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Asymptotical Shock Wave Model for Acceleration Flow

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요 약

충격파모형은 교통류에서 운동학적 파동이 전파되는 속도이며, Lighthill과 Whitham(L-W)에 의해 처음 제시되 이래 지금까지 많은 교통류 문제에 적용되어지고 있다. 최근의 한 논문은 실재상황에서 발생되지 않는 충격파가 L-W모형에서 예측되는 모순을 지적하였고, 이러한 모순이 발생되는 원인과 이를 해소하는 새로운 점진적충격파모형을 제시한 바 있다. 그러나 이 모형은 교통류 흐름 중 감속하는 교통류에 대해 한정하여 유도 되었으며 반대상황 즉 가속하는 교통류에 대한 모형은 아직 제시되지 못하고 있다. 본 연구에서는 가속 교통류에 대한 점진적 충격파모형을 유도하고 이를 검증하고자 한다. 이를 위해 가속상태의 교통류에서 추종차량의 가속에 따른 차량간의 간격이 Greenshield의 모형을 충실히 따르도록 한정하고 이를 바탕으로 충격파모형을 유도하였다. 그 결과 본 연구에서 제시된 모형은 L-W모형의 모순이 해소됨을 확인하였고, 사례교통량을 적용해 기존모형들과의 결과 차이를 정량적으로 확인하였다. 한편 모형간의 차이가 분명하고도 구조적인 것을 확인하였고 이에 대한 추가적인 향후 연구의 필요성을 제시하였다.

Abstract

Shock wave model describes the propagation speed of kinematic waves in traffic flow. It was first presented by Lighthill and Whitham and has been deployed to solve many traffic problems. A recent paper pointed out that there are some traffic situations in which shock waves are not observable in the field, whereas the model predicts the existence of waves. The paper attempted to identify how such a counterintuitive conclusion results from the L-W model, and resolved the problem by deriving a new asymptotical shock wave model. Although the asymptotical model successfully eliminated the paradox of the L-W model, the validation of the new model is confined within the realm of the deceleration flow situation since the model was derived under such constraint. The purpose of this paper is to derive the remaining counter asymptotical shock wave model for acceleration traffic flow. For this, the vehicle trajectories in a time-space diagram modified to accommodate the continuously increased speed at every instant in such a way that the relationship between the spacing from the preceding vehicle and the speed of the following vehicle strictly follows Greenshield's model. To verify the validity of the suggested model, it was initially implemented to a constant flow where no shock wave exists, and the results showed that there exists no imaginary shock wave in a homogeneous flow. Numerical applications of the new model showed that the shock wave speeds of the asymptotical model for the acceleration flow tend to lean far toward the forward direction consistently. This means that the asymptotical models performs in a systematically different way for acceleration and for declaration flows. Since the output difference among the models is so distinct and systematic, further study on identifying which model is more applicable to an empirical site is recommended.

Key words : Shock wave, traffic flow theory, mathematical modeling, asymptotical model, model symmetry

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Ⅰ. Introduction

1. Background Information

Shock wave model was introduced five decades ago by Lighthill and Whitham [1]. It has been deployed to solve many traffic problems including the traffic behaviors at signalized intersection[2] and recurrent and no recurrent queuing on a highway. There have been many attempts to modify or transform the Lighthill-Whitham's shock wave model (L-W model) such as the works of Daganzo [3, 4], Newell [5, 6], Zhang [7], and Michalopoulos et al. [8]. Despite the efforts of many researchers, L-W's model has remained unchanged until recently.

A recent paper pointed out that the L-W model is self-contradict in a specific traffic condition [9]. According to the L-W model, there are some traffic situations in which shock waves are not observable in the field, whereas the model predicts the existence of waves [1, 9]. An example is the shock wave in a homogeneous speed condition. Lighthill and Whitham referred to this wave as unobservable; that is, analogous to a radio wave that cannot be seen. Gerlough and Huber [10] also described this wave as imaginary, but useful as an analytical tool.

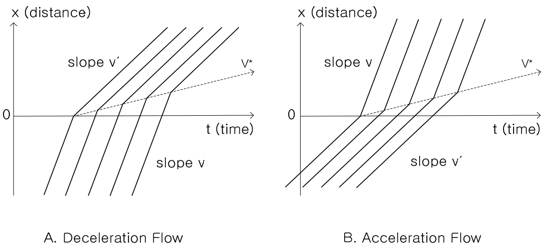
Cho [9] suggested that there is no logical reason why this particular wave is unobservable or imaginary while all other waves are observable in the field. He attempted to resolve the problem by deriving a new asymptotical shock wave model. Although Cho's revised model successfully eliminated the paradox of the L-W model, the validation of the new model is confined within the realm of the deceleration flow situation since the model was derived under such constraint[9, 11]. He called for the derivation of the counterpart model for the acceleration flow situation but it has not been published yet.

