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신경망 기법을 이용한 새로운 반응함수 추정 방법에 관한 연구

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Study on a New Response Function Estimation Method Using Neural Network

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Abstract

Purpose: The main objective of this paper is to propose an RD method by developing a neural network (NN)-based estimation approach in order to provide an alternative aspect of response surface methodology (RSM).

Methods: A specific modeling procedure for integrating NN principles into response function estimations is identified in order to estimate functional relationships between input factors and output responses. Finally, a comparative study based on simulation is performed as verification purposes.

Results: This simulation study demonstrates that the proposed NN-based RD method provides better optimal solutions than RSM.

Conclusion: The proposed NN-based RD approach can be a potential alternative method to utilize many RD problems in competitive manufacturing nowadays.

Key Words : Robust Design, Neural Network, Response Surface Methodology, Ordinary Least Square, Estimation, Simulation

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1. Introduction

As a systematic method for applying experimental designs and results, robust design (RD) is one of the most significant process design concepts for quality improvement and process optimization (Taguchi 1986). While maintaining required specifications, the primary issue of quality improvement is to obtain the optimal input factor settings by reducing the sensitivity from unknown and uncontrollable noise factors. RD techniques are the useful off-line tools of such quality improvement with many widespread applications that have resulted in significant improvement in product quality, manufacturability and reliability at low cost (Shin and Cho 2005).

An important step in RD procedure requires identifications of the relationships between input control factors and their associated output responses. Response surface methodology (RSM) is a useful approach for modeling process relationships by separately estimating the response functions of dual responses, such as the process mean and variance (Vining and Myers 1990). A number of RSM approaches based on conventional statistical estimation methodologies, such as least square method (LSM), maximum likelihood, and Baysian estimations were developed by Vining and Myers (1990), and Box and Draper (1987) in order to replace signal to noise ratio approach proposed by Taguchi (1986). Although RSM has provided significant advantages in product/process design and quality improvement, RSM has still room for improvement: Accuracy of fitted response models and basic error assumptions (i.e.. zero error expectation, constant error variance, uncorrected error, and normal error). An alternative approach based on a non-functional-formed estimation aspect was introduced by using NN concept applications (Truong et al. 2011). Because of the effectiveness of using soft-computing NN techniques, a number of researchers presented many applications of the artificial NN approach to RD optimization problems (Rowlands et al. 1996, Chiu et al. 1997, Su and Chang 1998, Cook et al. 2000, and Chow et al. 2002). A number of NN applications to control chart problems mostly associated with pattern recognition approaches have reported (Pham and Oztemel 1994, Wani and Pham 1999, and Ebrahimzadelh and Ranaee 2010).

The primary objective of RD optimization problems is to simultaneously minimize the process bias (i.e., deviation from the target value) and variability of a product/process. In an early attempt, Vining and Myers (1990) proposed a dual response approach based on RSM as an alternative to the Taguchi approach by separately estimating the response functions of the process mean and standard deviation. Hence, the mean squared error (MSE) model from relaxing the zero-bias assumption was proposed by Lin and Tu (1995). While allowing some process bias, the resulting process variance is less than the variance obtained from Vining and Myers' model. Many modifications and extensions of the MSE model have been discussed in RD literature, such as Jayaram and Ibrahim 1999, Cho et al. 2000, Kim and Cho (2000, 2002), Yue 2002, Miro-Quesada and Del Castillo 2004, Košksoy 2006, and Shin and Cho 2009.

The aim of this paper is to identify the advantages of using NN in RD problems by comparing conventional RSM-based estimation methods. First of all, the “black-box” relationship between input factors and their associated output responses of a product/process can be estimated by employing the NN approach in order to estimate the process mean and variance functions. In addition, the optimal factor settings can be obtained by using existing RD optimization models, such as dual response model (Vining and Myers 1990), MSE model (Lin and Tu 1994), bias specified robust design (VSRD) model (Shin and Cho 2004), and variability specifeid robust design (VSRD) model (Shin and Cho 2004). Finally, a comparative simulation study is performed in order to demonstrate the efficiency of the proposed NN-based RD method. The expected quality loss (EQL) criterion, which is one of the most popular comparative criteria, is utilized to compare RD optimal solutions. The primary difference between existing NN approaches and the proposed approach can be associated with the statistical estimation of response functions to RD problems. An overview of the proposed NN-based RD procedure including experimental design, estimation, and optimization is illustrated in Figure 1.

