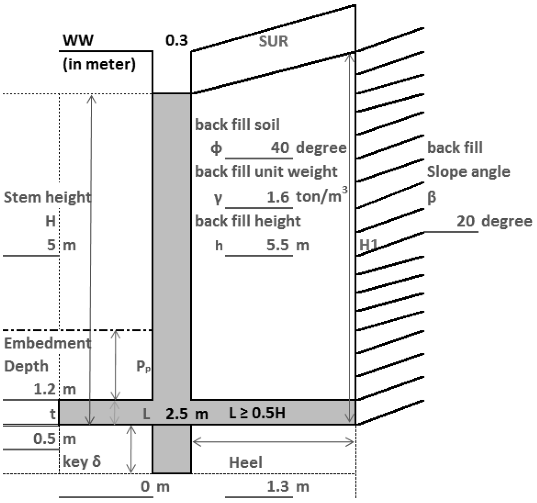



**Fig. 3** Exemplary T-wall dimension for Estimation of Factor of Safety against Sliding.

angle varied by construction section. Probabilistic analysis was necessary to find out the minimum footing width per design backfill slope angle which also meets variety of observed foundation soil condition, especially for effective internal friction angle and cohesion, related to long term stability. Sixteen (16) CU tests were performed to especially evaluate the drained strength parameters of site clayey/ organic foundation soils. The drained strength parameters of project site soils estimated form statistical analysis are summarized in Table 1. Model input parameters for beta distribution were also evaluated based on the assessed statistic parameters. Fig. 4 shows probability density functions of beta distributions on internal friction angle and cohesion used in the Monte Carlo simulation.

Stockpiling of gypsum material above existing site soils requires a strength analysis on the clayey soil against sliding. Site clayey foundation soils were found to have medium stiff to very stiff consistency, but no one could ensure the averaged drained shear strength of the native soil at locations of T-wall. Therefore, probabilistic analysis can support distributional information of sliding resistance, governed by various internal friction angle ($\phi$) and cohesion (c) values of clay material for the proposed back-slope angle of gypsum backfill. In the analysis, Monte Carlo simulation was implemented to assess distributional feature of factor of safety against sliding controlled by estimated $\phi$ and c variations with design back-slope angle of gypsum. Factor of safety (FS) against sliding was evaluated using below relationships.

$$FS = FR / LL$$
$$FR = PR(\phi, c) + BR(\phi, c)$$
$$LL = H1 + H2$$

**Table 1.** Statistical Data for Drained Soil Parameter and Estimated Beta Distribution Parameters

| Statistic Data on Lab Test Results | Internal Friction Angle, $\phi$ (degree) | Cohesion, c (kPa) |
|---|---|---|
| Minimum Value | 16 | 0 |
| Maximum Value | 26 | 4.7 |
| Mean Value | 21 | 2.4 |
| Standard Deviation | 1.0 | 0.5 |
| Estimated Beta Distribution Input Parameters | | |
| X | 0.5 | 0.5 |
| Y | 0.5 | 0.5 |
| Alpha ($\alpha$) | 11 | 11 |
| Beta ($\beta$) | 11 | 11 |

