Http://dx/doi.org/10.7735/ksmte.2013.22.3.394 *ISSN 2233-6036*

Rockwell Hardness Modeling Using Volumetric Variable

Do-hun Chin[a], Sang-rok Oh[b], Moon-chul Yoon[c*]

체적변수를 이용한 로크웰 경도 모델링

진도훈[a], 오상록[b], 윤문철[c*]

[a] *Zibtek, Republic of Korea*
[b] *Korea polytechnic(VII), Republic of Korea*
[c] *Pukyong National University, 365, Sinseon-ro, Nam-Gu, Busan, 608-739, Republic of Korea*

ARTICLE INFO

Article history:

Received	22	January	2013
Revised	8	March	2013
Accepted	25	March	2013

Keywords:
Confocal laser scanning microscope
Fractal interpolation
HEI
Indented volume
Least square method
Rockwell hardness

ABSTRACT

A new Rockwell hardness (HRC) model using a volumetric parameter by a least square and fractal interpolation method is suggested. The results are also investigated in comparison to real measured hardness data. For this purpose, the measurement of an indented volume is performed using a confocal laser scanning microscope (CLSM), and the captured height encoded image (HEI) is used as an original surface for the calculation of the indented volume. After configuring the surface, the constructed volume is calculated and used as an independent variable for HRC hardness modeling. The hardness model is established using an experimental modeling technique involving a least square algorithm and fractal interpolating model, and this suggested model can be used to reliably predict the Rockwell hardness. These techniques can also be applied to the modeling of the Brinnell and Vickers hardnesses using a volumetric variable.

1. Introduction

Until recently, a study about Rockwell hardness (H_{RC}) modeling was never performed and it is difficult to make a plan about the experimental hardness modeling. Other research about the hardness is just the investigation of measured hardness data considering general and modified mathematical function[1,2]. The H_{RC} hardness is widely used to measure hardness of materials, especially in steel. To measure the H_{RC} hardness of steels, the depth of indented mark is generally used to predict the H_{RC} hardness of the materials. The depth is an independent variable in hardness modeling and they are used to calculate the Rockwell hardness. In the past, to measure the Rockwell hardness, its measuring was performed by using a parameter conditions with depth of indented mark and indenting force of 150 kg_f. However, looking at the distribution of hardness data, it shows some distortion in data and different scattering along the indenter and load shape in experiments. When experimental hardness data are plotted in *x-y* coordinate, the scattering points of depth–hardness (*x-y*) are not linear in their shape (Fig. 1). It locates along the curve. If the relationship of volume and

---

* Corresponding author. Tel.: +82-51-629-6160
Fax: +82-51-629-6150

E-mail address: mcyoon@pknu.ac.kr (Moon-chul Yoon).

hardness is considered, the scattering can be located along a line or curve in measured data. So, a more reliable model must be established considering the experimental data. In this sense, the Rockwell hardness model using volume parameter is suggested to represent this H_{RC} model in a more practical and reliable way. With the development of noncontact measuring technique, the indented volume can be measured using *CLSM* (confocal laser scanning microscope) or other microscope in a micro scale[3-5]. Also, it is possible to represent the Rockwell hardness with an independent parameter of volume in a model. This study shows a new developed technique for measuring the Rockwell hardness by using a volumetric parameter. It has some merits in the sense that it can suggest the mathematical model in 1st, 2nd and 3rd order polynomial model simultaneously. According to a different material, an appropriate model of different order can be used and it also can be selected as a formula to predict the Rockwell hardness in a more reliable way. To include the experimental data, the fractal interpolated model which represents the measured data in a more realistic way is used and compared together. This phenomenon is shown well in this paper. In measuring a volume of indented surface, an optical measurement of *CLSM* is used. Because it used the image data and involves many noise properties in the original indented image. It must be excluded. The HEI data of this generated surface has many noise properties in acquired indented image or its surface. So this noise problem must be overcome by some filtering technique. In this paper, a wavelet filtering technique is considered to remove the dynamic noise properties in the indented surface. In the previous works[3-5], it is well proven that the exact volume can be acquired by wavelet filtering in a lower level without severe deviation in calculating a volume. So this technique is adopted in estimating the volume of indented surface and it is used for the modeling of Rockwell hardness. At last, this result enables the tool developer or its practicing engineers to predict and compare the mechanical property of materials in a more effective way and it can be used to understand the different physical property of new developing materials.

