## 스테레오 카메라의 최적 위치 및 방향

# An Optimal Position and Orientation of Stereo Camera 

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## 요 약

모션 및 무인 차량의 깊이 제어를 위해 스테레오 비전 분석을 하였다. 스테레오 비전에서 3 차원 좌표의 깊 이 정보는 스테레오 이미지 사이의 포인트를 식별한 후 삼각 측량을 통해 얻을 수 있다. 그러나 항상 몇몇 이 유 때문에 삼각 측량의 오류가 발생한다. 비전 삼각 측량의 이러한 오류는 카메라의 위치와 방향에 주의하여 배열함으로써 완화 될 수 있다. 본 논문에서는 무인 차량을 위해 카메라의 최적의 위치와 방향을 결정하는 방 법을 제시하였다.


#### Abstract

A stereo vision analysis was performed for motion and depth control of unmanned vehicles. In stereo vision, the depth information in three-dimensional coordinates can be obtained by triangulation after identifying points between the stereo image. However, there are always triangulation errors due to several reasons. Such errors in the vision triangulation can be alleviated by careful arrangement of the camera position and orientation. In this paper, an approach to the determination of the optimal position and orientation of camera is presented for unmanned vehicles.


Key words : sensor system(센서 시스템), active stereo vision system(능동 스테레오 비전), stereo vision (스테레오 비전), unmanned vehicle(무인 차량)

## I. Introduction

A vision system is an important sensor for the unmanned vehicle such as the unmanned forklift for approaching destination. In order for the unmanned forklift to move with loads without an human beings, they need a number of sensors such as the gyro sensor, supersonic wave sensor, infrared rays sensor, and vision sensor. Out of the sensors, one of the most important
sensor is the vision sensor. A number of researches were performed in application of vision sensor to the unmanned forklift [1],[2]. However, the vision sensors in the research are single vision system such that it should be used with other sensors to find depth information. However, there are always triangulation errors due to several reasons and those have been studied [3]-[6].

We focus on a stereo vision system with position and orientation errors due to several reasons. In section 2, Sensitivity for the stereo vision sensor system is

[^0]explained. Also, System error equation is explained in section 3.

## II. Sensitivity for the stereo vision sensor system

The stereo vision sensor system is composed of two cameras capable of yawing and pitching motion as shown in Fig. 1. Each camera has two degree-of- freedom (DOF). The stereo camera is used to find the depth information between the unmanned vehicle and an object.

In this section, the system sensitivity equation to measure triangulation error of the stereo vision system is derived.


그림 1. 스테레오 비전 시스템 사진
Fig. 1 A picture of the stereo vision system

In the following two equations, x and y can be set as functions of $\mathrm{X}, \mathrm{Y}$, and Z by fixing the camera orientation variables, $\theta$, a constant. Denoting them in function formation, we get

$$
\begin{gather*}
x=\left(X_{r} c \theta+Y_{r} s \theta\right) / k=f(X, Y, Z)  \tag{1}\\
y=\left(-X_{r} s \theta c a Y_{r} c \theta c a Z_{r} S a\right) / k=f(X, Y, Z) \tag{2}
\end{gather*}
$$

Substituting $\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{X}_{\mathrm{r}}, \mathrm{Y}_{\mathrm{rl}}$, and $\mathrm{Z}_{\mathrm{rl}}$ into $\mathrm{x}, \mathrm{y}, \mathrm{X}_{\mathrm{r}}, \mathrm{Y}_{\mathrm{r}}$, and $Z_{\mathrm{r}}$, respectively, in order to indicate the position and orientation of camera one, yields

$$
\begin{align*}
& x_{l}=\left(X_{r l} c \theta_{l}+Y_{r l} S \theta_{l}\right) / k_{l}=f_{l l}(X, Y, Z)  \tag{3}\\
& y_{l}=\left(-X_{r I} S \theta_{l} C a_{l}+Y_{r l} C \theta_{l} C a_{l}+Z_{r l} S a_{1 l}\right) / k_{l} \\
&=f_{12}(X, Y, Z) \tag{4}
\end{align*}
$$