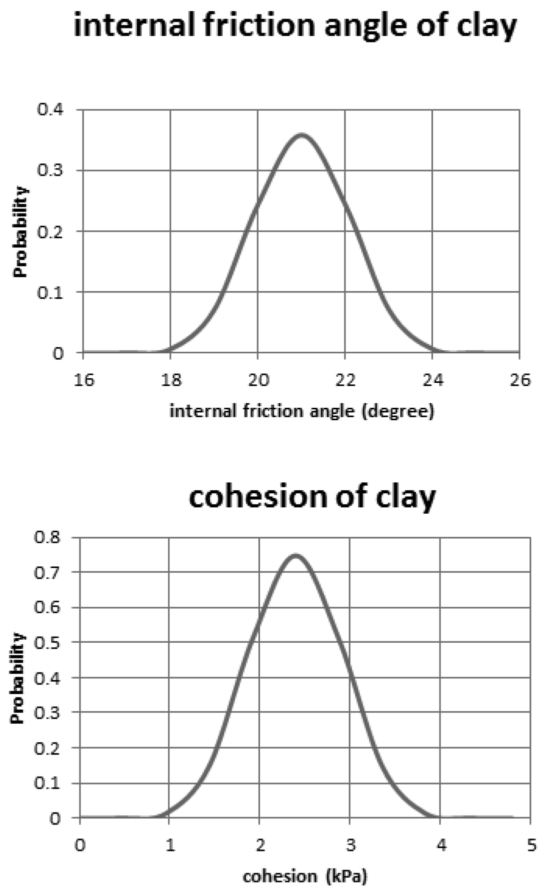



**Fig. 4** Probability density function of drained soil input parameters for monte carlo simulation

where FR: Footing Resistance against Sliding; LL: Lateral Load Acting behind Wall; PR: Passive Resistance from Embedded Front Soil Depth; BR: Base Resistance of Footing; H1:Lateral Load from Self Weight of Retained Gypsum; H2: Lateral Load from Surcharge on Top of

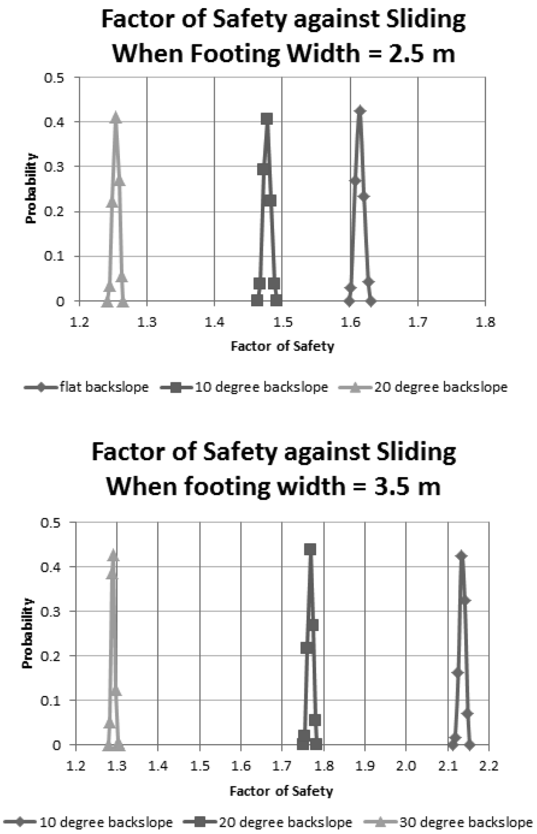



**Fig. 5** Monte carlo simulation results for factor of safety against sliding with varying back slope of retained gypsum

Retained Gypsum

In the analysis, wall height and front embedment were assumed with each 5 m and 1.2 m, and internal friction angle and unit weight of synthetic gypsum were assumed consistently with 40 degree and 1.6 tons cubic meter, respectively as quality control is conducted on the backfill during construction. Two wall footing widths of 2.5 m and 3.5 m were examined to calculate factor of safety against sliding with assumed design back-slope angles (flat, 10, 20 and 30 degree). Monte Carlo simulation was conducted with 500 iterations, and the analysis results for different back slope angle of retained gypsum are compared in Fig. 5. From the analysis, the design back slope angle is considered a key parameter to determine the wall footing width whereas variation of drained soil parameter, observed in project area did not influence much on the factor of safety against sliding. Analysis results, presented in Fig. 5 indicated a footing with of 2.5 m limits a back slope angle below 10 degree, but a footing width of 3.5 m allows a back slope angle up to 25 degree. While no seepage condition was included in this analysis, groundwater level can be considered as an additional input parameter in Monte Carlo simulation if significant seepage flow is expected behind wall.

### 3.2 Probabilistic long term settlement of organic soil

Accurate estimation of time-rate consolidation will be a big concern when a railway structure is seated on organic silt notorious for a large compression amount and extended creep deformation. Preloading fill or light weight fill placement after excavation below subgrade (EBS) can be proposed as for a remedial construction method to reduce the railroad settlement after construction. If general depth and thickness of organic soil are identified from site soil borings, long term settlement behaviors of the organic soil per each construction stage can be predicted based on the measured organic soil properties (initial void ratio, over-consolidation ratio, compression indices, creep index and coefficient of consolidation). Additional probabilistic approach was also developed to estimate range of compressible soil thickness from field settlement data if organic material is not clearly delineated from other soil layers or alternatively laminated between native soils (Lee and Masud, 2011).