2. Data acquisition and analysis

2.1 Rockwell hardness

Tradition representation of hardness

$$H_{RC} = 100 - 500 \cdot t \qquad (150\ kg) \qquad (1)$$

Suggested model

$$\rightarrow H_{RC} = a_1 + a_2V + a_3V^2 + a_4V^3$$

Rockwell hardness is generally represented with Eq. (1). Theoretical data must be represented with dashed line as Fig. 1. However, the real measured data scatters along the curve of square points. So there are a large amount of errors between measured and theoretical hardness. Fig. 1 shows the Rockwell hardness according to the parameter of indented depth. The hardness can be represented with volume.

Also, Fig. 2 shows the HRC (Rockwell hardness) according

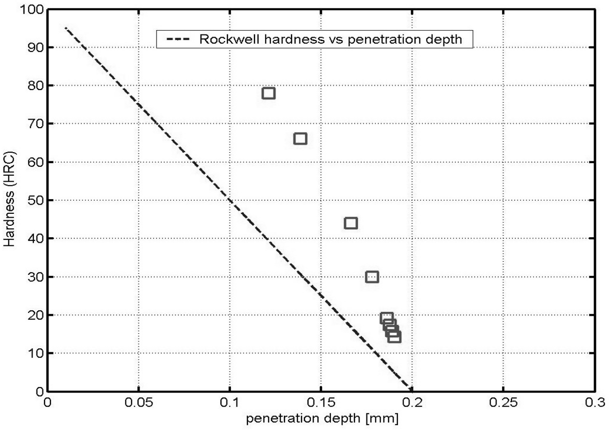


Fig. 1 H_{RC} versus penetration depth

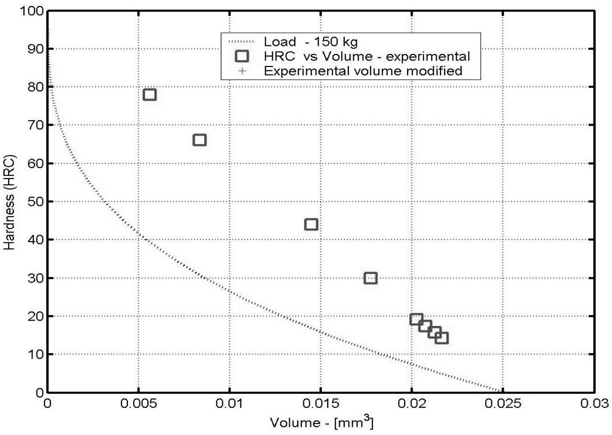


Fig. 2 H_{RC} versus penetrated volume

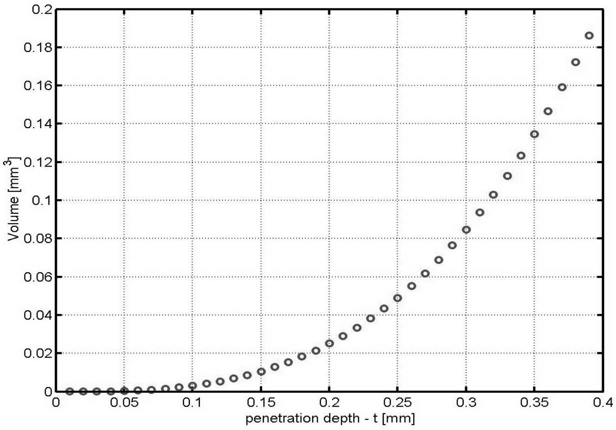


Fig. 3 Volume vs. penetration depth for H_{RC} indenter

to its volume after calculation. Considering above transformation of Eq. (1), the hardness data with depth can also be represented with volume. The volume of an indented specimen is measured by the technique of references[5] and it is used for this process. The measured hardness can be represented in Fig. 1 and Fig. 2.

In comparing the ideal indented line, the measured hardness shows a steeper and negative gradient in shape in Fig. 1. Also the relationship between indented volume and penetrated depth may be represented as Fig. 3.