Substituting $\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{X}_{\mathrm{r}}, \mathrm{Y}_{\mathrm{r} 2}$, and $\mathrm{Z}_{\mathrm{r} 2}$ into $\mathrm{x}, \mathrm{y}, \mathrm{X}_{\mathrm{r}}, \mathrm{Y}_{\mathrm{r}}$, and $Z_{r}$, respectively, in order to indicate the position and orientation of camera two, yields

$$
\begin{equation*}
x_{2}=\left(X_{r 2} c \theta_{2} Y_{r 2} \theta_{2}\right) / k_{2}=f_{21}(X, Y, Z) \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
y_{2}=\left(-X_{12} s \theta_{2} c a_{2} Y_{r 2} c \theta_{2} c a_{2}+Z_{r 2} s a_{2}\right) / k_{2}=f_{22}(X, Y, Z) \tag{6}
\end{equation*}
$$

There are three ways to derive the system error equation to measure the triangulation error. These ways are dependent on which camera coordinates on the image plane are used. If only $\mathrm{xx}, \mathrm{yx}$, and $\mathrm{x}_{2}$ are used to determine the coordinates of the task, $\mathrm{X}, \mathrm{Y}$, and Z , equations (3), (4), and (5) are used. Taking partial differentiations of those equations, and expressing in matrix notation yield

$$
\left[\begin{array}{l}
\partial x_{1}  \tag{7}\\
\partial y_{1} \\
\partial x_{2}
\end{array}\right]=J_{1}\left[\begin{array}{l}
\partial X \\
\partial Y \\
\partial Z
\end{array}\right]
$$

Assuming that very small errors occur in locating a scene on both of the stereo image, equation (7) can be expressed as

$$
\left[\begin{array}{l}
\Delta x_{1}  \tag{8}\\
\Delta y_{1} \\
\Delta x_{2}
\end{array}\right]=J_{1}\left[\begin{array}{l}
\Delta X \\
\Delta Y \\
\Delta Z
\end{array}\right]
$$

where

$$
J_{1}=\left[\begin{array}{ccc}
\partial f_{12} / a X & \partial f_{11} / \partial Y & \partial f_{11} / \partial Z  \tag{9}\\
\partial f_{12} / a X & \partial f_{12} / \partial Y & \partial f_{12} / \partial Z \\
\partial f_{12} / a X & \partial f_{21} / \partial Y & \partial f_{21} / \partial Z
\end{array}\right]
$$

in which

$$
\begin{equation*}
\partial f_{I l} \partial X=c \theta_{l} k_{l}+\left(X_{r l} c \theta_{l}+Y_{r l} \theta_{l}\right)\left(s \theta_{I} s a_{l} \lambda\right) / k_{l}^{2}(1 \tag{10}
\end{equation*}
$$

$$
\begin{equation*}
\partial f_{l l} / \partial Y=S \theta_{l} / k_{l}-\left(X_{r l} C \theta_{1}+Y_{r l} S \theta_{1}\right)\left(C \theta_{I} S a_{l} / \lambda\right) / k_{l}^{2} \tag{11}
\end{equation*}
$$

$$
\begin{align*}
& \partial f_{l l} / \partial Z=\left(X_{r l} c \theta Y_{r l} S \theta_{l}\left(c a_{l} / \lambda\right) / k l_{l}^{2}\right. \\
& \partial f_{12} / \partial X=-S \theta_{l} C a_{l} / k_{l}+ \\
&\left(-X_{r l} S\right.\left.\theta_{1} c a_{l}+Y_{r l} c \theta_{l c} c a_{l}+Z_{r l} S a_{l}\right)\left(s \theta_{1} s a_{l} / \lambda\right) / k_{l}^{2} \tag{13}
\end{align*}
$$

$$
\begin{align*}
\partial f_{12} / \partial Y= & c \theta_{l} C a_{l} / k_{l} \\
& \left(-X_{r l} S \theta_{1} c a_{1}+Y_{r l} c \theta_{l} c a_{1}+Z_{r l} S a_{1}\right)\left(c \theta_{1} S a_{l} \lambda\right) / k_{l}^{2} \tag{14}
\end{align*}
$$