In a construction portion of the planned high speed railway, a highly compressible soil, mainly consisted of normally consolidated organic silt and elastic silt was observed below silty sand layer at approximately 7 meters below the surface and an averaged thickness of organic soil was estimated to be 6 meters. Moisture and organic contents of the organic soils were generally over 100%wt. and above 15%wt., respectively. Total eight (8) 1-D consolidation tests were performed on the organic soil samples to evaluate the range of virgin compression and creep indices. The lab tests indicated the virgin compression index of organic soil ranges from 0.2 to 0.8 whereas the creep index ranges from 0.001 to 0.01.

Organic soils or peat material, showing high moisture contents exhibits no clear distinction of time period between primary and secondary consolidation during settlement. Hence, for railroad construction sites containing organic soils, creep deformation also need be accounted to predict accurate settlement amount during and after the railway construction. Governing soil parameters in determining the primary settlement amount are initial void ratio and virgin compression index of organic soil. Secondary settlement of organic soil was known mainly governed by creep compression index, ending time of primary consolidation and void ratio after the primary consolidation. The distributional feature of input parameters were optimized with a best fitting probability density function (PDF) based on one-dimensional consolidation test results on site

**Table 2** Statistical Data for Primary Settlement Soil Parameter and Estimated Beta Distribution Parameters

| Statistic Data on Lab Test Results | Compression Index (Cc) | Initial Void Ratio (e0) |
|---|---|---|
| Minimum Value | 0.2 | 1.0 |
| Maximum Value | 0.8 | 2.0 |
| Mean Value | 0.5 | 1.5 |
| Standard Deviation | 0.1 | 0.1 |
| Estimated Beta Distribution Input Parameters | | |
| X | 0.5 | 0.5 |
| Y | 0.2 | 0.1 |
| Alpha (α) | 3 | 11 |
| Beta (β) | 3 | 11 |

organic soils. Table 2 shows statistic values of the input parameters on primary settlement analysis, estimated from 1-D consolidation test results and assigned beta distribution parameters.

Primary settlement amount (S1) was probabilistically calculated using a conventional equation as shown in following.

$$S1 = \frac{\hat{C_c}H}{(1+\hat{e}_0)}Log\left(\frac{P_0+\Delta P}{P_0}\right)$$

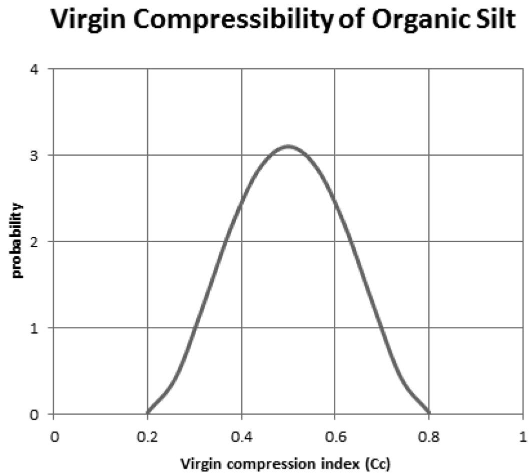
where $C_c$ and $e_0$ are random variables of site soil compression index and initial void ratio; H is organic silt layer thickness; $P_0$ is in-situ overburden pressure at middle of layer; ΔP is effective stress increase at middle of layer by load addition at surface.

Actual long term settlement calculation was conducted from 7 meters below surface and at each sub layer level, divided by 0.5 meter in thickness and summed for site averaged thickness of organic soil (6 meters). Load increase (ΔP) within organic soil layer was also estimated at a different depth increased by 0.5 meter interval using Boussinesq's solution for a strip or rectangular load.

Evaluation of secondary consolidation (creep deformation) parameters for organic soil takes extensive test period (up to 10 days per each test loading stage). Therefore, limited number of test data on creep index ($C_\alpha$), void ratio after primary consolidation ($e_p$) and coefficient of consolidation ($C_v$) was insufficient to estimate PDF of each input parameter, and the minimum values of creep index and 90% consolidation time would be hardly estimated. For the reason, those two input parameter PDFs were assumed to follow a beta distribution skewed to zero axis, similar to log-normal distribution or Poisson distribution. Table 3 summarizes the beta distribution input parameters for secondary settlement estimate (additional creep deformation after 20 years) using Monte Carlo simulation.