2.2 Data acquisition of indented mark

Both hardness and their depth are measured by the Rockwell hardness tester and their specimens. Also this specimen is prepared for volumetric measurement by CLSM. After generating the indented surface by wavelet filtering using the HEI data, its volume of indented mark can be calculated using the Eq. (2). For measuring the hardness and its volume of several materials, AL7075, SM45C, SKD61, heat treated SM45C, HP steel for mould, NAK, heat treated SKD61 and Carbides specimen were used. For measurements of hardness at first, the indented specimen is indented and prepared for measuring. Then its image of the indented mark is measured by optical measuring instrument and its topographic image is captured with CLSM as a HEI data of Fig. 6(b) and its volume of indented mark is calculated using the Eq. (2). The basic configuration of CLSM is shown in Fig. 4. In a CLSM system, the key component is a pinhole or confocal aperture which reduces needless lights that exist above or below a focal plane of workpiece. In scanning a specimen in a z-direction, if a maximum intensity at any height is detected, then the data of height at that position is supposed to be saved in a matrix form. Using this matrix, the indented surface position at each pixel is configured with a focus detection technique which equates vertical position. If a maximum light intensity is detected at the arbitrary position of the surface, the z-position can be detected at that pixel location (x_i, y_j) in indented surface and it constitutes a so-called z-matrix, $z = z\ (x_i,\ y_j)$, which is stored as a HEI (Height Encoded Image) data. However, for a wavelet filtering, the HEI data are used for generating a filtered surface of Fig. 7(b) in these studies[3-5].

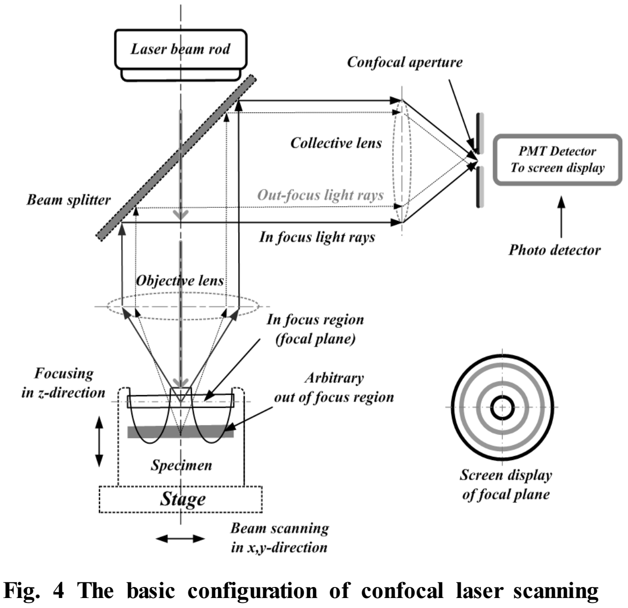


Fig. 4 The basic configuration of confocal laser scanning microscope

2.3 Volume measurement of indented mark

Using the HEI data, the height of a surface is already stored and the volume can be calculated by summation of the differential volume as shown in Fig. 5. The differential volume at arbitrary pixel point (x_i, y_i, z_i), which is represented with the discretized form of Eq. (2) may be integrated as shown in Fig. 5. The number of element into x, y direction is M and N in Eq. (2). Using this equation, the total volume of indented mark may be calculated as[5];

$$Vol_{indented mark} = \sum_{i,j=1}^{M,N} \Delta V_{i,j} = C_1 \sum_{i=1}^{M}\sum_{j=1}^{N} \Delta x \cdot \Delta y \cdot (\bar{z}_{i,j} - z_{i,j}) \qquad (2)$$

Both MBI (maximum brightness image) and HEI (height

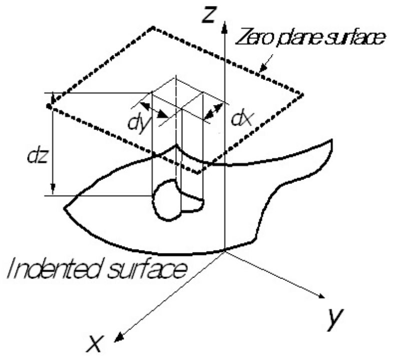
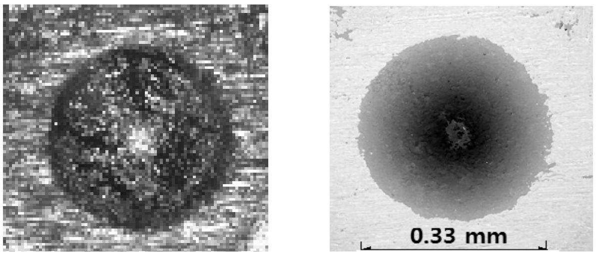