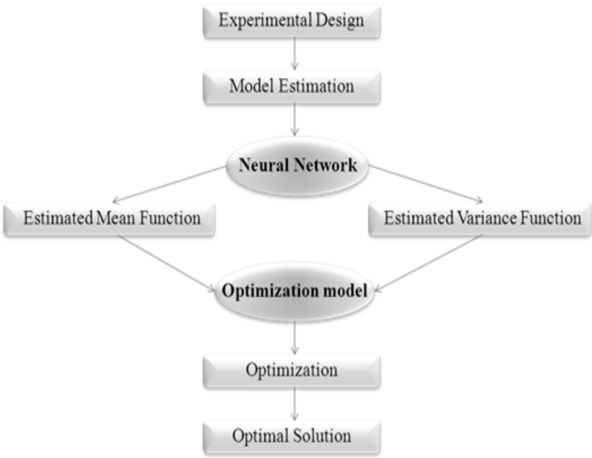
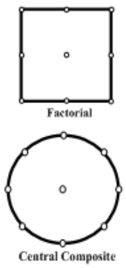


Figure 1. An overview procedure of the proposed NN-based RD method

2. Neural network (NN) estimation approach to Robust Design (RD)

In experimental design phase, an experimental design such as full or fractional factorial designs, central composite designs is selected and the response of interest is measured under the

Table 1. A standard experimental design format for RD problems

Runs	x	y (Replications)					$\bar{y}$	s	s^2
1		y_{11}	y_{12}	$\cdots$ y_{1v}	$\cdots$	y_{1m}	$\overline{y_1}$	s_1	s_1^2
2		y_{21}	y_{22}	$\cdots$ y_{2v}	$\cdots$	y_{2m}	$\overline{y_2}$	s_2	s_2^2
$\vdots$		$\vdots$	$\vdots$	$\ddots$ $\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
u		y_{u1}	y_{u2}	$\cdots$ y_{uv}	$\cdots$	y_{um}	$\overline{y_u}$	s_u	s_u^2
$\vdots$		$\vdots$	$\vdots$	$\dots$ $\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n		y_{n1}	y_{n2}	$\cdots$ y_{nv}	$\cdots$	y_{nm}	$\overline{y_n}$	s_n	s_n^2

selected design. In this stage, n runs of experiments are performed with m replications at each run. Then, the vector of mean, the vector of standard deviation and the vector of variance of product/process can be calculated.

A standard experimental format for RD format is introduced in which the noise factors are not included and the replications of responses are investigated as in Table 1.

2.1. Conventional RSM

RSM is a collection of mathematical and statistical techniques that are useful for modeling and analyzing problems when the response of interest is influenced by several factors. When the exact functional relationship is not known or very complicated, the conventional least squares method is typically used to estimate the input-response functional forms of responses in RSM (Box and Draper 1987, Khuri and Cornell 1987, Myers and Montgomery 2002). The response (y) which depends on the level of p control factors $(x_1,x_2,...,x_p)$ is modeled based on the following assumptions:

The response is modeled by the function $y(x)=f(x_1,x_2,\dots,x_p)$,

The levels of x_i for $i=1,2,...,p$ are quantitative and continuous,

The levels of x_i for $i=1,2,...,p$ can be controlled by the experiment.