**Table 3** Statistical Data for Secondary Settlement Soil Parameter and Assumed Beta Distribution Parameters

| Statistic Data on Lab Test Results | Creep Index (Cα) | Void Ratio ($e_p$) after Primary Consol. | Elapsed Time ($t_{90}$) for 90% Consol. (month) |
|---|---|---|---|
| Minimum Value | 0.001 | 0.5 | 0.5 |
| Maximum Value | 0.01 | 1.1 | 10 |
| Estimated Beta Distribution Input Parameters | | | |
| Alpha (α) | 0.8 | 3 | 0.7 |
| Beta (β) | 1.2 | 3 | 8.3 |



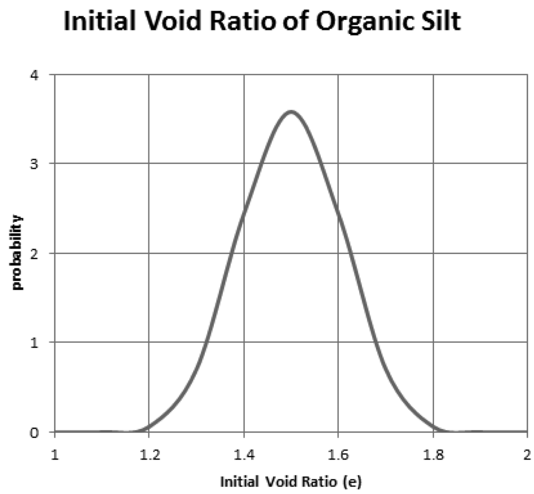




**Fig. 6** Probability density functions of input parameters for primary settlement calculation

Secondary settlement amount (S2) was probabilistically calculated using a following equation.

$$S2 = \frac{\hat{C_\alpha}}{(1+\hat{e}_P)}H\cdot Log\left(\frac{t}{\hat{t}_{90}}\right)$$