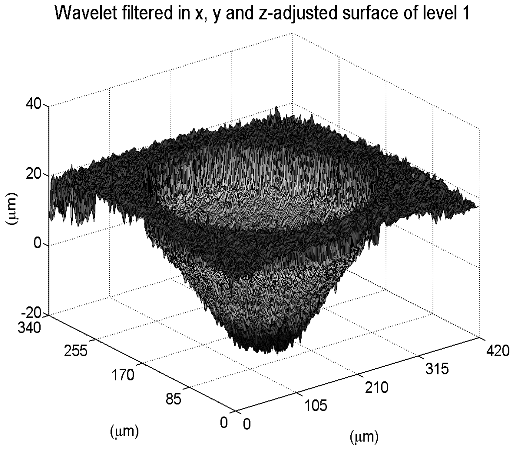
Fig. 5 Differential volume and its estimation



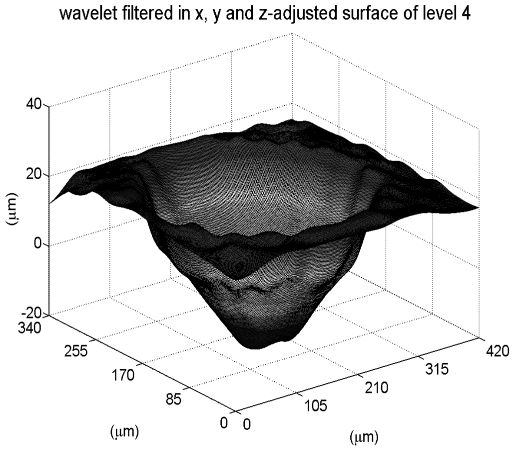
(a) MBI (b) HEI

Fig. 6. Measured indented image by CLSM

encoded image) of micro indented surface are shown in Fig. 6. Using HEI image, its predicted surface can be generated as shown after wavelet filtering (Fig. 7(b)) or not (Fig. 7(a)). To get a higher level surface of wavelet filtering effect, a lower level surface of wavelet filtering is used as original data for calculating the volume of indented mark. The involved noise of photo image processing of HEI into surface is supposed to be removed by wavelet filtering and it generates a smoother approximate surface finally. So for generating a smoother surface, its original raw surface data acquired from a measurement using noncontact CLSM instrument must be filtered with above technique.



(a) at level 1



(b) at level 4

Fig. 7 Construction of indented surface by wavelet filtering

The noise spike of the marked surface is filtered using the 2-dimensional wavelet filtering which is discussed in references [3-5]. If a photo of HEI (Fig. 6(b)) is to be generated as a micro indented surface[3], its surface height data may be may be stored as a z-data, 8 bit value ranging from 0~255, and the resolution of the data into z direction is 0.2 μm. And this information is used for the height compensation of surface in converting the image data. If a small specimen is mounted on the table of microscope, the mounting error of the specimen inevitably occurs in a measured surface. This problem also is solved by tilting compensation technique[3-5]. Referring to

Table 1 Measured volume and its Rockwell hardness

Materials item	AL7075	SM45C	SKD61	heat treated SM45C	HP steel for mould	NAK	heat treated SKD61	Carbides
Volume (mm^3)	0.0056	0.0084	0.0145	0.0178	0.02	0.021	0.0213	0.0216
H_{RC} (Kgf/mm^2)	78	66	44	30	19	18	16	14

these articles, it is well known that any surface which is wavelet filtered results in a same volume regardless of its level if the level is ranging from 1st to 5th. The final indented volumes of H_{RC} indenter mark are generated in various levels considering a wavelet filtering and reliably estimated volume that is indented for the modeling of Rockwell hardness[5]. Using the CLSM measurement and volume calculation technique together, the total volume of indented mark of different specimen can be calculated and it is summarized in Table 1. In calculating the volume with indented depth, the relationship between volume and penetration depth is plotted in Fig. 1 and Fig. 2. As a depth increases, the volume is increasing in a more accelerated slope as shown in Fig. 3. According to these results, it is well known that the volume of the filtered surface below the fourth level is almost equal to its original one which is not filtered. So any volume calculated with the surface below level 4 results in the same volume in a measurement. So any volume can be used as an indented volume for the modeling of hardness.

