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A Second-Order Design Sensitivity-Assisted Monte Carlo Simulation Method for Reliability Evaluation of the Electromagnetic Devices

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Abstract – In the reliability-based design optimization of electromagnetic devices, the accurate and efficient reliability assessment method is very essential. The first-order sensitivity-assisted Monte Carlo Simulation is proposed in the former research. In order to improve its accuracy for wide application, in this paper, the second-order sensitivity analysis is presented by using the hybrid direct differentiation-adjoint variable method incorporated with the finite element method. By combining the second-order sensitivity with the Monte Carlo Simulation method, the second-order sensitivity-assisted Monte Carlo Simulation algorithm is proposed to implement reliability calculation. Through application to one superconductor magnetic energy storage system, its accuracy is validated by comparing calculation results with other methods.

Keywords: Finite element method, Reliability evaluation, Second-order design sensitivity analysis, Sensitivity-assisted Monte Carlo simulation, Uncertainty.

1. Introduction

To deal with uncertainty of design variables, the reliability-based design optimization (RBDO) algorithm has been developed to improve constraint feasibility [1]. The performance analysis of the electromagnetic device by the finite element method (FEM) needs a huge computational cost. Therefore, it is very essential to develop the accurate and numerically efficient reliability calculation method. The first-order sensitivity-assisted Monte Carlo Simulation (FS-MCS) method is proposed in [2] to efficiently implement reliability analysis; however, it appears insufficient for problems involving large variations or having strongly nonlinear performance function. Therefore, the application of reliability analysis is still restricted especially in the RBDO of real electromagnetic problems.

As it is well known, the second-order design sensitivity can improve the convergent efficiency of optimization process and can provide more accurate information than the first-order one [3]. In fact, the second-order sensitivity analysis by the FEM is complicate and computationally expensive. For the electromagnetic application problems, methods utilizing the second- or higher-order sensitivities are much less developed than that in the mechanical engineering [4], and therefore are not yet often applicable. It is worth mentioning, recently, researchers set about studying the second-order sensitivity in the electrical engineering. One finite difference technique is used to calculate the second-order sensitivity [5]; however, the proper step size is difficult to be determined for a higher accuracy. In the electric circuit system, the second-order capacitance sensitivity is studied by the combination of the derivative method and the adjoint variable method. [6].

Therefore, in this paper, a hybrid direct differentiation-adjoint variable method is presented to implement the second-order sensitivity analysis of the electromagnetic problems by the FEM. Its numerical efficiency is qualitatively compared with other methods. Furthermore, the second-order sensitivity-assisted Monte Carlo simulation (SS-MCS) method is proposed to expedite reliability calculation in the RBDO. Its accuracy and numerical efficiency is widely discussed with the MCS, the FS-MCS, and the reliability index approach (RIA) [1] through numerical examples.

2. Design Sensitivity Analysis Based on the FEM

2.1. First-order sensitivity analysis

In the FEM, the system equation of the electromagnetic problem from Galerkin approximation is written as follows:

$$[K][A] = \{Q\} \tag{1}$$

where $[A]$ is the magnetic vector potential, $[K]$ and $\{Q\}$ are the global stiffness matrix and forcing vector, respectively. Usually, the design target of electromagnetic problem such as system energy and torque has explicit or implicit relations with geometric variables and the state variable $[A]$. The real geometric dimensions (height and width) and physical parameter (current density) are normally transferred

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into design parameters [p] related with nodal mesh. Therefore, the derivative of performance function $g([p], A(p))$ with respect to the ith design parameter p_i is calculated as follows:

$$\frac{dg}{dp_i} = \left.\frac{\partial g}{\partial p_i}\right|_{A=const.} + \left.\frac{\partial g}{\partial [A]^T}\right|_{p=const.} \frac{d[A]}{dp_i} \tag{2}$$

where terms $\partial g/\partial p_i$ and $\partial g/\partial [A]^T$ are problem dependent and can be calculated analytically. For the calculation of $d[A]/dp_i$, there are following two methods.