2. Problem statement and study purpose

Shock wave model describes the propagation speed of kinematic waves in traffic flow. It was first presented by Lighthill and Whitham and virtually all the contemporary traffic engineering textbooks [2, 10, 12, 13] described it as Eq. (1):

$$V^* = \frac{q_2 - q_1}{k_2 - k_1} \tag{1}$$

where q_i and k_i are the flow rate and density of region $\boldsymbol{i}$, respectively. In Eq.(1), simultaneous switching the positions of $\boldsymbol{i}$ ($\boldsymbol{i}$=1, 2) respectively does not make any difference to the shock wave speed. This means that Eq.(1) is symmetric that the shock wave speeds of deceleration flow and acceleration flow are identical. Fig. 1 illustrates such symmetry of the L-W model.



〈Fig. 1〉 The symmetry of L-W model

In Fig. 1-A, vehicles decelerate speed from **v** to **v′** (**v** > **v′**) which forms a shock wave propagating at a speed of $\mathbf{v}^*$. In Fig 1-B, vehicles accelerate speed from **v′** to **v** which also forms an identical shock wave speed of $\mathbf{v}^*$. Such nature of symmetry of L-W model is self-evident in Eq.(1) since the simultaneous change of the locations of q_i and k_i ($\boldsymbol{i}$ = 1, 2) makes no difference for all q_i and k_i of the given highway.

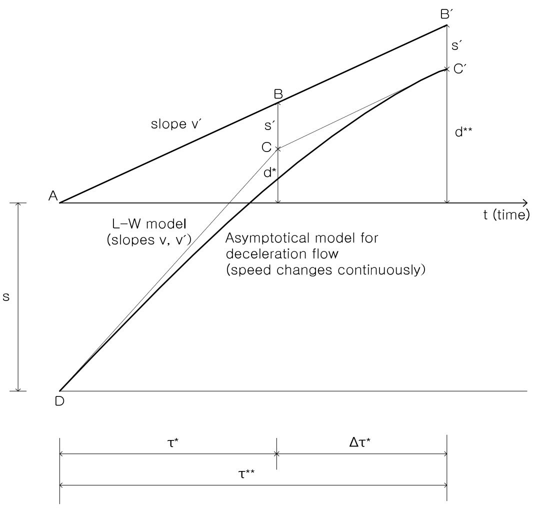
On the contrary, in the current asymptotical shock wave model, which was derived only for the deceleration flow, such symmetry as in L-W model is not necessarily guaranteed [9, 11].

$$v^{**} = \frac{v_f}{k_j}\left\{k_j - k' - \frac{k'}{\frac{k'-k}{k} + \ln\frac{k'-k}{(\alpha-1)k}}\right\} \tag{2}$$

In Eq. 2, simultaneous change of the locations of **k** and **k**′ will not yield the same shock wave speed $\mathbf{v}^{**}$ as long as $\mathbf{k} \neq \mathbf{k}'$. Such dissymmetry of Eq. 2 was anticipated since it was derived for the deceleration flows only. Cho [9] left the derivation of the asymptotical shock wave model for acceleration flow for future research. The purpose of this paper is to derive the remaining asymptotical shock wave model for acceleration traffic flow and to test the suggested model.

Ⅱ. Review of the Asymptotical Shock Wave Model for Deceleration Flow

Before the derivation of the asymptotical shock wave model for the acceleration flow, this section briefly reviews the graphical derivation procedure of Cho's current shock wave model, which is dedicated for the deceleration flow. The review is focused on the continuously changing trajectory of the following vehicle which distinguishes the model with that of Lighthill and Whitham.



〈Fig. 2〉 Trajectories of L-Ws linear model and asymptotical model for deceleration

While deriving the L-W model, it was assumed that a driver traveling along a highway at a constant speed **v** suddenly changes speed to **v**′ and maintains this speed for an arbitrarily long time [9]. A following driver may increase or decrease his/her speed in some manner but, if unable to pass, will also adjust to the new speed **v**′. In L-W model, the details of the transition trajectory were disregarded and it extrapolated the trajectory at a speed **v** and **v**′ until the two asymptotes intersect as shown by the two articulated dashed lines in Figure 2.