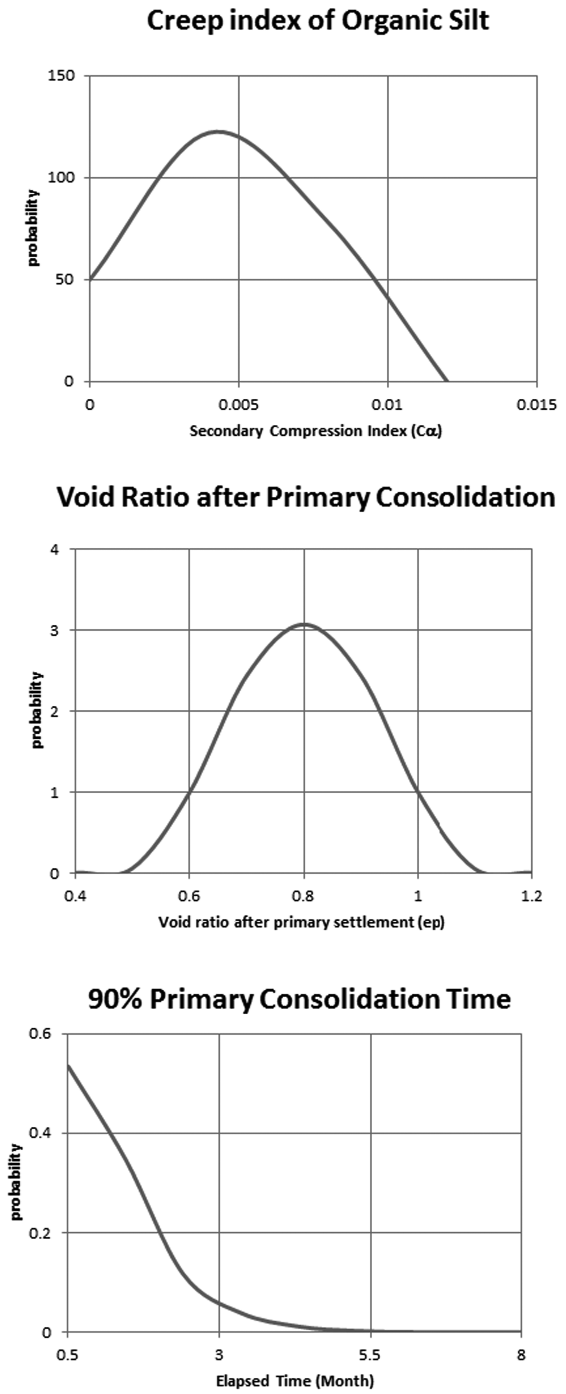



**Fig. 7** Probability density functions of input parameters for secondary settlement calculation

**Table 4** Probabilistic Settlement Estimates of Organic Soil Using Point Estimation Method

| Statistical Parameter | Primary Settlement, S1 (centimeter) | Creep Settlement, S2 (centimeter) |
|---|---|---|
| Mean, $\bar{x}$ | 14.5 | 4.3 |
| Standard Deviation, s | 2.1 | 2.5 |
| 16 Percentile | 10.9 | 2.5 |
| 84 Percentile | 17.8 | 8.9 |
| For Perfectly Correlation btw. S1 and S2 ($\rho$ = 1.0) | | |
| $\bar{x}$ (S1+S2) | 20.2 cm | |
| $\sigma$ (S1+S2) | 4.6 cm | |
| For No Correlation btw. S1 and S2 ($\rho$ = 0.0) | | |
| $\bar{x}$ (S1+S2) | 20.1 cm | |
| $\sigma$ (S1+S2) | 3.3 cm | |

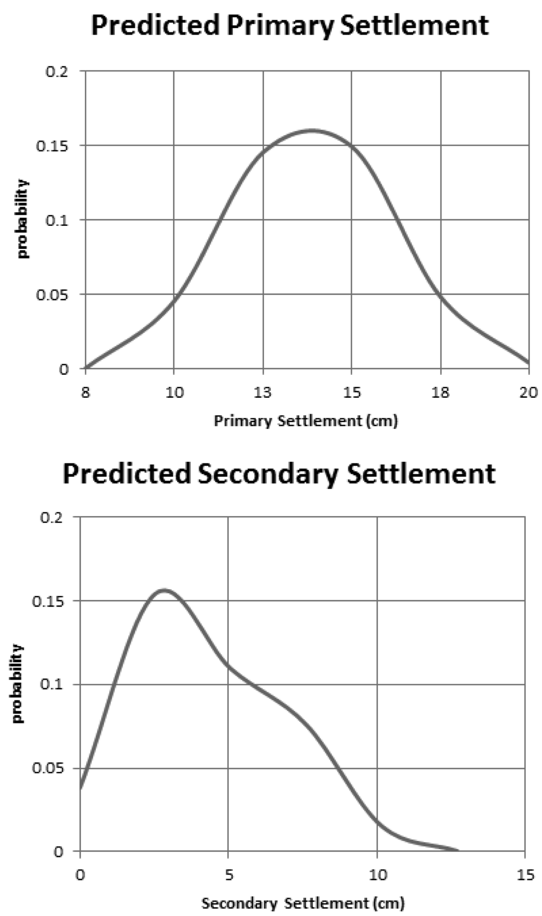



**Fig. 8** Probability density functions of primary and secondary (Creep) settlements of organic soil obtained from Monte Carlo simulation

where $\hat{C}_\alpha$, $\hat{e}_P$ and $\hat{t}_{90}$ are random variables of creep (secondary compression) index, void ratio after primary consolidation and the month taken for 90% consolidation of the organic soil, respectively; H is the estimated organic soil thickness (6 meters); t is time elapse for creep deformation estimate (e.g., 240 months for 20 years)

Figs. 6 and 7 show input data distribution shapes used for calculation of primary and secondary settlements, respectively.

Fig. 8 shows different PDF shapes of the primary and secondary settlement estimates. Thus, total settlement combining primary and secondary consolidation of organic soil can be evaluated using bivariate or quarter point estimate method provided the means and standard deviations of primary and secondary settlements are assessed as shown in Table 4.

Confidence interval of total settlement ($\hat{\chi}$) based on a correlation factor between primary settlement and creep deformation, known as 0.5 from simulation results could be suggested as following.

For confidence level of 68%, $16.2cm \leq \hat{\chi} \leq 24.2cm$

For confidence level of 95%, $14.2cm \leq \hat{\chi} \leq 28.2cm$

## 4. Conclusion

Monte Carlo simulation with optimized beta distribution input parameters must be a powerful approach able to predict probabilistic range of analysis results related to railway infrastructure construction (i.e., stability and settlement) on unpredictable soils, characterized with a non-uniform but bounded range of soil properties based on the statistical data (minimum and maximum values, mean and standard deviation of random variables) of input parameters (soil internal friction angle, cohesion, compression index, initial void ratio, creep index and coefficient of consolidation, etc.). Add-in program introduced in this research can help geotechnical and civil engineer design and produce a prediction model of target analysis output as for a risk management or construction cost estimate tool simply by connection to customized spreadsheet programs.

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