Therefore, RSM is critically dependent on the chosen functional model form and the mean and variance are estimated in the same way. The second-order polynomial models of mean process, standard deviation and variance process can be expressed respectively as follows:

$$\bar{y}(x)=f_1(x)=\beta_0+\sum_{i=1}^{p}\beta_i x_i+\sum_{i=1}^{p}\beta_i x_i^2+\sum_{j=1}^{p}\beta_{ij}x_i x_j+\epsilon_k \quad (1)$$

$$s(x)=f_2(x)=\delta_0+\sum_{i=1}^{p}\delta_i x_i+\sum_{i=1}^{p}\delta_i x_i^2+\sum_{j=1}^{p}\delta_{ij}x_i x_j+\epsilon_l \quad (2)$$

$$s^2(x)=f_3(x)=\alpha_0+\sum_{i=1}^{p}\alpha_i x_i+\sum_{i=1}^{p}\alpha_i x_i^2+\sum_{j=1}^{p}\alpha_{ij}x_i x_j+\epsilon_m \quad (3)$$

where the $\beta's$, $\delta's$, and $\alpha's$ are the unknown coefficients, and ϵ_k, ϵ_l, and $\delta's$ are the random errors.

In matrix notation, the response is generalized:

$$y=f(x)=X\beta+\epsilon \quad (4)$$

where β, X and ϵ denote a vector of model parameters, n×q designed matrix, and random error respectively.

The estimated $\hat{\beta}$ can be computed by using the ordinary least squares method

$$\hat{\beta}=(X^TX)^{-1}X^Ty \quad (5)$$

2.2. The proposed NN estimation approach

NN is a typical stochastic model based on "black box" approach in which the knowledge of the phenomenon is unknown. NN is an information processing model derived from the information processing method of a biological neural system. This model is created by a large number of elements (i.e., neurons or processing elements) connected by weighted links that work as an integrated system to solve specific issues. In terms of an RD problem, NN can be used to illustrate the unknown functional relationship between the input factors and output responses without any assumptions. The input layer and output layer in NN can be considered similarly as the input factors and output responses in RD, respectively. The functional relationship between input layer and output layer may be achieved after transformations through n - 1 hidden layers. This transformation is illustrated in Figure 2. In NN, each neuron is a unit of computation in which a simple mathematical model is used. The connection among neurons can generate a powerful computational NN. Figure 3 depicts a general architecture of a neuron system.

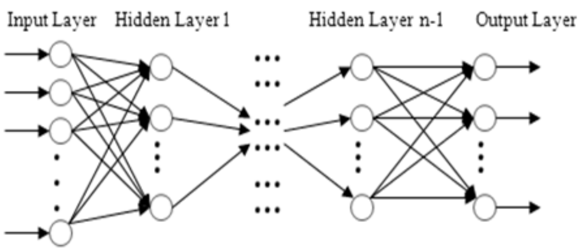


Figure 2. A multi-layers NN.

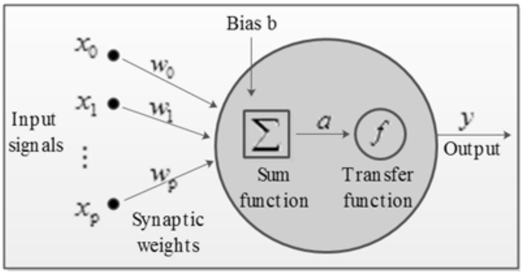


Figure 3. Architecture of a neuron system

An NN approach is configured for a specific application through a learning process from the set of training samples. The essence of learning process is the adjustment of the connected weights between neurons in a system. For both functional fitting and pattern recognition problems, based on the comparison of network outputs and their targets, the neural network is adjusted until they are well matched. Each learning process will generate a complicated function between input factors and output responses with arbitrary errors. The output y(x) of a neuron can be calculated as follow:

$$y = f(x) = f(\sum_{i=0}^{p} \omega_i x_i)|_{x_0 = 1} = f(a) \tag{6}$$

where f and ω_i denote a transfer function and weights of the corresponding input signal x_i, respectively. In many cases, This transfer function in the hidden layers can often be a sigmoid transfer function such as