A. Direct Differentiation Method (DD)

Firstly, differentiating both sides of (1) with respect to p_i,

$$[K]\frac{d[A]}{dp_i} = -\left\{\frac{\partial [K]}{\partial p_i}[\tilde{A}] - \frac{\partial \{Q\}}{\partial p_i}\right\} \tag{3}$$

where $[\tilde{A}]$ is the converged solution of (1). The numerical analysis method such as incomplete Cholesky-Conjugate gradient is applied to solve (3). Then substituting the solution into (2), the sensitivity can be obtained.

B. Adjoint Variable Method (AV)

For simplification, one adjoint variable [λ] is defined as :

$$[K][\lambda] = \partial g / \partial [A] \tag{4}$$

and then substitute (4) and (3) into (2), the sensitivity is expressed by the following equation:

$$\frac{dg}{d[p]^T} = \left.\frac{\partial g}{\partial [p]^T}\right|_{A=const.} - [\lambda]^T \left\{\frac{\partial [K]}{\partial [p]^T}[\tilde{A}] - \frac{\partial \{Q\}}{\partial [p]^T}\right\} \tag{5}$$

Assuming the number of design parameters is n, the DD method totally needs ($1+n$) times FEM analysis: one of (1) and n of (3). However, without any relation with the quantity of design parameters, the AV method only needs (1+1) times FEM analysis: one of (1) and one of (4). Therefore, the AV method is superior to the DD method.

2.2. Second-order sensitivity analysis by Hybrid Direct Differentiation-adjoint Variable Method (HDD-AV)

Based on (2), the second-order sensitivity of performance function with respect to p_i and p_j is obtained by using the chain rule of differentiation as follows:

$$\frac{d}{dp_j}\left(\frac{dg}{dp_i}\right) = \frac{d}{dp_j}\left(\left.\frac{\partial g}{\partial p_i}\right|_{A=const.}\right) + \frac{d}{dp_j}\left(\left.\frac{\partial g}{\partial [A]^T}\right|_{p=const.} \frac{d[A]}{dp_i}\right) \tag{6a}$$

where the calculation of right two parts is driven as follows:

$$\frac{d}{dp_j}\left(\frac{\partial g}{\partial p_i}\right) = \frac{\partial^2 g}{\partial p_i \partial p_j} + \frac{\partial}{\partial [A]^T}\left(\frac{\partial g}{\partial p_i}\right)\frac{d[A]}{dp_j} \tag{6b}$$

$$\begin{aligned} &\frac{d}{dp_j}\left(\frac{\partial g}{\partial [A]^T}\frac{d[A]}{dp_i}\right) \\ &= \frac{d}{dp_j}\left(\frac{\partial g}{\partial [A]^T}\right)\cdot\frac{d[A]}{dp_i} + \frac{\partial g}{\partial [A]^T}\frac{d}{dp_j}\left(\frac{d[A]}{dp_i}\right) \\ &= \frac{\partial^2 g}{\partial [A]^T \partial p_j}\cdot\frac{d[A]}{dp_i} + \frac{d[A]^T}{dp_j}\cdot\frac{\partial^2 g}{\partial [A]^2}\cdot\frac{d[A]}{dp_i} \\ &\quad + \frac{\partial g}{\partial [A]^T}\cdot\frac{\partial^2 [A]}{\partial p_i \partial p_j} \end{aligned} \tag{6c}$$

Finally, the second-order sensitivity is summarized as:

$$\begin{aligned} \frac{d^2 g}{dp_i dp_j} &= \frac{\partial^2 g}{\partial p_i \partial p_j} + \frac{\partial^2 g}{\partial p_i \partial [A]^T}\frac{d[A]}{dp_j} + \frac{\partial^2 g}{\partial [A]^T \partial p_j}\frac{d[A]}{dp_i} \\ &\quad + \frac{d[A]^T}{dp_j}\frac{\partial^2 g}{\partial [A]^2}\frac{d[A]}{dp_i} + \frac{\partial g}{\partial [A]^T}\frac{\partial^2 [A]}{\partial p_i \partial p_j} \end{aligned} \tag{6d}$$

where the second-order derivative of [A] with respect to design parameters p_i and p_j is very difficult to calculate.