On the contrary, Cho's approach intended to eliminate the distortion in relationships among flow-density-speed from L-W model derivation procedure by modifying the vehicle trajectories in a time-space diagram to accommodate the changing speed at every moment as the spacing changes from **s** to **s**′. In other words, when a preceding vehicle changes speed from **v** to **v**′, the following vehicle continuously changes speed in the way that the relationship between the spacing of the vehicles and the speed of the following vehicle strictly follow the presumed relationships of the Greenshield's model. The resulted was the asymptotical curve trajectory in Figure 2. As a result, the time required for the following vehicle to change its spacing from **s** to **s**′ is different in each model, i.e., one is τ^{*} and the other is $\tau^{**}(=\tau^{*}+\triangle^{*})$. It should be noted here that, in case of the asymptotical model, the speed decelerates continuously as the spacing changes from **s** to **s**′ and, therefore, the relationships among flow-density-speed are satisfied and the modeling distortion of the L-W model is eliminated [9].

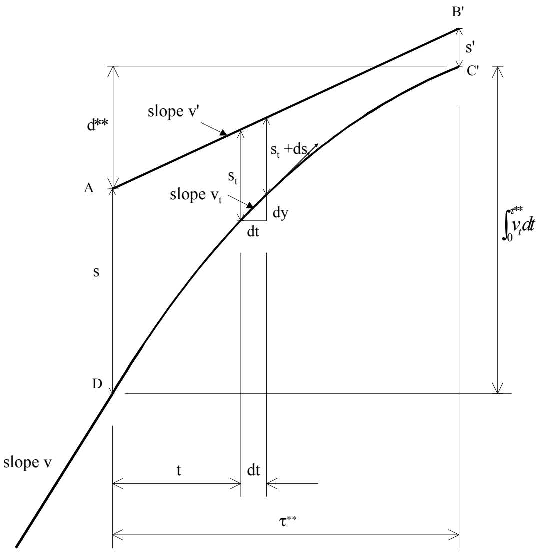
From this revised trajectory, asymptotical shock wave speed for the deceleration flow was derived. Considering the geometric conditions of Figure 3, Cho formulated following equation;

$$s' = s + v'\tau^{**} - \int_0^{\tau^{**}} v_t \, dt \tag{3}$$

where τ^{**} is the time for a wave to propagate from one car to the next. Equation (2) was the final revised shock wave speed for the deceleration flow in Cho's asymptotical model [9].

$$v^{**} = \frac{v_f}{k_j}\left\{k_j - k' - \frac{k'}{\frac{k'-k}{k} + \ln\frac{k'-k}{(\alpha-1)k}}\right\} \qquad (2)$$

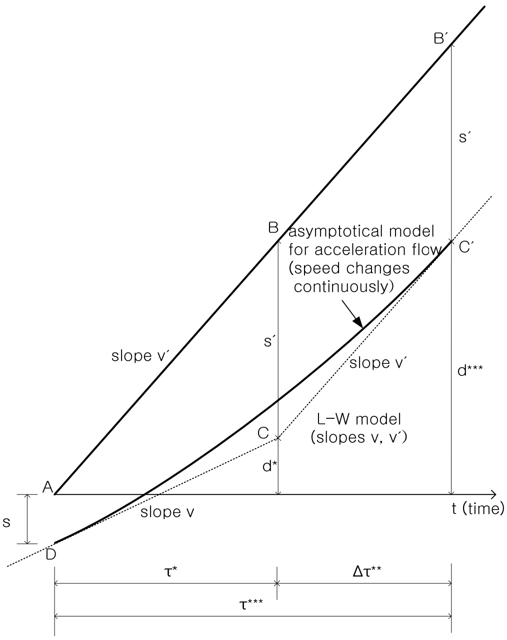
In Equation (2), $\mathbf{v}^{**}$ is the modified shock wave speed in a deceleration flow, k_j and $\frac{v_f}{k_j}$ are constants under given highway conditions, and α is a model parameter that can be empirically decided for any subject highway[9].