$$f(a) = \frac{1}{1 + e^{-a}}. \tag{7}$$

In the experimental design stage, information about the relationships between input factors and output responses of a system/process is exploited. A series of structured tests are designed in which planned changes to the input variables of a system/process are made. Therefore, the effects of these changes on a pre-defined output are assessed. Then, NN is utilized in a model estimation stage to extract the complicated relationship between control factors and their associated responses in order to estimate process mean and variance models. In this estimation stage, two important steps to obtain a suitable NN for each problem are (1) selecting the NN structure including input nodes, hidden players, hidden nodes, and output nodes and (2) setting the required parameters, such as learning rate, iterations, and initializing weights. The trained NN serves as the estimated mean and variance functions. These functions are employed in the optimization stage in order to achieve robust factor settings that are less sensitive to unknown and unpredictable variations. The general framework of the proposed NN-based RD approach can be defined as follows:

General framework

Step 1: Identify the potential control variables, noise variables, responses and target values for individual response.

Step 2: Execute the experiment design.

Step 3: Estimate the response functions (mean and variance) of the process parameters by using NN.

Step 4: Generate optimization criterion, set up an optimization problem for quality characteristic and eventually find out the optimal solutions.

3. Comparative simulation study: RSM and NN

The main purpose of the simulation study is to compare two methods (i.e., conventional RSM and NN-based estimation). In many real wold industrial situations, the "true" function of the phenomenon is often unknown and unpredictable. In this simulation study, we assume that the "true" relationship was known and is presented in sinusoidal function as

$$\begin{aligned} & y = 2\sin(x_1 + x_2) \\ & s.t \ \ x_1, x_2 \in [0,\ 2\pi]. \end{aligned} \tag{8}$$

A full factorial design is conducted in order to investigate the effects of input factors to their associated output responses as well as the effects of interactions between input factors and output responses. Based on "true" function values, responses are sampled according to a set of input factors. In this study, each input factor has 5 levels, and thus the total experimental runs are $5^2 = 25$. In addition, at each sampling point, variance is added by number of replications. Hence, the noise factors are included in the variance function and the empirical data can be generated as: $y_i = y_{true_i} + N(0,\ \sigma_i^2)$. A further assumption is made that a noise factor has the "true" form: $\sigma_i = e^{(x_1 - x_2)/10}$. This "true" form is directly proportional to $x_1's$ level and inversely proportional to $x_2's$ level. The sample "approximated" value of responses are, then, used to estimate the mean and variance functions. Based on the equation (4), the proposed mean and variance functions of the interested responses in the quadratic form can be obtained as follows:

$$y = f(x) = \beta_0 + \beta_1 x + \beta_2 x^2$$
$$x = (x_1, x_2), x \in [0, 2\pi] * [0, 2\pi] \tag{9}$$

where $\beta's$ represent the unknown coefficients.

In NN, the assumption of the relationship between a sampled response and input factors is not necessary. In this proposed procedure, the NN architecture for estimating the mean and variance functions consists of 2 neurons (x_1, x_2) in the input layer, 3 neurons in the hidden layer, and 1 neuron (mean or variance) in the output layer (called, topology 2-3-1). The experimental data, "true" response, "true" variance value and corresponding empirical data for estimation are performed in Table 2.