$$
\begin{align*}
& \partial f_{12} / \partial Z=s a_{1} / k_{l}+ \\
& \quad\left(-X_{r l} \mid \theta_{1} c a_{l}+Y_{r l} c \theta_{l} c a_{l}+Z_{r I} s a_{l}\right)\left(C a_{l} / \lambda\right) / k_{l}^{2} \tag{15}
\end{align*}
$$

$$
\begin{equation*}
\partial f_{21} \partial X=c \theta_{2} / k_{2}+\left(X_{r 2} c \theta_{2}+Y_{r 2} s \theta_{2}\right)\left(s \theta_{2 s} s a_{2} / \lambda\right) / k_{2}^{2} \tag{16}
\end{equation*}
$$

$$
\begin{equation*}
\partial f_{21} / \partial Y=s \theta_{2} / k_{2}-\left(X_{r 2} c \theta_{2}+Y_{r 2} s \theta_{2}\right)\left(c \theta_{2} S a_{2} \lambda\right) / k_{2}^{2} \tag{17}
\end{equation*}
$$

$$
\begin{equation*}
\partial f_{21} \partial \partial_{z}=\left(X_{r 2} c \theta_{2}+Y_{r 2} s \theta_{2}\right)\left(c c_{2} / \lambda\right) / k_{2}^{2} \tag{18}
\end{equation*}
$$

The measurement errors in three-dimensional coordinates due to the input errors, $\Delta x_{1}, \Delta y_{1}, \Delta x_{2}$ in locating the projection of robot task on the image plane, are then easily determined by the following equation as

$$
\left[\begin{array}{l}
\Delta x  \tag{19}\\
\Delta y \\
\Delta x
\end{array}\right]=J_{1}^{-1}\left[\begin{array}{l}
\Delta X_{1} \\
\Delta Y_{1} \\
\Delta Z_{2}
\end{array}\right]
$$

Similarly, the measurement errors in three-dimensional coordinate due to the input errors, $\Delta x_{1}, \Delta y_{1}, \Delta y_{2}$ in locating the projection of robot task, are determined by following equation.

$$
\left[\begin{array}{l}
\Delta x  \tag{20}\\
\Delta y \\
\Delta x
\end{array}\right]=J_{2}^{-1}\left[\begin{array}{l}
\Delta X_{1} \\
\Delta Y_{1} \\
\Delta Z_{2}
\end{array}\right]
$$

where

$$
J_{1}=\left[\begin{array}{ccc}
\partial f_{11} / \partial x & \partial f_{11} / \partial Y & \partial f_{11} / \partial Z  \tag{21}\\
\partial f_{12} / \partial x & \partial f_{12} / \partial Y & \partial f_{12} / \partial Z \\
\partial f_{22} / \partial x & \partial f_{22} / \partial Z & \partial f_{22} / \partial Z
\end{array}\right]
$$

in which

$$
\begin{gather*}
\partial f_{22} / \partial X=-s \theta_{2} a_{2} / k_{2}+\left(-X_{12} s \theta_{2} c a_{2}+\right. \\
\left.Y_{12} c \theta_{2} c a_{2}+Z_{r 2} s a_{2}\right)\left(s \theta_{2} S a_{2} / \lambda\right) / k_{2}^{2}  \tag{22}\\
\partial f_{22} / \partial Y=c \theta_{2} c a_{2} / k_{2}-\left(-X_{22} s \theta_{2} c a_{2}+\right. \\
\left.Y_{r 2} c \theta_{2} c a_{2}+Z_{r 2} s a_{2}\right)\left(c \theta_{2} S a_{2} / \lambda\right) / k_{2}^{2} \tag{23}
\end{gather*}
$$

$$
\begin{align*}
& \partial f_{22} / \partial Z=s a_{2} / k_{2}+\left(-X_{r 2} s \theta_{2} c a_{2}+Y_{22} c \theta_{2} c a_{2}+\right. \\
&\left.\left.Z_{r 2} s a_{2}\right)\left(c a_{2} / \lambda\right) / k_{2}^{2}\right) \tag{24}
\end{align*}
$$