〈Fig. 3〉 Curved trajectory for the asymptotical model for deceleration Flow

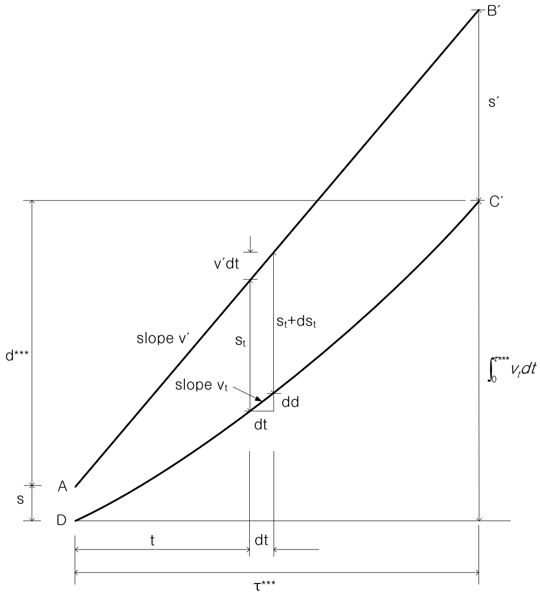
Ⅲ. Derivation of Asymptotical Shock Wave Model for Acceleration Flow

This section attempted to derive the remaining half of the existing asymptotical shock wave model: model for acceleration flow. This approach also intends to eliminate the distortion in relationships among flow-density-speed from L-W model derivation procedure by modifying the vehicle trajectories in a time-space diagram to accommodate the changed speed at every instant, as the spacing changes from **s** to **s′**. That is, when a preceding vehicle changes speed from **v** to **v′** (**v** < **v′**), the following vehicle continuously accelerates speed in such a way that the relationship between the spacing of the vehicles and the speed of the following vehicle strictly follow Greenshield's model. Figure 4 compares the trajectories of L-W model and the suggested model.



〈Fig. 4〉 Comparison of trajectories: asymptotical and L-W models for acceleration flow

In Figure 4, the trajectory of the preceding vehicle is the same in both models. However, the trajectories of the following vehicle in L-W's model and the suggested model differs; one is piecewise linear lines, while the other is a monotonic concave curve. The curve is asymptotical to the two dashed lines in the figure which represent the trajectories of the following vehicle of L-W model, whereas the solid curve above it represents the trajectory in the suggested model. Thus, the time required for the following vehicle to change its spacing from **s** to **s′** is different in each model, i.e., one is τ^{*} and the other is τ^{***}. It should be noted here that, in the case of the suggested approach, the speed changes continuously as the spacing changes from **s** to **s′** and, therefore, the relationships among flow-density-speed satisfy the presumed relationships to eliminate the modeling distortion of L-W approach.



〈Fig. 5〉 Curved Trajectory : Asymptotical Model for Acceleration Flow

Derivation of the new shock wave speed for acceleration flow utilize this revised trajectory. Let us consider a time instance *t* and a small time segment **dt** between **t** = 0 and **t** = τ^{***}. The spacing and the speed of the following vehicle at time **t** (0<**t**< τ^{***}) are depicted by $\mathbf{s_t}$ and $\mathbf{v_t}$, respectively. Geometric condition in Figure 5 validates the following;

$$s' = s + v'\tau^{***} - \int_0^{\tau^{***}} v_t dt \tag{4}$$

We get k = k_j (1-v/v_f) from Greenshield's model and k_t = 1/s_t, thus v_t can be represented as

$$v_t = v_f\left(1 - \tfrac{1}{k_j s_t}\right) \tag{5}$$

Substituting $\mathbf{v_t}$ in Equation(4) with Equation (5) becomes

$$s' = s + v'\tau^{***} - v_f \int_0^{\tau^{***}} \left(1 - \tfrac{1}{k_j s_t}\right) dt \tag{6}$$

or

$$s' = s + v'\tau^{***} - v_f \tau^{***} + \tfrac{v_f}{k_j} \int_0^{\tau^{***}} \frac{1}{s_t} dt \tag{7}$$

Now, let $\mathbf{ds_t}$ and **dd** represent the change of spacing between the two consecutive vehicles and the change of spatial location of the following vehicle during a small time period **dt**, respectively. From the geometric conditions of Figure 5, the Equation [8] is obtained:

$$\begin{aligned} s_t + v'\,dt &= s_t + ds_t + dd \\ &= s_t + ds_t + v_t dt \\ \therefore v'\,dt - v_t dt &= ds_t \\ (v' - v_t)dt &= ds_t \end{aligned}$$

Therefore,

$$dt = \frac{ds_t}{v' - v_t}, \quad v_t \neq v' \quad . \tag{8}$$

By substituting **dt** in Equation(7) with Equation (8) and changing the integration range with spacing terms,

$$s' = s + v'\tau^{***} - v_f \tau^{***} + \frac{v_f}{k_j} \int_s^{s'} \frac{1}{s_t(v' - v_t)} ds_t \quad . \tag{9}$$

In Equations (8) and (9), it should be noted that $v_t \neq v'$ to prevent zero denominators and, therefore $s_t \neq s'$. This means that the final term of Equation (9) cannot be integrated as it is, since the upper integration boundary is **s′**, which makes the denominator 0. Cho [9] also confronted the same computational problem during the derivation of the model for deceleration.