Starting from the DD method, differentiating both sides of (3) with respect to p_j, the second-order derivative of [A] can be calculated as follows:

$$\frac{d}{dp_j}\left\{[K]\frac{d[A]}{dp_i}\right\} = \frac{\partial [K]}{\partial p_j}\frac{d[A]}{dp_i} + [K]\frac{\partial^2 [A]}{\partial p_i \partial p_j} \tag{7a}$$

$$\frac{d}{dp_j}\left\{\frac{\partial [K]}{\partial p_i}[\tilde{A}] - \frac{\partial \{Q\}}{\partial p_i}\right\} = \frac{\partial^2 [K]}{\partial p_i \partial p_j}[\tilde{A}] + \frac{\partial [K]}{\partial p_i}\frac{d[A]}{dp_j} - \frac{\partial^2 \{Q\}}{\partial p_i \partial p_j} \tag{7b}$$

$$\begin{aligned} \frac{\partial^2 [A]}{\partial p_i \partial p_j} = -[K]^{-1} &\left\{\frac{\partial^2 [K]}{\partial p_i \partial p_j}[\tilde{A}] - \frac{\partial^2 \{Q\}}{\partial p_i \partial p_j}\right. \\ &\left. + \frac{\partial [K]}{\partial p_i}\frac{d[A]}{dp_j} + \frac{\partial [K]}{\partial p_j}\frac{d[A]}{dp_i}\right\}. \end{aligned} \tag{7c}$$

Substituting (7c) and (4) into (6d), the total second-order derivative of performance function with respect to p_i and p_j can be expressed as follows:

$$\begin{aligned} \frac{d^2 g}{dp_i dp_j} &= \frac{\partial^2 g}{\partial p_i \partial p_j} + \frac{\partial^2 g}{\partial p_i \partial [A]^T}\frac{d[A]}{dp_j} + \frac{\partial^2 g}{\partial [A]^T \partial p_j}\frac{d[A]}{dp_i} \\ &\quad + \frac{d[A]^T}{dp_j}\frac{\partial^2 g}{\partial [A]^2}\frac{d[A]}{dp_i} - [\lambda]^T \cdot Tmp \end{aligned} \tag{8a}$$

$$Tmp = \frac{\partial^2 [K]}{\partial p_i \partial p_j}[\tilde{A}] - \frac{\partial^2 \{Q\}}{\partial p_i \partial p_j} + \frac{\partial [K]}{\partial p_i}\frac{d[A]}{dp_j} + \frac{\partial [K]}{\partial p_j}\frac{d[A]}{dp_i} \tag{8b}$$

where the second-order derivatives of the global stiffness

Table 1. Comparison of computational cost [7]

Method	DD-DD	AV-AV	HDD-AV
Cost	$(n+1)(n+2)/2$	$2n+2$	$n+2$

matrix and the forcing vector ($\partial^2[K]/\partial p_i\partial p_j$ and $\partial^2\{Q\}/\partial p_i\partial p_j$) are accomplished by the local Jacobian derivatives [6], other terms are problem dependent and can be calculated directly.

Since the utilization of (3) and (4), this method is called the hybrid direct differentiation-adjoint variable (HDD-AV) method. Its computational complexity is (n+2) times FEM analysis (one of (1), one of (4), and n of (3)), which is linear to number of design parameters. As compared with other combinations [7] in Table 1, the HDD-AV method is the most efficient one so that it will speed up reliability analysis and enhance convergence of the RBDO.

3. Reliability Calculation Based on First- and Second-order Sensitivity Analysis

For the reliability analysis, assuming all uncertain variables are independent with each other and follow the Gaussian distribution [8], the uncertainty set is defined as:

$$U(\mathbf{x}) = \left\{ \boldsymbol{\xi} \in R^N \middle| \mathbf{x} - k\boldsymbol{\sigma} \le \boldsymbol{\xi} \le \mathbf{x} + k\boldsymbol{\sigma} \right\} \tag{9}$$