Also all the $\mathrm{Ax}_{1}, \mathrm{Ay}_{1}, \mathrm{Ax}_{2}$, and $\mathrm{Ay}_{2}$ can be used to determine the measurement errors in three-dimensional coordinate as

$$
\left[\begin{array}{l}
\Delta x_{1}  \tag{25}\\
\Delta y_{1} \\
\Delta x_{2} \\
\Delta y_{2}
\end{array}\right]=J_{3}\left[\begin{array}{l}
\Delta X \\
\Delta Y \\
\Delta Z
\end{array}\right]
$$

where

$$
J_{3}=\left[\begin{array}{ccc}
\partial f_{11} / a X & \partial f_{11} / \partial Y & \partial f_{11} / \partial Z  \tag{26}\\
\partial f_{12} / a X & \partial f_{12} / \partial Y & \partial f_{12} / \partial Z \\
\partial f_{21} / a X & \partial f_{21} / \partial Y & \partial f_{21} / \partial Z \\
\partial f_{22} / a X & \partial f_{22} / \partial Y & \partial f_{22} / \partial Z
\end{array}\right]
$$

The $\mathrm{J}_{3}$ in Eq. (26) is a four by three matrix. If the matrix $\left(J_{3}{ }^{\mathrm{T}} \mathrm{J}_{3}\right)^{-1} \mathrm{~J}_{3}{ }^{\mathrm{T}}$ is applied on both sides of equation (25), we obtain

$$
\left[\begin{array}{c}
\Delta X  \tag{27}\\
\Delta Y \\
\Delta Z
\end{array}\right]=\left(J_{3}^{T} J_{3}\right)^{-1} J_{3}^{T}\left[\begin{array}{l}
\Delta x 1 \\
\Delta y 1 \\
\Delta x 2 \\
\Delta y 2
\end{array}\right]
$$

Three ways of derivation of the system sensitivity equation to measure the triangulation error have been
made so far. Equations (19), (20), and (27) are the matrices to measure the error in three-dimensional coordinate by the input error on the image planes. Similarly, the system error equation to measure the triangulation error is derived in next section

## III. System error equation

The depth information of the dimensional coordinates can be obtained based on the projected scene on the stereo image. Because of image resolution and possible computation errors in image correlation and human error in locating ascene on the camera image, a certain degree of inaccuracy in determining the correct depth information of the robot task in three- dimensional coordinate always occurs. In other words, if we assume that those input errors, $x_{1}{ }^{\prime}, y_{1}{ }^{\prime}, x_{2}{ }^{\prime}$, and $y_{2}{ }^{\prime}$ on the image planes occur, the corresponding errors $\mathrm{X}_{1}{ }^{\prime}, \mathrm{Y}_{1}{ }^{\prime}$, and $\mathrm{Z}_{1}{ }^{\prime}$ are caused in the three-dimensional coordinate. In this approach, the system error equation to measure the triangulation error is derived in this section.

The system error equation is based on equations (6) and (7). If only $\mathrm{x}_{1}, \mathrm{y}_{1}$, and $\mathrm{x}_{2}$ are used, the system error equation can be expressed as the following matrix by setting $x_{1}=x_{1}+x_{1}{ }^{\prime}, y_{1}=y_{1}+y_{1}{ }^{\prime}, x_{2}=x_{2}+x_{2}^{\prime}$, and $y_{2}$ $=y_{2}+y_{2}{ }^{\prime}$,

$$
A^{\prime}\left[\begin{array}{l}
X_{r} 1+X_{1}^{\prime}  \tag{28}\\
Y_{r} 1+Y_{1}^{\prime} \\
Z_{r} 1+Z_{1}^{\prime}
\end{array}\right]=\left[\begin{array}{l}
\lambda x x_{1} \\
\lambda x x_{1} \\
\lambda x x_{2}+P^{\prime}
\end{array}\right]
$$

where $\quad A^{\prime}=$

$$
\left[\begin{array}{c}
x x_{1} s \theta_{1} s \alpha_{1}+\lambda c \theta_{1},-x x_{1} c \theta_{1} s \alpha_{1}+\lambda s \theta_{1}, \quad x x_{1} c \alpha_{1}  \tag{29}\\
y y_{1} s \theta_{1} s \alpha_{1}-\lambda s \theta_{1} c \alpha_{1},-y y_{1} c \theta_{1} s \alpha_{1}+\lambda c \theta_{1} c \alpha_{1}, y y_{1} c \alpha_{1}+\lambda s \alpha_{1} \\
x x_{2} s \theta_{2} s \alpha_{2}+\lambda c \theta_{2},-x x_{2} c \theta_{2} s \alpha_{2}+\lambda s \theta_{2}, x x_{2} c \alpha_{2}
\end{array}\right]
$$