The problem is depicted [9] as "...this problem stemmed from the modified assumption that the time-space trajectory of the following vehicle strictly satisfies the presumed flow-density-speed relationships. In figure, the slope of the trajectory of the modified model constantly changes as the spacing decreases. Thus, although $\mathbf{v}_t$ approaches $\mathbf{v}'$, it can never merge to $\mathbf{v}'$, as long as the volume-speed relationship is strictly satisfied. Since drivers cannot achieve such extremely precise spacing and speed adjustment, it is necessary to approximate the upper bound of the integration range. The assumption to accommodate this problem is as follows:

...the following vehicle stops spacing and speed adjustment when the speed sufficiently approaches $\mathbf{v}'$."

The same relaxed assumption is adopted here which enables to substitute $\mathbf{s}'$ in Equation (9) with $\beta\mathbf{s}'$, where β is a number slightly smaller than 1.0 in the case where $\mathbf{v} < \mathbf{v}'$ and $\mathbf{s} < \mathbf{s}'$ (acceleration flow). Substituting $\beta\mathbf{s}'$ for $\mathbf{s}'$ in Equation (9) gives Equation (10):

$$s' = s + v'\tau^{***} - v_f\tau^{***} + \frac{v_f}{k_j}\int_s^{\beta s'} \frac{1}{s_t(v' - v_t)} ds_t. \quad (10)$$

Except the parameter β instead of α, Equation (10) is identical to the counter equation of the deceleration flow model. Thus the remaining computation process is exactly same to that of the deceleration flow model, and we get:

$$v^{***} = \frac{v_f}{k_j}\left\{k_j - k' - \frac{k'}{\frac{k'-k}{k} + \ln\frac{k'-k}{(\beta-1)k}}\right\} \quad (11)$$

Equation (11) is the revised shock wave speed for the acceleration flow. In Equation (11), $\mathbf{v}^{***}$ is the shock wave speed of the asymptotical model for acceleration flow, where k_j and $\frac{v_f}{k_j}$ are constants under given highway conditions, and β can be empirically determined from field observation.

Ⅲ. Model validation and numerical test

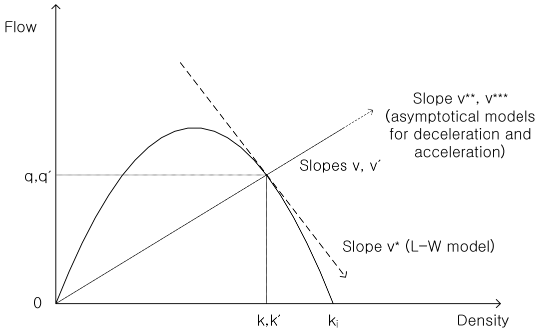
1. Shock wave in a homogeneous flow

In his previous work, Cho [9] demonstrated with the deceleration shock wave model that the shock wave speed in a homogeneous traffic stream is always identical to the ambient vehicle speed. Theoretically, the asymptotical model for the acceleration flow, Eq. (11), should yield same result. To verify the validity of the model for acceleration flow, we computed the shock wave speed in a homogeneous stream in general terms. In a homogeneous traffic stream, $\mathbf{v}$ and $\mathbf{v}'$, and $\mathbf{s}$ and $\mathbf{s}'$ are all identical and constant, respectively. The wave velocity $\mathbf{v}^{***}$ in such case can be represented by letting k' approach to k in Eq. (11) as follows:

$$\lim_{k'\to k}(v^{***}) = \lim_{k'\to k}\left[\frac{v_f}{k_j}\left\{k_j - k' - \frac{k'}{\frac{k'-k}{k} + \ln\frac{k'-k}{(\beta-1)k}}\right\}\right]$$

$$= \frac{v_f}{k_j}\left\{k_j - k - \frac{k}{-\infty}\right\} = v \quad (12)$$

Equation (12) shows that, with the model for acceleration flow, the shock wave speed in a homogeneous traffic stream is always identical to the ambient speed, or equivalently, $\mathbf{v} = \mathbf{v}' = \mathbf{v}^{***}$. Figure 6 graphically represented the same result where the slopes of the asymptotical models are all the same, whereas the shock wave speed $\mathbf{v}^*$ of L-W model differs from the ambient traffic speed ($\mathbf{v}$ or $\mathbf{v}'$). Thus the paradox of L-W model is also resolved for the acceleration flow case.