where N is the number of real design variables, ξ is the perturbed design of $\mathbf{x}$, k is a constant decided by the required confidence level (CL) (e.g, k=1.96 if CL=95%) and the standard deviation σ is set zero for a deterministic design variable. Once the first- and second-order sensitivity information is obtained by the AV and the HDD-AV methods based on the FEM, respectively, the performance constraint, $g(\mathbf{x})\geq 0$, in the $U(\mathbf{x})$ can be approximated by the Taylor series expansion as follows:

$$g(\boldsymbol{\xi}) \cong g(\mathbf{x}) + \nabla g(\mathbf{x})\cdot(\boldsymbol{\xi}-\mathbf{x}) + (\boldsymbol{\xi}-\mathbf{x})^T H(\mathbf{x})(\boldsymbol{\xi}-\mathbf{x})/2 \tag{10}$$

where $\nabla g(\mathbf{x})$ is the gradient vector and $H(\mathbf{x})$ is the Hessian matrix including all the second-order derivatives. Until now, the performance constraint function can be treated as an analytic function in the uncertainty set. Then the conventional Monte Carlo Simulation (MCS) method can be applied to evaluate reliability by the following formula:

$$R\left(g(\mathbf{x}) \ge 0\right) = m/M \tag{11}$$

where m is the number of test designs satisfying the constraint in (11) among total M random test designs. The flowchart of reliability calculation is shown in Fig. 1. The SS-MCS is expected to be more accurate than the FS-MCS.

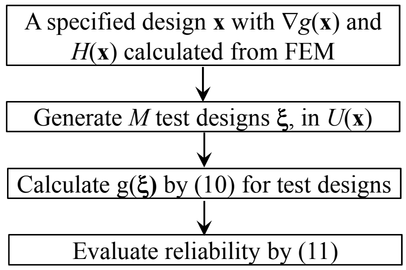


Fig. 1. Flowchart of the sensitivity-assisted MCS method.

4. Numerical Results

4.1 Analytic test problem 1

An analytic constraint function with two uncertain design variables as shown in Fig. 2, is applied to illustrate the necessity of the second-order derivative as follows:

$$g(\mathbf{x}) = x_1^2 x_2/20 - 1 \ge 0\,. \tag{12}$$

The corresponding reliabilities of design A (3.16, 2.15) and design B (3.297, 2.905) as marked in Fig. 2, under different uncertainties are calculated by using the RIA [8], the conventional MCS, the FS-MCS, and the SS-MCS methods, respectively. The maximum test designs and CL in the MCSs are set one million and 0.95, respectively. The results are shown in Table 2. No matter for design A or design B, the reliability from the SS-MCS shows better consistence with that from the MCS method. When the design is close to the constraint boundary such as design A, the FS-MCS and RIA methods show lower accuracy as the

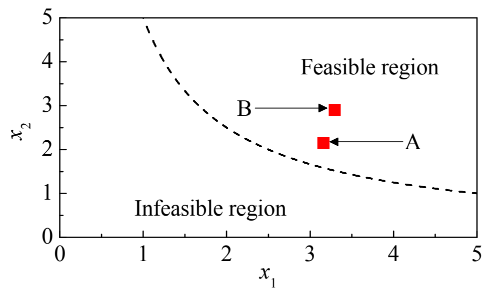


Fig. 2. Analytic constraint function.

Table. 2 Reliability of different designs

Design	Method	Different standard deviation (σ)				
		0.1	0.2	0.3	0.5	0.8
A	RIA	0.8138	0.6722	0.6169	0.5708	0.5443
	FS-MCS	0.8327	0.6834	0.6246	0.5752	0.5476
	SS-MCS	0.8357	0.6776	0.6119	0.5504	0.5074
	MCS	0.8355	0.6779	0.6118	0.5503	0.5062
B	RIA	1.0000	0.9988	0.9782	0.8870	0.7754
	FS-MCS	1.0000	0.9999	0.9808	0.8791	0.7525
	SS-MCS	1.0000	1.0000	0.9939	0.9051	0.7699
	MCS	1.0000	1.0000	0.9934	0.9052	0.7714