Table 2. Experimental data

Coded Input		"True" response $y = 2\sin(x_1+x_2)$	Empirical mean	"True" noise $\sigma = e^{(x_1 - x_2)/10}$	Empirical standard deviation
x_1	x_2				
-0.5	0.5	-1.4142	-1.4069	0.9245	1.0571
1	-0.5	1.4142	1.6386	1.7329	1.7748
0.5	1	2	2.0722	1.1701	1.0459
-0.5	-0.5	1.4142	1.4795	1.0817	1.1537
-0.5	-1	2	1.9779	1.1701	1.2501
0.5	0	0	-0.1485	1.3691	1.4902
-1	-0.5	1.4142	1.449	0.9245	0.8838
0.5	-1	-2	-1.6374	1.602	1.4355
0	0.5	-1.4142	-1.3984	1.0817	1.0753
0.5	0.5	1.4142	1.5221	1.2657	1.1365
0.5	-0.5	-1.4142	-1.4674	1.481	1.3493
0	1	0	0.0875	1	0.9506
-1	0.5	1.4142	1.4108	0.7901	0.8016
1	0.5	1.4142	1.3923	1.481	1.6492
-1	-1	0	-0.0145	1	1.1061
-0.5	0	0	0.0067	1	0.8706
1	0	2	2.13	1.602	1.6488
-1	1	0	-0.1006	0.7304	0.7148
1	-1	0	0.1896	1.8745	1.8941
-0.5	1	-2	-1.9597	0.8546	0.872
1	1	0	0.13	1.3691	1.2763
0	-1	0	0.0951	1.3691	1.286
0	-0.5	-1.4142	-1.3989	1.2657	1.2452
-1	0	2	2.006	0.8546	0.8833
0	0	-2	-1.9725	1.1701	1.1398

In term of regression, the empirical and predicted data of mean and standard deviation based on each method are demonstrated in the visual of Figures 4 and 5. These figures show that the proposed NN estimation approach gives the better results than the RSM based on LSM where the empirical and predicted data points in NN method are mostly closer to the line of ideal estimation than in LSM based RSM. In addition, correlation coefficient R (i.e., one of significant statistical criteria in analysis of fitted response functions) of mean and variance using both LSM and NN are calculated as follows: R_mean for LSM = 0.60, R_mean for NN = 0.99, R_sd for LSM = 0.96, R_sd for NN = 0.99. Therefore, NN approach can generate the exact parameters for the model with the arbitrary error and without any assumptions of the true functional form.