2.4 Polynomial hardness modeling

For a polynomial Rockwell hardness model, the coefficients of several order models of Eq. (3) - Eq. (5) can be used. And it is supposed to minimize the error ε in Eq. (6). The coefficients of each model are obtained by least square method of Eq. (7) and the algorithm can be summarized as follows;

1st order model : $y = a_0 + a_1 V$ (3)

2nd order model : $y = a_0 + a_1 V + a_2 V^2$ (4)

3rd order model : $y = a_0 + a_1 V + a_2 V^2 + a_3 V^3$ (5)

$$y = X \cdot \hat{a} + \varepsilon \tag{6}$$

$$\epsilon = \begin{bmatrix} \epsilon_o \\ \epsilon_1 \\ \cdots \\ \cdots \\ \epsilon_n \end{bmatrix},\ y = \begin{bmatrix} H_{RCo} \\ H_{RC1} \\ \cdots \\ \cdots \\ H_{RCn} \end{bmatrix},\ \hat{a} = \begin{bmatrix} a_o \\ a_1 \\ \cdots \\ \cdots \\ a_n \end{bmatrix},\ X = \begin{bmatrix} 1 & V_1 & V_1^2 & V_1^3 \\ 1 & V_2 & V_2^2 & V_2^3 \\ \cdots & & \cdots & \cdots \\ \cdots & & \cdots & \cdots \\ 1 & V_n & V_n^2 & V_n^3 \end{bmatrix}$$

The coefficients can be calculated as,

$$\hat{a} = (X^t X)^{-1} X^t y \tag{7}$$

n : No. of exp., k : No. of exp. Variable

2.5 Fractal representation of H_{RC} hardness

The HRC hardness point can be interpolated using an experimental hardness and can generate many random hardness data. In this sense, the fractal interpolation of limited hardness data after coordinate transformation can make a new fractal region data of w_n. And it can be transformed and interpolated data may be generated in matrix form as follows[14] and this fractal transforma- tion is represented as Eq. (8). It shows the relationship of fractal interpolated model using measured data point (V(volume), H_{RC}). In this model, the fractal interpolated hardness point (V(volume), H_{RC}') can be generated with measured hardness point (V(volume), H_{RC}) by the following transformation[14] and its number is magnified fully by setting of the generated data number.

$$W_n\left\{\frac{V'}{H_{RC}'}\right\} = \begin{bmatrix} a_n & 0 \\ c_n & d_n \end{bmatrix}\left\{\frac{V}{H_{RC}}\right\} + \left\{\frac{e_n}{f_n}\right\} \tag{8}$$

Where a_n, c_n, d_n, e_n and f_n are the related transformation constant in the n-th interval and it must be given as a initial condition. The fractal interpolated equation is affine to geometric transformation using Eq. (8). In x-y(V-H_{RC}) plane, the line parallel to x′ and y′ axis is also parallel to x and y axis respectively after transformation. And if the lp is the parted distance of particle points parallel to y axis, then the following scaling factor d_n can be defined[14].

$$|d_n| = \frac{w_n(l_p)}{l_p} \tag{9}$$

Where w_n is the gradient of the geometry and the general fractal dimension D_F by contractive mapping can be written as follows[14].

$$\sum_{n=1}^{N} |d_n|\, a_n^{\ D_F - 1} = 1 \tag{10}$$

The left point w_n of Eq. (8) represents the H_{RC} hardness of fractal interpolated model and it generates the scattered hardness data following the measured hardness.

3. Discussion of H_{RC} model

In Eq. (7), the matrix y is an independent variable for hardness and V in X is the dependent variable of volume. Using this matrix, the coefficients of the polynomial model $\hat{a}$ can be obtained by Eq. (7). It is the coefficient of the model. By curve fitting using the given experimental data of Fig. 2, three models of 1st, 2nd and 3rd order polynomial model for Rockwell hardness is obtained respectively and their coefficients of models can be summarized as follows. Fig. 8 shows the first order model using the H_{RC} data of several specimens of Fig. 2. For this case, the estimated coefficients of first order model are a_0=100.2 and a_1=-3975. The experimental data of Rockwell hardness scatter around a linear line according to volumetric variable. Also they scatter along a convex curve with respect to penetration depth as shown in Fig. 1. As the volume increases, the Rockwell hardness decreases linearly according to volume unlike the case of indented depth.

