Wave Propagation in a Strip Plate with Longitudinal Stiffeners
보강재를 가진 무한길이 띠 평판의 진동해석

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ABSTRACT

It is important to understand the vibrating behavior of plate structures for its many engineering applications. In this study, the vibration characteristics of strip plates that have finite width and infinite length are investigated theoretically and numerically. The waveguide finite element(WFE) approach, which is an effective tool for studying waveguide structures, is used in this study. The WFE method requires only a cross-sectional finite element model, and uses theoretical harmonic solutions to assess wave propagation along the longitudinal direction. First, WFE results for a simple strip plate are compared with the theoretical results(i.e., dispersion diagrams and point mobilities) to validate the numerical model. Then, in the numerical analysis, different numbers of longitudinal stiffeners are included in the plate model to investigate the effects of stiffeners in terms of the dispersion curves and mobilities. Finally, the dispersion curves of a stiffened double plate are obtained to examine the characteristics of its wave propagation.

요 약

보강재를 가진 평판 구조로 이루어진 많은 구조물의 진동 현상을 해석하기 위해서는 평판 요소에 대한 진동 특성을 이해하는 것이 필요하다. 이 연구에서는 폭이 유한하고 길이가 무한한 띠 평판의 진동 특성을 이론 해석과 수치 해석을 통해 알아보고자 한다. 수치 해석 기법으로는 단면의 형상이 길이 방향으로 일정한 도파관 구조물의 진동 해석에 효과적인 도파관유한요소법(waveguide finite element method)을 사용한다. 도파관유한요소법은 구조물의 2차원 단면만을 유한요소 모델링하고, 길이 방향으로는 파동이 조화 진동하면서 전파한다고 가정한다. 이 논문에서는 먼저 띠 평판의 분산 선도와 가진점 모빌리티에 대한 수치 해석 결과를 이론 해석 결과와 비교하여 수치 해법의 타당성을 검증한다. 그리고 수치 해석을 이용해 보강재가 부착된 평판에 대한 분산 선도와 가진점 모빌리티를 구하고, 보강재가 띠 평판의 파동 전파 및 진동에 미치는 영향을 검토한다. 마지막으로 보강재가 부착된 이중 평판(double plate)에 대해 분산 선도를 구하고, 이로부터 파동 전파 특성을 살펴본다.

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1. Introduction

Many large structures, like ships and trains, are built using plates with complex stiffeners. These structures are often simplified as waveguides having uniform cross-sections along their lengths. To be able to predict the vibrational responses of these waveguide structures, it is necessary to understand the propagating behavior of the waves in stiffened plates as a basic elemental structure. The characteristics of wave propagation in a stiffened plate are investigated in Ref. (5) using the finite element method(FEM). However, this method is unsuitable for the forced response of an infinite-length structure. In this paper, the vibration characteristics of infinite-length strip plates are investigated numerically by means of the waveguide finite element(WFE) method. The WFE method models only 2D cross-sections of the waveguide structures but takes into account the 3D nature of the infinite extent of the waveguide. In the numerical analysis, different numbers of longitudinal stiffeners are combined with strip plates to investigate the effects of stiffeners on the wave propagation in plates in terms of the dispersion diagrams and mobilities. Finally, a stiffened double plate is formed to examine the characteristics of its wave propagation.

2. Theoretical Analysis

Before starting the numerical analysis of the strip plates, a theoretical analysis is performed for a simply supported strip plate without stiffeners to understand the general behavior of the base strip plate. The strip plate considered in this study has width l_y in the y direction and is infinitely long in the x direction, as illustrated in Fig. 1. The properties and dimensions of the strip plate are given in Table 1.

For a thin, undamped plate, the z-directional

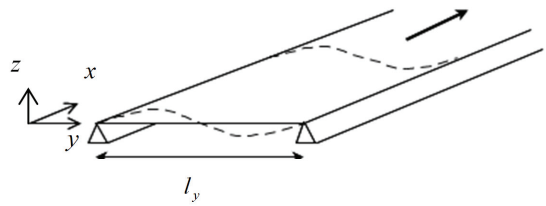

Fig. 1 A simply supported plate strip model

Table 1 Dimensions and material properties of the strip plate

Parameters	Value	Units
E	7.1×10^{10}	N/m^2
υ	0.332	
h	6	mm
l_y	1	m
ρ	2.7×10^3	kg/m^3
η	0.1	

displacement $w(x,y,t)$ satisfies the following differential equation.