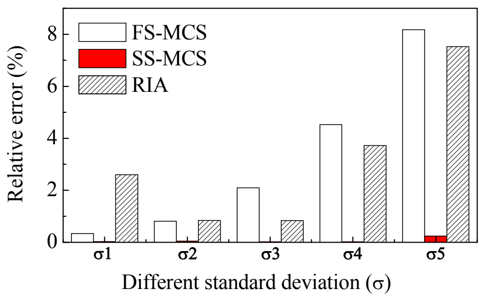


Fig. 3. Relative error of reliability for design A.

standard deviation increases. Fig. 3 compares the relative errors of different approximations. It is unquestionable that the SS-MCS is superior to the FS-MCS from the viewpoint of accuracy, for example, the relative error of the SS-MCS is only 0.24% while the FS-MCS is 8.19% and the RIA is 7.53% when $\boldsymbol{\sigma}$=0.8.

4.2 Analytic test problem 2

For a further investigation, a constraint function with strong nonlinearity shown in Fig. 4 is selected as follows:

$$g(\mathbf{x}) = -1 + (s-6)^2 + (s-6)^3 - 0.6(s-6)^4 + t \geq 0 \quad (13)$$

where $0 \leq x_1$, $x_2 \leq 10$, $s = ax_1 + bx_2$, and $t = bx_1 - ax_2$ (a=0.9063, b=0.4226). In the strongly nonlinear area, three different designs are selected as marked in Fig. 4. The results of reliability analysis by different methods are compared in Table 3. Taking the reliability of the conventional MCS method as a reference R_0, and the relative error (δ_R) of reliability R from other methods is defined as $\delta_R = |R - R_0| / R_0 \times 100\%$. From Table 3, it is obvious that due to the first-order Taylor approximation, the FS-MCS and RIA methods are out of operation even under a small uncertainty such as $\boldsymbol{\sigma}$=0.2, however, the SS-MCS can still give a higher accuracy with the maximum relative error of 3.87% when uncertainty is increased to $\boldsymbol{\sigma}$=0.3. In a word, the application space of the SS-MCS is wider than both RIA and FS-MCS.

From discussions through the above two different

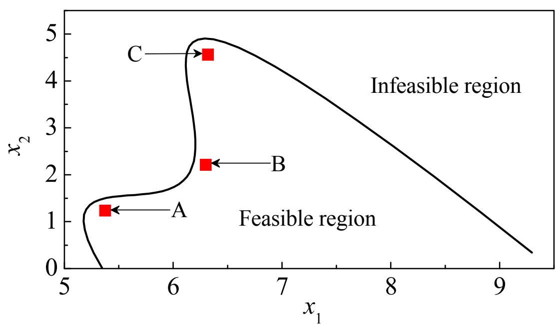


Fig. 4. Analytic constraint function where A(5.376, 1.236), B(6.30, 2.21), and C(6.32,4.56).

Table 3. Comparison of Reliability Calculation [a]

σ	Method	Design A		Design B		Design C	
		R	δ_R (%)	R	δ_R (%)	R	δ_R (%)
0.1	RIA	0.9612	1.908	0.8817	2.682	0.9757	2.205
	FS-MCS	0.9914	1.174	0.8863	2.174	0.9999	0.221
	SS-MCS	0.9830	**0.316**	0.9051	**0.099**	0.9944	**0.331**
	MCS	0.9799	—	0.9060	—	0.9977	—
0.2	RIA	0.8112	5.942	0.7230	3.484	0.8380	6.116
	FS-MCS	0.8662	13.125	0.7223	3.578	0.9284	17.564
	SS-MCS	0.7632	**0.327**	0.7471	**0.267**	0.7953	**0.709**
	MCS	0.7657	—	0.7491	—	0.7897	—
0.3	RIA	0.7218	15.952	0.6534	4.905	0.7446	27.282
	FS-MCS	0.7678	23.341	0.6525	5.036	0.8299	41.863
	SS-MCS	0.5984	**3.872**	0.6827	**0.640**	0.5994	**2.462**
	MCS	0.6225	—	0.6871	—	0.5850	—