〈Fig. 6〉 Shock wave speeds in homogeneous flow

2. Numerical model test

(1) Test data

To test the symmetry of the asymptotical shock wave model for acceleration flow, a set of numerical values are deployed into the model. For the convenient and consistent comparison and data accessibility, the numerical values are cited from one of the contemporary traffic engineering textbook written by Garber and Hoel [13]. The same data were also cited by Cho [11] for the tests of the asymptotical model for deceleration. For this reason, the distance scale uses mile (mi) instead of kilometer (km). The given numerical traffic values are as follows:

Saturation flow rate (q_{max}) : 2000 veh/hr/ln

Jam density (k_j) : 150 veh/ln/mi

From Greenshield's speed-density relationship and equation of $q=k \bullet v$, the free flow speed (v_f) is determined as follows:

Free flow speed (v_f) : 53.3mph

The Greenshield's speed-density relationship is also used for this numerical test since it was used by the two previous asymptotical model papers [9, 11]. Since the suggested asymptotical model in this paper was applicable only to the acceleration flow conditions, the speed before change (**v**) is smaller than that of after the change (**v′**). The default value of β for this model tests is 0.0095, which means that the following driver terminates his/her vehicle spacing adjustment with 0.05 percent margin. Other β values may be deployed but previous study result [10] indicates that the closer proximity to 1.0 enhances the model stability thus 0.0095, instead of 0.95 and 0.095, was deployed in this test.

(2) Model test 1: symmetry comparison

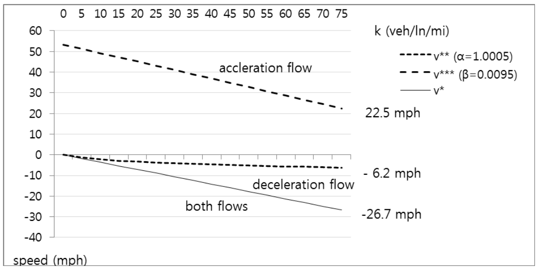
One of the most frequent observations of shock waves occurs at a congested urban intersection when the signal changes from a green light to a red light (deceleration) or when a red light changes to green (acceleration). In L-W model, as long as the approaching speed is identical to that of departure, the shockwave speed for deceleration is also identical to that of acceleration (Table 1). Deployment of actual numerical values to the models clearly distinguishes the outputs of each model. In Table 1. The shock wave speeds of L-W model for deceleration and acceleration are the same - 26.7 mph (backward). On the contrary, the shock wave speed of the asymptotical model for deceleration is - 13.3 mph (backward) while that of the acceleration is + 11.8 mph (forward). This example clearly shows the asymmetric nature of the asymptotical model.

〈Table 1〉 Symmetry and asymmetry examples of the models

Model (Equation Number)	v	v′	Shock wave speed
L-W model (1)	53.3 mph	0	v* = -27.6 mph (backward)
	0	53.3 mph	v* = -27.6 mph (backward)
Asymptotical model for deceleration (4)	53.3 mph	0	v** = -13.3 mph (backward)
Asymptotical model for acceleration (12)	0	53.3 mph	v*** = 11.8 mph (forward)

(3) Model test 2: acceleration from stop

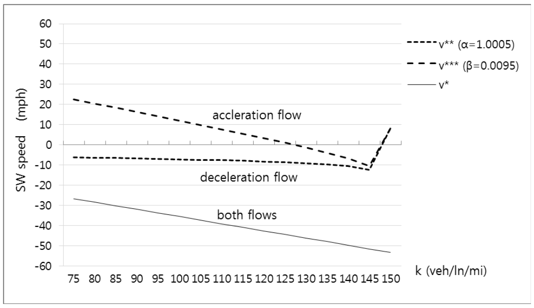
For a detailed illustration of the shock wave comparison, two identical numerical tests were conducted with different density ranges. Figure 7 compares the shock waves formed when the flow condition changed from stable to no flow (stop); thus, the initial density k ranged from 0 (free flow condition) to 75 veh/ln/mi with an increment of 5 while all terminal density is $\mathbf{k_j}$ (=**k**'). Figure 8 is the remaining half plotted for the initial density k from 75 with an increment of 5 up to 150 veh/ln/mi (jam density). L-W model was deployed twice; one for deceleration (**v**>0 and **v**′=0) and the other for acceleration flow (**v**=0 and **v**′>0). The deceleration and acceleration asymptotical models were deployed separately in accordance with the flow scenarios, i.e., deceleration model for v>0, and acceleration model for **v**′=0, and **v**=0 and **v**′>0).