If only $x_{1}, y_{1}$, and $y_{2}$ are used, the system error equation can be expressed as,

$$
B^{\prime}\left[\begin{array}{l}
X_{r 1}+X_{1}^{\prime}  \tag{30}\\
Y_{r 1}+Y_{1}^{\prime} \\
Z_{r 1}+Z_{1}^{\prime}
\end{array}\right]=\left[\begin{array}{l}
\lambda x x_{1} \\
\lambda x x_{1} \\
\lambda x x_{2}+q^{\prime}
\end{array}\right]
$$

where $\mathrm{B}^{\prime}=$

$$
\left[\begin{array}{ccc}
x x_{1} s \theta_{1} s \alpha_{1}+\lambda c \theta_{1}, & -x x_{1} c \theta_{1} s \alpha_{1}+\lambda s \theta_{1}, & x x_{1} c \alpha_{1}  \tag{31}\\
y y_{1} s \theta_{1} s \alpha_{1}-\lambda s \theta_{1} c \alpha_{1},-y y_{1} c \theta_{1} s \alpha_{1}+\lambda c \theta_{1} c \alpha_{1}, y y_{1} c \alpha_{1}+\lambda s \alpha_{1} \\
y y_{2} s \theta_{2} s \alpha_{2}-\lambda s \theta_{2} c \alpha_{2},-y y_{2} c \theta_{2} s \alpha_{2}+\lambda c \theta_{2} c \alpha_{2}, y y_{2} c \alpha_{2}+\lambda s \alpha_{2}
\end{array}\right]
$$

If all the variables, $\mathrm{xx}_{1}, \mathrm{yy}_{1}, \mathrm{xx}_{2}$ and $\mathrm{yy}_{2}$ are used, the system error equation can be expressed as,

$$
C\left[\begin{array}{l}
X_{r 1}+X_{1}{ }^{\prime}  \tag{32}\\
Y_{r 1}+Y_{1}^{\prime} \\
Z_{r 1}+Z_{1}^{\prime}
\end{array}\right]=\left[\begin{array}{l}
\lambda x x_{1} \\
\lambda y y_{1} \\
\lambda x x_{2}+p^{\prime} \\
\lambda y y_{2}+q^{\prime}
\end{array}\right]
$$

where $\mathrm{C}^{\prime}=$

$$
\left[\begin{array}{lc}
x x_{1} s \theta_{1} s \alpha_{1}+\lambda c \theta_{1}, & -x x_{1} c \theta_{1} s \alpha_{1}+\lambda s \theta_{1},  \tag{33}\\
y y_{1} s \theta_{1} s \alpha_{1}-\lambda s \theta_{1} c x_{1} c,-y y_{1} c \alpha_{1} s \alpha_{1}+\lambda c \theta_{1} c \alpha_{1}, y y_{1} c \alpha_{1}+\lambda s \alpha_{1} \\
x x_{1} s \theta_{1} s \alpha_{2}+\lambda c \theta_{2}, & -x x_{2} c \theta_{2} s \alpha_{2}+\lambda s \theta_{2}, \\
y y_{2} s \theta_{2} s \alpha_{2}-\lambda s \theta_{2} c \alpha_{2},-y y_{2} c \theta_{2} s \alpha_{2}+\lambda c \theta_{2} c \alpha_{2}, y y_{2} c \alpha_{2}+\lambda s \alpha_{2}
\end{array}\right]
$$

$$
\begin{array}{r}
P^{\prime}=\left(x x_{2} s \theta_{2} S a_{2}+\lambda c \theta_{2}\right)\left(X_{o 2}-X_{o l}\right)+\left(-x x_{2} C \theta_{2} S a_{2}+\right. \\
\left.\lambda s \theta_{2}\right)\left(Y_{o 2}-Y_{o l}\right)+x x_{2} c a_{2}\left(Z_{o 2}-Z_{o l}\right) \tag{34}
\end{array}
$$