Also the theoretical hardness with respect to volume is a little bit different than the measured one. So, this curve of polynomial model can represent the hardness more appropriately than that of theoretical one. In a second order polynomial model of Fig. 9, the hardness decreases linearly like the first order model. However, the curve is similar with first order linear line. The section in vertical axis is a little bit lower than the first order model and it is second order curve. Their estimated coefficients of fitted curve in second order model are estimated as a_0 = 97.8841, a_1 = -3555.8004 and a_2 = 15071.3287 respectively. However, they scatter around a line in linear scale and do around the slight convex curve in log scale with respect to indented volume as shown in Fig. 9. The third order model shows the similar decreasing trend for the hardness according to indented volume in Fig. 10. Also, the estimated polynomial coefficients of the curve are calculated as a_0 = 101.8533, a_1 = -4639.9769, a_2 = 70277.107 and a_3 = -2036461.8506 respectively.

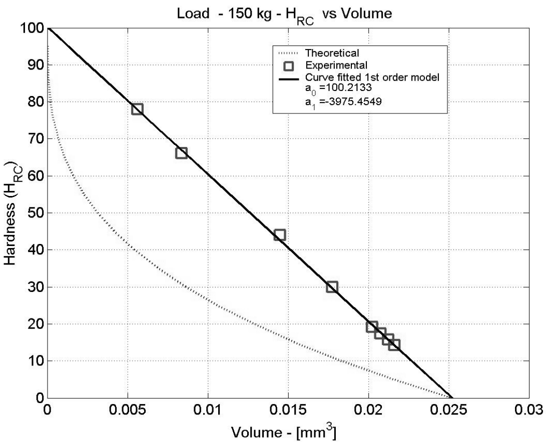


Fig. 8 1st order model (a_0 = 100.2, a_1 = -3975)

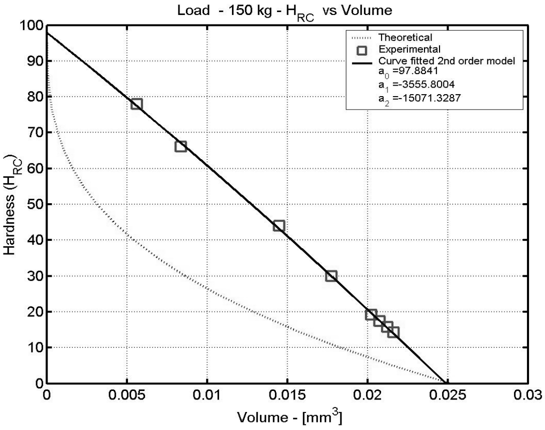


Fig. 9 2nd order model (a_0 = 97.884, a_1 = -3555.8, a_2 = 15071.3287)

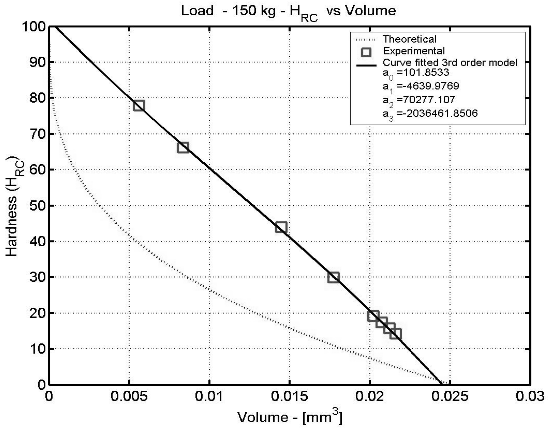


Fig. 10 3rd order model (a_0 = 101.85, a_1 = -4639.9769, a_2 = 70277,107, a_3 = -2036461.8506)

The section in vertical axis is a little bit higher than that of the first order model. Because the scattered data locates around the linear line, the fitted curve of third order seems like a linear line as Fig. 10 in linear scale. Even if the points (theoretical volume - H_{RC} hardness) locates along the solid curve, the real data looks like scattering points along a linear line. So these least square polynomial models of 1st~3rd order, which uses the real measured data, is more realistic and close to real hardness data. As a result, these developed models can be applied for representing several hardness, such as Brinnell, Vickers, Knoop easily without distortion.