〈Fig. 7〉 Comparison of shock waves at signalized intersection: stable flow to stop and stop to stable flow.

In Figure 7, the shock waves of the L-W model change linearly toward the backward direction as the initial density **k** increases for both deceleration and acceleration flows. For the deceleration flow condition, the shock wave speed of the asymptotical model is smaller than that given by the L-W model as was described by the previous work by Cho [9]. The shock wave speed of the asymptotical model for the acceleration flow, however, tends to lean far toward the forward direction consistently and systematically. This means that the performance difference between L-W model and asymptotical model is more distinctive in a acceleration flow than in the deceleration flow.

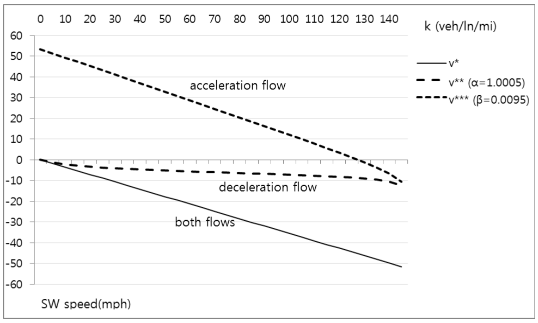
This comparison indicates that the two models perform in a very different way but we have little idea that one model performs better than the other. Since the output difference between the two models is distinct and systematic, further tests and inspection should follow to reveal which model is more applicable than the other. At this moment, however, it should be noted that L-W model is much stiffer since it ignores the speed spacing adjustment between two consecutive vehicles while a shock wave occurs while the asymptotical models incorporate speed-space relationship during the transition procedure.



〈Fig. 8〉 Comparison of shock waves at signalized intersection: forced flow to stop and stop to forced flow.

Figure 8 shows the remaining half of the density k spectrum : 75 to 150 veh/ln/mi for deceleration and for acceleration flows. The shock wave speeds of the asymptotical model for acceleration flow tend to be inclined to the forward direction more than those of the deceleration flow, which is similar to the aforementioned case described in Figure7. However, the shock wave speed differences between the two model are continuously decreasing as the density approaches to $\mathbf{k_j}$. The shock wave speed differences between the asymptotical model for acceleration flow and L-W model are virtually constant regardless of the range of **k**. Figure 8 also shows a outlying data points around the jam density where the initial density **k** is close to he final density **k**′.

Figure 9 combines Figures 7 and 8, excluding the near-jam density region. It shows more clearly that the asymptotical model yields different shock wave speeds for deceleration and acceleration flows thus the model is asymmetric. In addition, the difference between the asymptotical model and L-W model for acceleration flow is large but remained constant throughout the entire density spectrum. On the other hand. the difference between the asymptotical models for acceleration flow and for deceleration flow decreases gradually as the difference between **k** and **k**′ decreases.

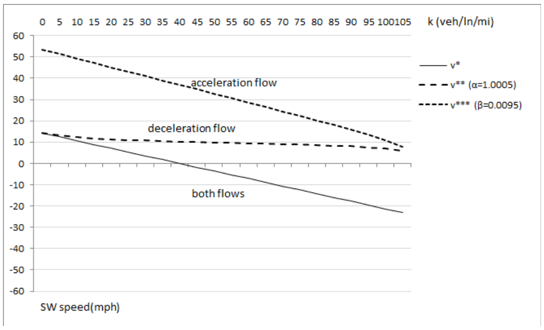


〈Fig. 9〉 Comparison of shock waves at signalized intersection: all flow to stop and stop to all flow.

(4) Model test 3: acceleration from a low speed

To test the asymptotical model for acceleration from a low speed, another set of numerical data are prepared. For this test, it is assumed that the flow speed changed from **v** = 14.2 mph with a density of 110 to higher speeds ranged from **v**′=16 mph to 53.3 mph, of which density ranged accordingly from 105 to 0.