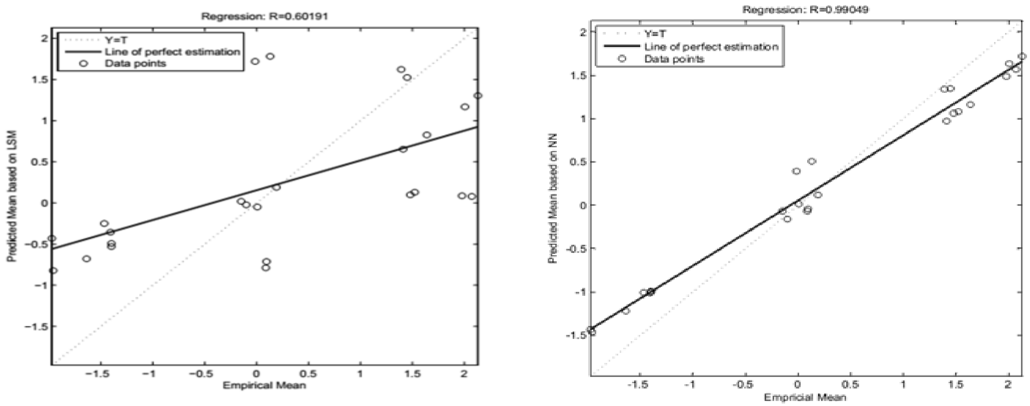


Figure 4. "True" and predicted mean based on LSM and NN methods

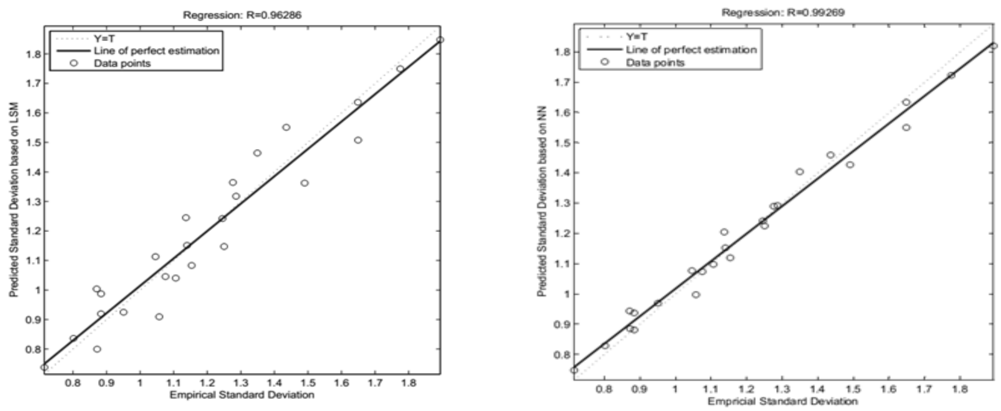


Figure 5. "True" and predicted standard deviation based on LSM and NN methods

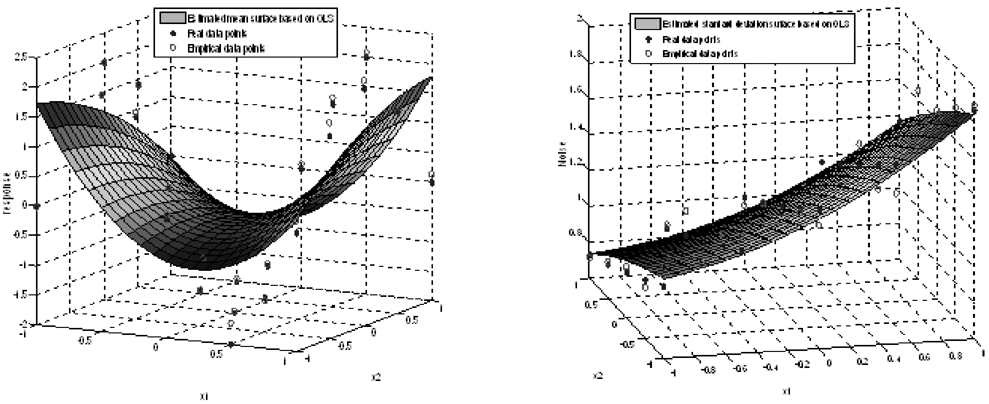


Figure 6. Estimated response surface plots based on LSM

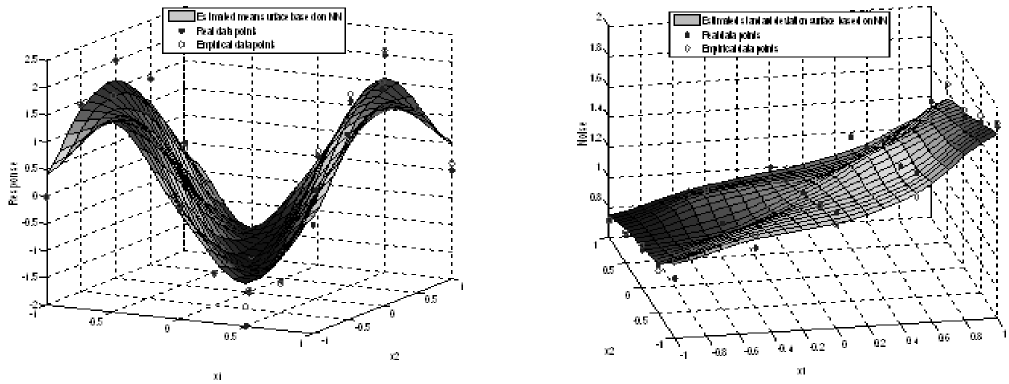


Figure 7. Estimated response surface plots based on NN

The estimated mean and standard deviation of responses based on LSM and NN are illustrated in surface plots as shown in Figure 6 and 7, respectively. As can seen from these figures, the surfaces in NN are tighter to the real data than in RSM. Therefore, NN approach might be a potential alternative method in functional estimation. In the final stage, a number of optimization models are chosen to find the optimal factor settings, as shown in Table 3. One of the most popular criteria to compare RD solutions can be the expected quality loss (EQL) denoted as

$$EQL = k(\widehat{\mu}(x) - \tau)^2 + \widehat{\sigma(x)}^2) \quad (10)$$

where k, τ, $\widehat{\mu}(x)$, and $\widehat{\sigma(x)}$ represent a positive loss coefficient, the target value which is set equal to