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\rho h\frac{\partial^4 w}{\partial t^4}=0, \qquad (1)$$

where D is the bending stiffness, E is Young's modulus, h is the plate thickness, υ is Poisson's ratio, and ρ is the mass density of the plate. Due to the simply supported boundaries at y=0 and y=l_y, the vertical displacement of the strip plate along the y direction has the shape of a sine wave. It is assumed that the vertical displacement has time and spatial harmonic motion in the x direction. Therefore, $w(x,y,t)$ is defined as Ref. (1)

$$w(x,y,t)=\sum_{m=1}^{\infty}\sin(\frac{m\pi}{l_y}y)e^{-ik_{x,m}x}e^{i\omega t}, \qquad (2)$$

where m is the order of the strip plate's mode in the y direction. Substituting this into Eq.(1) yields

$$(k_{x,m}^2+k_{y,m}^2)^2=k_B^4, \qquad (3)$$

$$k_{x1,m} = \pm\sqrt{k_B^2 - k_{y,m}^2}\ , \tag{4}$$

and

$$k_{x2,m} = \pm\sqrt{-k_B^2 - k_{y,m}^2}\ , \tag{5}$$

where k_B is the free bending wavenumber, and k_x and k_y are the wavenumbers of the plate in the x and y direction, respectively. Eq. (3) has four solutions for k_x, which are divided into two different wave solutions for each m, as given in Eqs. (4) and (5). The frequency at which k_B=k_y is referred to as the m^{th} cut-on frequency ω_m. When a force is applied to the plate, the structural responses can be calculated from the sum of all the wave components sustained in the strip plate. For a force acting at x=0 and y=y_0, the point mobility is written as Ref. (2)

$$Y(\omega)=\sum_{m=1}^{\infty}\frac{\omega}{Dl_y k_{x1,m}(k_{x1,m}^2-k_{x2,m}^2)}\left[1-\frac{k_{x1,m}}{k_{x2,m}}\right]\sin^2(k_y y_0), \tag{6}$$

where $D'=E(1+i\eta)h^3/12(1-\upsilon^2)$ and η is the damping loss factor. The experimental dispersion diagrams and point mobilities will be compared with the numerical values later in this paper to validate the numerical results.

3. Waveguide Finite Element Analysis

3.1 Equation of Motion

The WFE equation is given by Ref. (3)

$$\{\mathbf{K}_4(-i\kappa)^4+\mathbf{K}_2(-i\kappa)^2+\mathbf{K}_1(-i\kappa)+\mathbf{K}_0-\omega^2\mathbf{M}\}\tilde{\mathbf{\Phi}}=\mathbf{0}, \tag{7}$$

where κ is the x-directional wavenumber, $\mathbf{K}_4$, $\mathbf{K}_2$, $\mathbf{K}_1$ and $\mathbf{K}_0$ are the matrixes that come from the stiffness of the structure, $\mathbf{M}$ is the mass matrix, and $\tilde{\mathbf{\Phi}}$ is the displacement vector representing the shape of the cross-sectional deformation. Eq. (7) has two unknown parameters of frequency and wavenumber. If a wavenumber is given, Eq. (7) can be solved for the frequencies, which represent the dispersion relations of the propagating waves. Conversely, if a frequency is given, Eq. (7) can be solved to obtain wavenumbers for all the waves, including the near-field ones, which are required to predict forced responses.

Point mobility can be expressed at x=0 for x>0 Ref. (4)

$$Y(\omega) = i\omega\sum_{m=1}^{\infty}\mathbf{A_m}\tilde{\mathbf{\Phi}}_\mathbf{m}\ , \tag{8}$$

where $\mathbf{A_m}$ and $\tilde{\mathbf{\Phi}}_\mathbf{m}$ are the amplitude and deformation shape, respectively, of the m^{th} wave.

3.2 Modeling a Stiffened Plate

The cross-sectional model of a plate with three stiffeners is shown in Fig. 2. The base plate is simply supported at both ends, and the properties of the base plate are the same as those listed in Table 1. The thickness and height of the stiffeners are set to 0.6 cm and 5 cm, respectively. The distance between the adjacent stiffeners is set to 25 cm. In the cross-sectional FE modeling, plate elements are used as illustrated in Fig. 2. This WFE model consists of 55 elements with 56 nodes and has 218 degrees of freedom. Fig. 2 illustrates a strip plate with three stiffeners; however, in the analysis, plates with either a single or three stiffeners are investigated to evaluate the effects of the stiffeners.