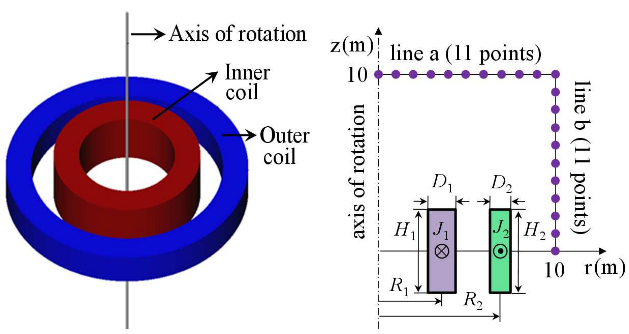
[a] Test designs of MCSs are 1,000,000 and confidence level is 0.95.

analytic examples, it can be concluded that the second-order sensitivity analysis is very essential for the strong nonlinear constraint function approximation. The SS-MCS can be expected to improve the quality of optimal design in the reliability-based design optimization.

4.3 Electromagnetic application to the Superconductor Magnetic Energy Storage System (SMES)

The superconductor magnetic energy storage (SMES) system is applied to guarantee power continuity for very sensible loads to deal with sudden perturbations, which are caused by the appearance or disappearance of a load on the line (voltage surge or sag) and the very short power failures. In a word, applications of SMES in power system can enhance the system stability and improve power quality [9].

Fig. 5 shows the configuration of one SMES system, which has been accepted as one benchmark optimization problem for testing of electromagnetic analysis method; it is composed of two concentric coils carrying current with opposite directions [10]. The optimal design of the SMES should couple the totally stored energy of E_0=180 (MJ) with a minimal stray field. To guarantee the inner and outer coils running under superconducting conditions, the quenching conditions shown in Fig. 6 are taken as constraints



(a) viewpoint in 3D (b) viewpoint in 2D

Fig. 5. Configuration of the SMES system.

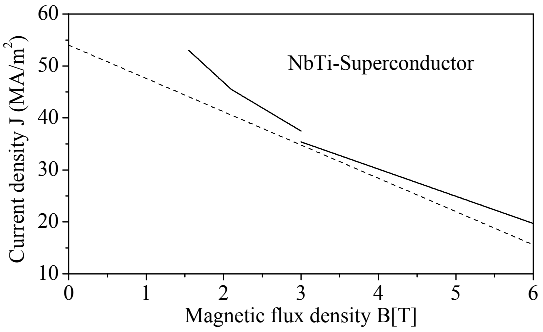


Fig. 6. Quenching curve of the superconductor.

Table 4. Result of reliability calculation

No.	Optimal design **x** [m]			Reliability of $g_2(\mathbf{x})$ [a]		
	R_2	$H_2/2$	D_2	FS-MCS	SS-MCS	MCS
1[10]	3.08	0.239	0.394	0.9805	0.9807	0.9807
2[12]	3.05	0.246	0.400	0.7200	0.7232	0.7231
3[13]	2.6602	0.5574	0.2218	0.9500	0.9512	0.9521
4[14]	3.0988	0.26435	0.3903	0.6679	0.6708	0.6716
5[15]	3.0197	0.3081	0.3496	0.5158	0.5215	0.5210

[a] Reliability of $g_1(\mathbf{x})$ for all cases is 1.0.

in the optimization process as follows:

$$g_i(\mathbf{x}) = 54.0 - 6.4 \cdot |B_{m,i}| - |J_i| \geq 0, \quad i = 1,2 \tag{14}$$

where J_i and $B_{m,i}$ are the current density and the maximum magnetic flux density in the ith coil, respectively.

A. Reliability Calculation

For the three-parameter (radius, height, and thickness of outer coil) SMES optimization problem [10], the three geometric parameters $\mathbf{x} = [R_2,\ H_2,\ D_2]^T$ are treated as uncertain variables. For the first- and second-order sensitivity analysis, the following terms of gradient vector and hessian matrix need to be calculated:

$$\nabla g_i(\mathbf{x}) = \left\{ \partial |B_{m,i}| / \partial R_2 , \partial |B_{m,i}| / \partial H_2 , \partial |B_{m,i}| / \partial D_2 \right\} \tag{15a}$$

$$H(\mathbf{x}) = \left\{ \partial^2 |B_{m,i}| / \partial R_2^2 , \partial^2 |B_{m,i}| / \partial H_2^2 , \partial^2 |B_{m,i}| / \partial D_2^2 \right\} \tag{15b}$$

where the crossed terms of $H(\mathbf{x})$ are ignored. The adjoint variable in (8) is obtained by solving equation as follows:

$$[K][\lambda]_{m,i} = \partial |B_{m,i}| / \partial [A], \qquad i = 1,2. \tag{16}$$