Table 3. RD Optimization models

Models	Author(s)	Year	Optimization models	Discussions
Dual response (DR)	Vining and Myers	1990	$Min\ \widehat{\sigma}(x)$ $s.t\ \ \widehat{\mu} = \tau$ $x \in \Omega$	- Dual response approach using RSM - Zero bias logic - Prioritized mean
	Del Castillo and Montgomery	1993		
MSE	Lin and Tu	1995	$Min\ (\widehat{\mu}(x) - \tau)^2 + \widehat{\sigma}^2(x)$ $s.t\ \ x \in \Omega$	- Simultaneous optimization of bias and variance - Equal weight
Bias specified model (BSRD)	Shin and Cho	2004	$Min\ \widehat{\sigma}(x)$ $s.t\ \ \lvert\widehat{\mu}(x) - \tau\rvert \le \epsilon_\mu$	- Specified maximum bias - Modification of dual response model - ε-constraint method
Variability specified model (VSRD)			$Min\ \lvert\widehat{\mu}(x) - \tau\rvert$ $s.t\ \ \widehat{\sigma}(x) \le \epsilon_\sigma$ $x \in \Omega$	- Specified maximum variance - Modification of dual response model - ε-constraint method

1 in this study, the estimated process mean unction, and the estimated process standard deviation function, respectively. The significant advantage of the EQL criterion is to decompose a quality loss in terms of the process bias (i.e., deviation between the process mean and the target) and standard deviation simultaneously. Based on the EQL criterion, the optimal solutions are evaluated and compared. The final optimization results by using LSM and NN approaches are demonstrated in Table 4. In BSRD and VSRD models, ϵ_μ and ϵ_σ are assigned by the average value of the bias and standard deviation of the DR and MSE models.

Table 4. Optimization results

RD Models		Optimal Solution		Mean $\hat{\mu}(\mathbf{x})$	Bias $\lvert\hat{\mu}(\mathbf{x}) - \mathbf{x}\rvert$	STD $\widehat{\sigma(\mathbf{x})}$	EQL
		$\mathbf{x_1}$	$\mathbf{x_2}$				
DR	Based LSM	0.7995	1.0000	1.0000	0.0000	1.2560	1.5776
	Based NN	-1.0000	0.4853	1.0000	0.0000	0.8303	0.6893
MSE	Based LSM	-1.0000	0.3151	0.8625	0.1375	0.8688	0.7738
	Based NN	-1.0000	0.5173	0.9416	0.0584	0.8261	0.6859
BSRD	Based LSM	0.7718	1.0000	0.9021	0.0979	1.2418	1.5517
	Based NN	-1.0000	0.5378	0.9021	0.0979	0.8233	0.6875
VSRD	Based LSM	-0.9760	0.1035	1.0000	0.0000	0.9060	0.8208
	Based NN	-0.7898	0.0555	1.0000	0.0000	0.8936	0.7985

As shown in Table 4, the EQL values of all optimization models using the NN approach are smaller than the ones using the LSM method. The optimal solutions (i.e., the process mean and standard deviation) are significantly similar to the "true" values. In this particular comparative study, the proposed NN-based response function estimation method may provide better results in terms of estimation and optimization view points. Based on this result, the proposed response function estimation method can be utilized in RD problems instead of using RSM.

4. Conclusion

An alternative approach to a RD problems is proposed in this paper by employing the NN method to identify relationships between input factors and output responses. A comparison study by using LSM and NN approaches based on simulations was performed for verification purposes. In simulation study, the proposed NN-based RD approach provides the better results in both terms of estimation and optimization based on based on the EQL comparative criterion. Without any assumptions of the true functional forms, the precise parameters of the models can be obtained in the proposed NN-based parameter estimation. As results, the proposed NN-based RD approach can be a potential alternative method to utilize many RD problems in real-world industrial situations. Based on the results of this research, a systemic screening method to evaluating transfer functions in NN methods can be a possible further study issue. For further verification and validation associated with the NN-based response function estimation, a number of estimation and optimization results by using different sets of simulated data may provide the better precision.