Furthermore, the polynomial model can be compared with fractal model as in Fig. 11. The dotted and scattered data is the hardness data generated by fractal interpolation. It also locates around the experimental data. In general, a model is generally represented by function. However, the fractal interpolation can also be used to construct a model that match with experimental data exactly. The scattering of the interpolated data of Fig. 11 is not clear. Also it can be represented in log scale as Fig. 12. If Fig. 11 can be zoomed, then the data can be plotted more clearly as Fig. 13. The least square polynomial model of 1st~3rd order, which uses the real data, can be compared with a fractal interpolated model. Fig. 11 shows the polynomial model and Fig. 11 also shows a fractal interpolated model data and H_{RC} hardness data simultaneously. Looking at the figure, the scattered data of fractal model is more close to measured data than that of any polynomial model. So the fractal interpolated model can represents the experimental H_{RC} hardness more close to real data. This phenomenon can be seen clearly with red point data in Fig. 12 and Fig. 13, which is represented in 3rd order polynomial model and fractal interpolated model together. Fig. 12 shows the deviation of scattered data around the polynomial curve. Even if the scattered H_{RC} hardness data of fractal model can't be represented as a function like polynomial model, the generated data of fractal model can represent the real hardness data more closely to a measured hardness.

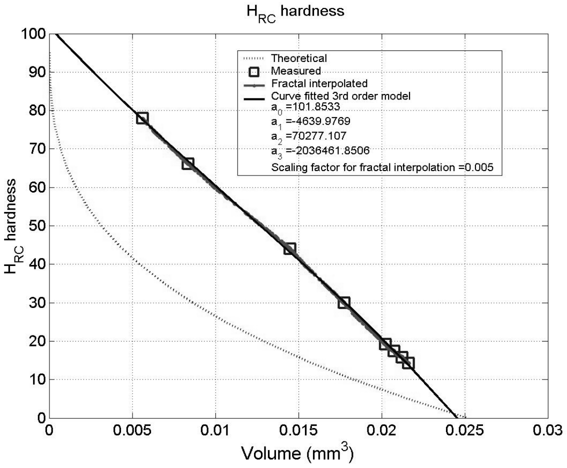


Fig. 11 H_{RC} curves of 3rd order polynomial order model with volume parameter

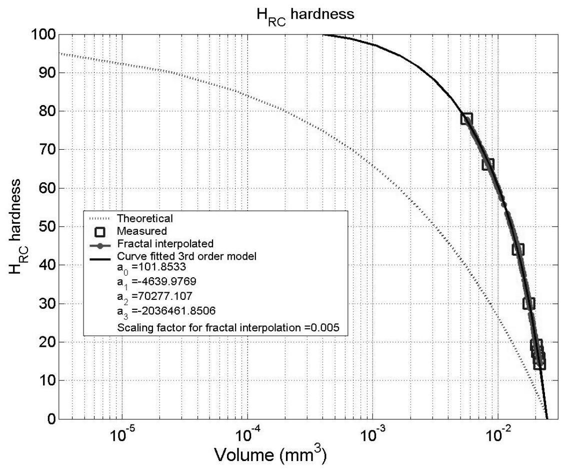


Fig. 12 H_{RC} curves of different order models with volume parameter

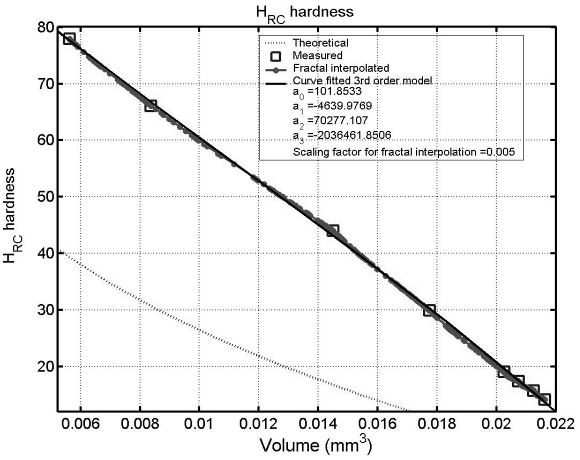


Fig. 13 H_{RC} scattering with fractal interpolation and 3rd order curve fitting model