The solid line and the crude dashed line in Figure 10 are the shock wave speeds of L-W model and the asymptotical model for deceleration flow, respectively.



〈Fig. 10〉 Comparison of shock waves: acceleration from a low speed to higher speeds. and deceleration from higher speeds to a lower speed.

When compared with the counter part lines in Figure 9, the slopes and shapes of the two lines in Figure 10 are very similar but they are shifted toward the downstream direction. The shock wave speeds of the acceleration model, which are plotted by the dense dashed line in Figure 10, are far more shifted toward the downstream direction. Overall, however, the shock waves of the deceleration and acceleration models are clearly different thus Figure 10 depicts the asymmetry of the asymptotical model.

3. Test of significance

The numerical tests of the asymptotical shock wave model acceleration in Section 2 showed that its outputs are different than the corresponding outputs of L-W model and deceleration model. To assess whether the outputs of three models are different significantly in a statistical sense, a matched difference *t-test* was repeatedly applied as shown in Table 1. The formulated null hypothesis for the significance test was defined as follows:

H_0: The observed average of the differences of both models is not significantly greater than the expected average of the difference (0).

The model output differences of each numerical

〈Table 2〉 Tests of significance of model performance

t-test	Model test No.	Comparison between	Sample size	Output difference average	Standard Deviation	*t-statistic*	Degree of freedom	Reference *p-value*
t-test 1	2	v^{*} and v^{***}	30	48.9	3.2	84.5	29	$p < 0.0005$
t-test 2	3	v^{*} and v^{***}	22	35.9	2.3	73.9	21	$p < 0.0005$
t-test 3	2	v^{**} and v^{***}	30	29.2	16.1	29.1	29	$p < 0.0005$
t-test 4	3	v^{**} and v^{***}	22	21.6	11.9	19.6	21	$p < 0.0005$

deployment, output difference average, the standard deviations, the *t-statistics*, and the *p-values* are computed and summarized in Table 2.

In the tests, all *p-values* are smaller than 0.0005; thus, the null hypothesis is rejected at the 0.05% level of significance. These tests of significance indicate that the asymptotical model for acceleration yielded significantly different outputs compared to both L-W model and the deceleration model.

Ⅴ. Conclusions

With L-W model, there exists a shock wave even in a constant traffic flow. Lighthill and Whitham and other researchers described this wave as imaginary, but useful as an analytical tool. Cho [9, 11] suggested that there is no logical reason why this particular wave is unobservable or imaginary while all other waves are observable in the field. He attempted to resolve the paradox by deriving a new asymptotical shock wave model and showed that the derivation process of the L-W model oversimplified the relationships among speed-density-flow.

Although the asymptotical model resolved the counterintuitive output of L-W model, it remained uncompleted since it incorporated the deceleration flow conditions only. The actual derivation of Cho's asymptotical model showed that it considered the vehicle trajectories in association with the deceleration flow only and he left the derivation of the companion model for the acceleration flow for further study. In this paper we derived the asymptotical model for the acceleration traffic flow. For this, to eliminate the distortion in relationships among flow-density-speed from L-W model derivation procedure, the vehicle trajectories in a time-space diagram modified to accommodate the changing speed at every instant. With the new model, when a preceding vehicle increases the speed, the following vehicle continuously accelerates speed in such a way that the relationship between the spacing from the preceding vehicle and the speed of the following vehicle strictly follows Greenshield's model. In spite of the different trajectories, the asymptotical model for the acceleration flow was identical to that of the deceleration flow except the parameter, β, which incorporates the spacing and speed adjustment threshold of the following vehicle under acceleration flow condition.

To verify its validity, the new model was initially implemented to a constant flow where no shock wave exists, and the results showed that there exists no imaginary shock wave in a homogeneous flow. Thus the paradox of L-W model was resolved. Numerical applications of the new model with a set of traffic flow scenario showed that the asymptotical model is not symmetric. For the acceleration flow condition, the shock wave speed of the asymptotical model is greater than that given by the model for deceleration flow as was demonstrated previous. The shock wave speed of the asymptotical model for the acceleration flow tends to lean far toward the forward direction consistently. This means that the asymptotical models performs in a systematically different way for acceleration and for declaration flows.

Although L-W model and the asymptotical model performed differently, this paper leaves the topic that which model performs better than the other for the future study since it exceeds the scope of the study. However, the output difference between the two models is so distinct and systematic that immediate further study on both models should follow to identify which model is more applicable to an empirical site. Thus this paper may serve as a starting point for such further researches.

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