$$
\begin{array}{r}
q^{\prime}=\left(y y_{2} s \theta_{2} s a_{2}-\lambda s \theta_{2} C a_{2}\right)\left(X_{o 2}-X_{o l}\right)+\left(y y_{2} c \theta_{2} s a_{2}+\right.  \tag{35}\\
\left.X c \theta_{2} c a_{2}\right)\left(Y_{o 2}-Y_{o 1}\right)+\left(y y_{2} c a_{2}+\lambda s a_{2}\right)\left(Z_{02}-Z_{01}\right)
\end{array}
$$

The procedure of deriving the system error equation is quite similar to that of system sensitivity equation, so that only equation (32) which uses all the input variables is derived.

In equation (33), the system error can be expanded into two equations as, $\mathrm{C}^{\prime}=\mathrm{C}+\mathrm{C}_{1}$ where C is decided (32) and (37), and $C_{1}$ matrix is expressed as,

$$
C_{1}=\left[\begin{array}{lll}
x_{1}{ }^{\prime} s \theta_{1} s \alpha_{1}, & -x_{1}{ }^{\prime} c \theta_{1} s \alpha_{1}, & x_{1}{ }^{\prime} c \alpha_{1}  \tag{37}\\
y_{1}{ }^{\prime} s \theta_{1} s \alpha_{1}, & -y_{1}{ }^{\prime} c \theta_{1} s \alpha_{1}, & y_{1}{ }^{\prime} c \alpha_{1} \\
x_{2}{ }^{\prime} s \theta_{2} s \alpha_{2}, & -x_{2}{ }^{\prime} c \theta_{2} s \alpha_{2} & x_{2}{ }^{\prime} c \alpha_{2} \\
y_{2}^{\prime} s \theta_{2} s \alpha_{2}, & -y_{2}^{\prime} c \theta_{2} s \alpha_{2}, & y_{2}^{\prime} c \alpha_{2}
\end{array}\right]
$$

The matrix on the left side of the equation (32) can be expanded into the following matrices,

$$
C\left[\begin{array}{c}
X_{r 1}+X_{1}^{\prime}  \tag{38}\\
Y_{r 1}+Y_{1}^{\prime} \\
Z_{r 1}+Z_{1}^{\prime}
\end{array}\right]=C\left[\begin{array}{c}
X_{r 1} \\
Y_{r 1} \\
Z_{r 1}
\end{array}\right]+C_{1}\left[\begin{array}{c}
X_{r 1} \\
Y_{r 1} \\
Z_{r 1}
\end{array}\right]+C\left[\begin{array}{c}
X_{r 1}{ }^{\prime} \\
Y_{r 1} \\
Z_{r 1}^{\prime}
\end{array}\right]
$$

From equation (38), the following equation yields

$$
C\left[\begin{array}{l}
X_{1}{ }^{\prime}  \tag{39}\\
Y_{1}^{\prime} \\
Z_{1}^{\prime}
\end{array}\right]=-C_{1}\left[\begin{array}{l}
X_{r 1} \\
Y_{r 1} \\
Z_{r 1}
\end{array}\right]+\left[\begin{array}{l}
\lambda x_{1}{ }^{\prime} \\
\lambda y_{1}{ }^{\prime} \\
\lambda x_{2}+p^{\prime}-p \\
\lambda y_{2}+q^{\prime}-q
\end{array}\right]
$$

where

$$
\begin{array}{r}
P^{\prime}-P=X_{2}{ }^{\prime} S \theta_{2} S a_{2}\left(X_{02}-X_{01}\right)-x_{2}^{\prime} c \theta_{2} S a_{2}\left(Y_{02}-Y_{01}\right)+ \\
x_{2}^{\prime} c a_{2}\left(Z_{02}-Z_{01}\right) \tag{40}
\end{array}
$$

$$
\begin{array}{r}
q^{\prime}-q=y_{2}^{\prime} s \theta_{2} s a_{2}\left(X o_{2}-X o_{1}\right)-y_{2}^{\prime} c \theta_{2} a_{2}\left(Y o_{2}-Y o_{1}\right)+ \\
y_{2}^{\prime} c a_{2}\left(Z o_{2}-Z o_{1}\right) \tag{41}
\end{array}
$$