4. Conclusions

The Rockwell hardness model using volumetric parameter is suggested and following conclusion can be drawn from this study;

(1) The characteristics of H_{RC} model are analyzed and its model represented with indented depth appears in a convex curve that is quite different with theoretical representation. And the distribution of Rockwell hardness data using indented volume appears along a linear line and it can alternatively be represented in a polynomial 1st, 2nd and 3rd order models by least square method.

(2) If this technique can be embedded in H_{RC} hardness tester, it can represent the hardness behavior better than the conventional one regardless of depth or volume. The 1st order model is enough in representing the Rockwell hardness than other 2nd and 3rd order model. Also its representation with volumetric variable may also be adopted in a higher order polynomial model.

(3) The fractal hardness model suggested is compared with polynomial one and the hardness model by fractal interpolation can represent real experimental data with less error than the polynomial model. And it can also be used to represent the scattered experimental H_{RC} hardness data.

(4) Further study on error of this polynomial of higher order and fractal model according to the experimental testing of different materials must be studied in detail using this volume variable. Also this technique can be applied to other Brinnell, Vickers and Knoop hardness modeling easily.

Acknowledgements

This work was supported by a Research Grant of Pukyong National University (2013)

References

[1] Jin, J. W, Kwak, S. J., Yoo, D. O., Kim, T. S., Kang, K. W., 2012, Hardness Evaluation of Spot Welding Using Instrumented Indentation Technique, Transactions of the KSME A, 36:9 1081-1086.

[2] Cha, S. I., Hong, S. H., Ha, K. H., Kim, B. K., 2002, Model on the hardness of nanocrystalline WC/Co cemented carbides, Proceeding of the KPMIC, Spring conference. 37.

[3] Yoon, M. C., 2008, Droplet geometry and its volume analysis, J. of KSTLE, 24:6 320-325.

[4] Jeong, J. S., Cho, H. G., Yoon, M. C., 2009, Crater wear volume calculation and analysis, J of KSMTE, 18:3 248-254.

[5] Yang, J. Y., Yoon M. C., 2012, Indented surface configuration and its volume calculation, J. of KSMTE, 21:5 708-713.

[6] Yang, J. Y., Yoon, M. C., 2011, Machined surface generation using wavelet filtering, J. of MST, 25:3 639-645.

[7] Yuan, C., Peng, Z., 2005, Surface Characterization using wavelet theory and confocal laser scanning microscopy, Trans. of the ASME, 127 394-404.

[8] Corle, T., Kino, G., 1996, Confocal Scanning Optical Microscopy and Related Imaging Systems, San Diego: Academic Press.

[9] Lee, H. W., Kwon W. T., 2010, Determination of the minute range for RSM to select the optimum cutting conditions during turning on CNC lathe, J. of MST, 28:8 1637-1645.

[10] Park, K. H., Jorge, O. Y., Yoon, M. C., Kwon, P., 2010, A study on droplets and their distribution for minimum quantity lubrication (MQL), I. J. MTM, 50 824-833.

[11] Fuh, K., Chang, H., 1977, An accuracy model for the peripheral milling of aluminum alloys using response surface design, J. of MPT, 72:1 42-47.

[12] Montgomery, D. C., 2001, Design and analysis of experiments (5th edition), New York, Wiley, USA.

[13] Cho, H. G., Chin, D. H., Yoon, M. C., 2009, Dynamic filtering of end-milling force using wavelet filter bank, J. of KSMTE, 18:4 381-387.

[14] Barnsley, M. F., 1993, Fractals everywhere, Academic Press Inc.

[15] Yoon, M. C. Kim, B. T., Chin, D. H., 2006, Roundness modelling by Fractal interpolation, J. of KSMTE, 15:3 67-72.

[16] Yoon, M. C., Chin, D. H., 2008, Fractal roundness modelling of a measured profile of a cylindrical object, Int. J. of AMT, 35:2 1156-1165.