Considering the computational burden of the FEM analysis, the proposed SS-MCS, the conventional MCS, and the FS-MCS are applied with 10,000 maximum test designs and *CL* of 0.95. Table 4 shows the reliability calculation result of optimal designs selected from published papers about the SMES when $\boldsymbol{\sigma}$=[15.3, 10, 10]^T (mm). Taking the reliability from the MCS as a reference value, Fig. 7 shows the relative errors by the FS-MCS and SS-MCS methods. It can be seen that the results of the SS-MCS method match well with the MCS method.

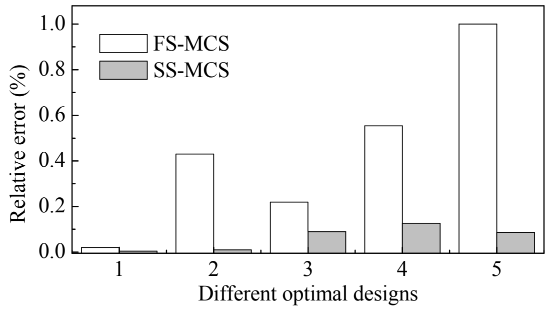


Fig. 7. Relative errors of different approximations.

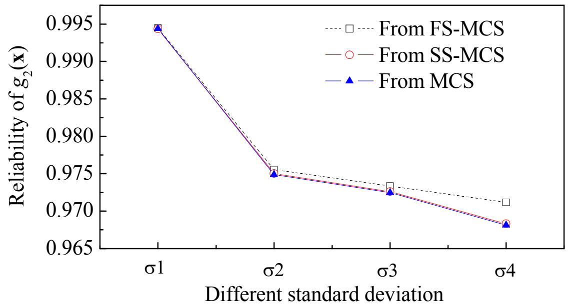


Fig. 8. Comparison of different uncertainties

Table 5. Different uncertainties in Fig. 8

case	Uncertainty of different variables [mm]		
	$\sigma(R_2)$	$\sigma(H_2/2)$	$\sigma(D_2)$
σ1	15.3	10	10
σ2	20	10	15
σ3	30	10	15
σ4	40	15	15

The performance of the FS-MCS and the SS-MCS under different standard deviations listed in Table 5 is investigated by a design $\mathbf{x}$= [3.093, 0.239, 0.391]^T (m). From Fig. 8, it can be seen that the FS-MCS and SS-MCS methods can give exactly same results as the MCS in the narrow uncertainty set such as **σ**1 and **σ**2. The FS-MCS method, however, results in bigger error as the standard deviation increases such as **σ**4. On the other hand, the SS-MCS shows higher accuracy even under bigger uncertainty.

B. Reliability-Based Design Optimization (RBDO)

In the optimization process, the objective function is formulated by combing system energy (E) and stray field (B_s) as follows:

$$f(\mathbf{x}) = \frac{B_s^2}{B_n^2} + \frac{|E - E_0|}{E_0}, \quad B_s^2 = \frac{1}{22} \sum_{i=1}^{22} B_{s,i}^2 \tag{17}$$

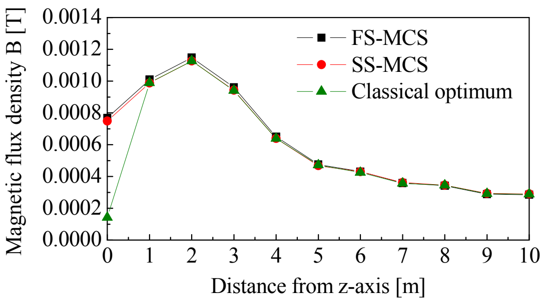
Table 6. Design space and values used in optimization