All the terms of equation (39) have four rows. Dividing the first, second, third, and fourth row of both sides of equation (39) by $\mathrm{x}_{1}^{\prime}$, $\mathrm{y}_{1}^{\prime}$, $\mathrm{x}_{2}{ }^{\prime}$, and $\mathrm{y}_{2}{ }^{\prime}$ respectively yields

$$
\begin{align*}
& {\left[\begin{array}{l}
c_{1}^{\prime} / x_{1}^{\prime} \\
c_{2}^{\prime} / y_{1}^{\prime} \\
c_{3}^{\prime} / x_{2}^{\prime} \\
c_{4}^{\prime} / y_{2}^{\prime}
\end{array}\right]\left[\begin{array}{l}
X_{1}^{\prime} \\
Y_{1}^{\prime} \\
Y_{1}^{\prime}
\end{array}\right]=\left[\begin{array}{l}
-s \theta_{1} s \alpha_{1,} c \theta_{1} s \alpha_{1,}, \\
-s \theta_{1} s \alpha_{1,} \\
-s \theta_{1}, c \theta_{1} s \alpha_{1,}, \\
-s \theta_{1} s \alpha_{1,}, c \theta_{1} s \alpha_{1,}, \\
-c \alpha_{1,},
\end{array}\left[\begin{array}{c}
X_{r 1} \\
Y_{r 1} \\
Y_{1} s \alpha_{1,} c \theta_{1} s \alpha_{1,} \\
Z_{r 1}
\end{array}\right]\right.} \\
& +\left[\begin{array}{l}
\lambda \\
\lambda \\
\lambda+\left(P_{1}^{\prime}-P\right) / Y_{2}^{\prime} \\
\lambda+\left(q^{\prime}-q\right) / Y_{2}^{\prime}
\end{array}\right] \tag{42}
\end{align*}
$$

where we set the matrix $\mathrm{C}^{\prime}$ as

$$
C^{\prime}=\left[\begin{array}{l}
C_{1}^{\prime}  \tag{43}\\
C_{2}^{\prime} \\
C_{3}^{\prime} \\
C_{4}^{\prime}
\end{array}\right]
$$

We set $\mathrm{k}^{\prime}$ as

$$
\begin{equation*}
k^{\prime}=\lambda k \tag{44}
\end{equation*}
$$

Substituting $\theta_{1}, a_{1} X_{01}, Y_{01}$, and $Z_{01}$ into $\theta, a, X_{0}, Y_{0}$, and $Z_{0}$, respectively, to indicate camera one coordinate, yields

$$
\begin{equation*}
k_{l^{\prime}}=\lambda k_{1}=-s \theta_{1} S a_{l} X_{r l}+c \theta_{1} s a_{l} Y_{r l}-c a Z_{r l}+\lambda \tag{45}
\end{equation*}
$$

Substituting $\theta_{2}, a_{2}, X_{02}, Y_{02}$, and $Z_{02}$ into $\theta, a_{1} X_{c}, Y_{c}$, and $Z_{0}$, respectively, to indicate the camera two coordinate, yields

$$
\begin{equation*}
k_{2}^{\prime}=\lambda k_{2}=-s \theta_{2} S a_{2} X r_{2}+c \theta_{2} S a_{2} Y_{r_{2}}-c a_{2} Z_{r 2}+\lambda \tag{46}
\end{equation*}
$$

Rearranging equation (42) yields

$$
\left[\begin{array}{l}
C_{1}^{\prime} / x_{1}^{\prime}  \tag{47}\\
C_{2}^{\prime} / y_{1}^{\prime} \\
C_{3}^{\prime} / x_{2}^{\prime} \\
C_{4}^{\prime} / y_{2}^{\prime}
\end{array}\right]\left[\begin{array}{c}
X_{1}^{\prime} \\
Y_{1}^{\prime} \\
Z_{1}^{\prime}
\end{array}\right]=\left[\begin{array}{c}
k_{1}^{\prime} \\
k_{1}^{\prime} \\
k_{2}^{\prime}+\left(p^{\prime}-p\right) / x_{2}^{\prime} \\
k_{2}^{\prime}+\left(q^{\prime}-q\right) / y_{2}^{\prime}
\end{array}\right]
$$