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REFERENCES

Box, and Draper. 1987. Empirical model building and response surface. New York: John Wiley & son.

Chiu, C. C., Su, C. T., Yang, G. H., Huang, J. S., Chen, S. C., and Cheng, N. T. 1997. "Selection of optimal parameters in gas-assisted injection moulding using neural network model and the Taguchi method." International Journal of Quality Science 2:106-120.

Cho, B. R. 1994. "Optimization issues in quality engineering." PhD diss., University of Oklahoma.

Cho, B. R., Kim, Y. J., Kimber, D. L., and Phillips, M. D. 2000. "An integrated joint optimization procedure for robust and tolerance design." International Journal of Production Research 38:2309-2325.

Chow, T. T., Zhang, G. Q., Lin, Z., and Song, C. L. 2002. "Global optimization of absorption chiller system by genetic algorithm and neural network." Energy and Building 34:103-109.

Cook, D. F., Ragsdale, C. T., and Major, R. L. 2000. "Combining a neural network with a genetic algorithm for process parameter optimization." Engineering Application of Artificial Intelligence 13:391-396.

Ebrahimzadelh, A., Ranaee, V. 2010. "Control chart pattern recognition using an optimized neural network and efficient features." The International Society of Automation 49:387-393.

Jayaram, J. S. R., and Ibrahim, Y. 1999. "Multiple response robust design and yield maximization." International Journal of Quality and Reliability Management 6:826-837.

Kim, Y. J., and Cho, B. R. 2000. "Economic consideration on parameter design." Quality and Reliability Engineering International 16:501-514.

Kim, Y. J., and Cho, B. R. 2002. "Development of priority-based robust design." Quality Engineering 14:355-363.

Lin, D. K. J., and Tu, W. 1995. "Dual response surface optimization: A fuzzy modeling approach." Journal of Quality Technology 27:34-39.

Miro-Quesada, G., and Del Castillo, E. 2004. "Two approaches for improving the dual response method in robust parameter design." Journal of Quality Technology 36:154-168.

Košksoy, O. 2006. "Multiresponse robust design: Mean square error (MSE) criterion." Applied Mathematics and Computation 175:1716-1729.

Pham, D. T., and Oztemel, E. 1994. "Control chart pattern recognition using combinations of multi-layered perceptrons and learning-vector quantisation neural networks." Journal of Process Mechanical Engineering 207:113-118.

Rowlands, H., Packianather, M. S., and Oztemel, E. 1996. "Using artificial neural networks for experimental design in off-line quality." Journal of Systems Engineering 6:46-59.

Shin, S., and Cho, B. R. 2005. "Bias-specified robust design optimization and its analytical solutions." Computers and Industrial Engineering 48:129-140.

Shin, S., and Cho, B. R. 2009. "Studies on a bi-objective robust design optimization problem." IIE Transactions 41:957-968.

Su, C. T., Chiu, C. C., and Chang, H. H. 1998. "Neural networks procedures for Taguchi's dynamic problem." Journal of the Chinese Institute of Industrial Engineerings 15:457-463.

Taguchi, G. 1986. Introduction to Quality Engineering. NY: Asian Productivity Organization, UNIPUB, White Plains.

Truong, N. K. V., Shin, S., and Nha, V. T. 2011. "Robust design by using neural network." Department of Systems

Management & Engineering. International Conference of Next Generation Information Technology 170-174.

Vining, G. G., and Myers, R. H. 1990. "Combining Taguchi and response surface philosophies: A dual response approach." Journal of Quality Technology 22:38-45.

Wani, A., and Pham, D. T. 1999. "Efficient control chart pattern recognition through synergistic and distributed artificial neural networks." Journal of Engineering Manufacture 213:157-169.

Yue, R. X. 2002. "Model-robust designs in multiresponse situations." Statistics & Probability Letters 58:369-379.