	Inner coil			Outer coil		
[m]	R_1	$H_1/2$	D_1	R_2	$H_2/2$	D_2
Min	-	-	-	2.6	0.204	0.1
Max	-	-	-	3.4	1.100	0.4
Fix	2.0	0.8	0.27	-	-	-

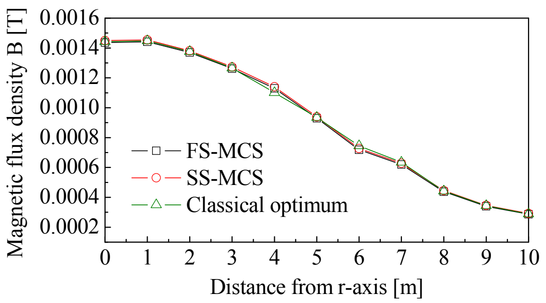
Table 7. Results of reliability-based design optimization

Typical design[a]	Classical	R^t=0.8		R^t=0.9	
		FS-MCS	SS-MCS	FS-MCS	SS-MCS
R_2 [m]	3.0819	3.0827	3.0829	3.0898	3.0895
$H_2/2$ [m]	0.2439	0.2475	0.2472	0.2679	0.2693
D_2 [m]	0.3849	0.3793	0.3795	0.3486	0.3472
$f(\mathbf{x})\times 10^{-2}$	8.7719	8.7732	8.7731	8.8211	8.8246
B_s^2 [10^{-7}T^2]	7.8948	7.8958	7.8958	7.9390	7.9421
E [MJ]	180.0000	179.9999	180.0000	180.0000	179.9999
$g_1(\mathbf{x})$	-7.8959	-7.7884	-7.8813	-7.7919	-7.7991
$g_2(\mathbf{x})$	-1.3835	-1.4764	-1.4775	-2.1140	-2.1310

[a] Each optimal design is selected among 20 independent runs.



(a) Stray field distribution on line a



(b) Stray field distribution on line b

Fig. 9. Stray field distribution of optimal designs (R^t=0.9)

where B_n=3 (mT), and $B_{s,i}$ is the magnetic flux density of the ith sampling point. Then the RBDO is formulated [8, 11] as follows:

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) \\ \text{subject to} & R\left(g_i(\mathbf{x}) \geq 0\right) \geq R_i^t, \qquad i = 1,2 \end{array} \qquad (18)$$

where R^t is the target reliability. Uncertainties are considered in both geometric and physical parameters as $\mathbf{x} = [R_2, H_2, D_2, J_1, J_2]^T$, where $\boldsymbol{\sigma} = [40, 15, 15]^T$ (mm) for geometric parameters, and the current densities follow Gaussian distribution with mean value of $\boldsymbol{\mu} = 22.5$ (MA/m^2) and standard deviation of $\boldsymbol{\sigma} = 0.179$ (MA/m^2), respectively. Other parameters and design range are listed in Table 6 [10].

The single objective particle swarm optimization algorithm is applied to (18) with 30 particles and 300 maximum iterations. The optimization results are shown in Table 7. It can be seen that, with the same target reliability, the reliable designs from different approximations can give similar objective values. All the reliable designs can satisfy the energy requirement; furthermore, as shown in Fig. 9, the solutions from RBDO can also guarantee a smaller stray field compared with the classical optimum. Since the accurate reliability calculation of the SS-MCS, the obtained design is more reliable to give bigger margins for constraints than that from the FS-MCS and the classical optimization. Therefore, the SS-MCS method can guarantee the more reliable solutions in the RBDO.

5. Conclusion

This paper presents the second-order sensitivity analysis by the HDD-AV method based on the FEM. The proposed second-order sensitivity-assisted Monte Carlo simulation method (SS-MCS) is successfully applied to the reliability calculation of electromagnetic device. The numerical results of analytic functions and the SMES system show that the second-order sensitivity can definitely improve the accuracy of the reliability calculation. The SS-MCS method can be used to efficiently improve the feasibility robustness of constraints in the RBDO of the electro-magnetic devices.

This paper clearly brings out problem domains in which the sensitivity-assisted MCS reliability calculation algorithms (FS-MCS and SS-MCS) will have superiority over other counterparts (MCS and RIA) and should encourage motivations to the reliability-based design optimization.

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