If we set

$$
D=\left[\begin{array}{l}
C_{1}^{\prime} / x_{1}{ }_{1}^{\prime}  \tag{48}\\
C_{2}^{\prime} / y_{1}^{\prime} \\
C_{3}^{\prime} / x_{2}^{\prime} \\
C_{4}^{\prime} / y_{2}^{\prime}
\end{array}\right]
$$

the $D$ matrix is a four by three matrix. In order to get the errors, $X_{1}{ }^{\prime}, Y_{1}{ }^{\prime}$, and $\mathrm{Z}_{1}$ of depth informations in the three-dimensional coordinate affected by the input errors, $\mathrm{X}_{\mathrm{l}}{ }^{\prime}, \mathrm{Y}_{\mathrm{l}}{ }^{\prime}$ and $\mathrm{Z}_{1}{ }^{\prime}$ on the image plane in the camera coordinates, the least squares method should be applied. The least squares method is applied by pre- multiplying on each side of the above equation by $\left(\mathrm{D}^{\mathrm{T}} \mathrm{D}\right)^{-1} \mathrm{D}^{\mathrm{T}}$, which
yields

$$
\left[\begin{array}{c}
X_{1}^{\prime}  \tag{49}\\
Y_{1}^{\prime} \\
Z_{1}^{\prime}
\end{array}\right]=\left(D^{T} D\right)^{-1} D^{T}\left[\begin{array}{c}
k_{1}^{\prime} 1 \\
k_{1}^{\prime} 1 \\
k_{2^{\prime}}+\left(p^{\prime}-p\right) / x_{2}{ }^{\prime} \\
k_{2}^{\prime}+\left(q^{\prime}-q\right) / y_{2}^{\prime}
\end{array}\right]
$$

The purpose of the optimal arrangement of position and orientation of stereo camera is to decide the best stereo camera arrangements to produce the least triangulation error as mentioned before. The depth error $\mathrm{X}_{1}{ }^{\prime}, \mathrm{Y}_{1}^{\prime}$, and $\mathrm{Z}_{1}{ }^{\prime}$ in three-dimensional coordinates is affected by the variation of the ten explicit variables, Xrl, $Y_{\mathrm{r}}, Z_{\mathrm{r}}, X_{\mathrm{r} 2}, Y_{\mathrm{r} 2}, Z_{\mathrm{r} 2}, x_{1}, y_{1}, x_{2}, y_{2}$, and four implicit variables, $a_{1}, \theta_{1}, a_{2}$, and $\theta_{2}$. Here the aiming angles of the stereo camera to the task and the mapped position of robot task on the camera image plane are dependent on each other. In other words, $\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{x}_{2}, \mathrm{y}_{2}$ can be treated as implicit variables and at the same time $\alpha_{1}, \theta_{1}, \alpha_{2}$ and $\theta_{2}$ can be set as explicit variables.

In order to solve the optimization problem, the limits of the image plan should be set within the image plane size, because the image of task can not be mapped out of the image plane. The smallest absolute value of the error, $X_{1}{ }^{\prime}, Y_{1}{ }^{\prime}$, and $Z_{1}{ }^{\prime}$ in three-dimensional coordinate in equation (49) should be obtained. If the absolute value of the error is set ERR, the expression is

$$
\begin{equation*}
E R R=\operatorname{sqrt}\left(X_{1}{ }^{\prime 2}+Y_{1}^{\prime 2}+Z_{1}^{\prime 2}\right) \tag{50}
\end{equation*}
$$

In order to find the minimum value of the equation (50), a program based on the "complex" method of M. J. Box is used [6]. The program finds the minimum of a multi variable, nonlinear function subject to nonlinear inequality constraints.

## IV. Conclusion

A stereo vision analysis was performed for motion and depth control of unmanned vehicles. In this paper,
a system error equation for the stereo vision is derived. Also, an approach to the determination of the optimal position and orientation of camera which produce the least triangulation error equation is presented.

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