3.3 Validation of WFEM

To justify the use of the WFE method, a base

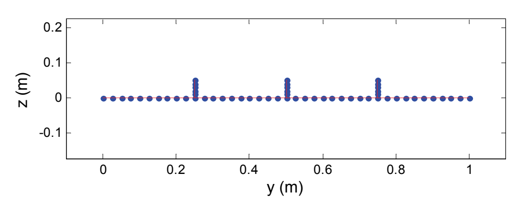

Fig. 2 Model of a cross-section of a stiffened plate

plate is investigated, and the investigation results are compared with theoretical results in this section.

The propagating features of the waves along the base plate can be understood using the dispersion relations obtained from Eqs. (4) and (5) theoretically and Eq. (7) numerically. These dispersion curves for the base plate without stiffeners are compared in Fig. 3. In Fig. 4, the point mobility predicted by the WFE method is illustrated together with the theoretical one. The excitation was applied at x=0 and y=0.425 m.

The results in Figs. 3 and 4 show that the WFE results coincide well with the theoretical ones. Therefore, it can be said from Figs. 3 and 4 that the WFE approach can accurately predict the structural response of the strip plate.

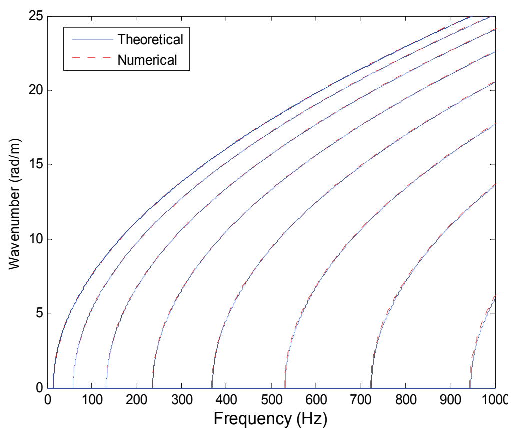

Fig. 3 The dispersion diagram of a strip plate

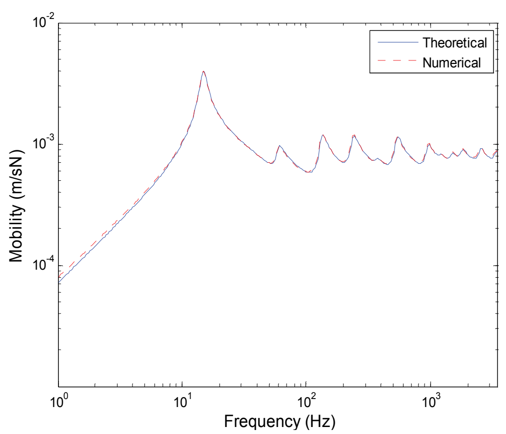

Fig. 4 The point mobilities of a strip plate

3.4 Results for Characteristics of a Plate with Stiffeners

The dispersion diagram in Fig. 3 shows that there are many propagating waves, which are distinguished by the modes in the y direction as given in Eq. (2). As frequency increases, higher-order modes are cut-on consecutively. The point mobility calculated at x=0 and y=0.425 m shows that it has a maximum response at the cut-on frequency of the first mode.

For a plate with a single stiffener, the dispersion diagrams can be obtained separately using the symmetric and anti-symmetric boundary conditions at the middle of the plate where the geometry becomes symmetric. To investigate the effect of a stiffener on the plate's dynamic response, a single stiffener with a height of 12 cm is used.

In the case of a symmetric boundary condition in the middle of the plate, the dispersion relations are shown in Fig. 5, which correspond to waves possessing odd numbers of mode m in the y direction. It can be seen from Fig. 5 that each dispersion curve can be divided into three regions of different slopes. In order to determine the physical behavior of the each region, three equivalent

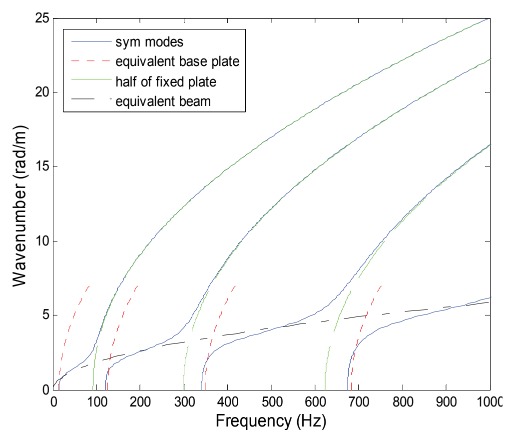

Fig. 5 Dispersion diagrams of symmetric modes of a stiffened plate with a single stiffener

systems are introduced; their dispersion curves are compared in Fig. 5 with those of the stiffened plate. The three systems regarded are an equivalent plate with a smeared stiffener mass, an equivalent beam Ref. (5) and a half width plate having a clamped support at one end and a simple support at the other end. It can be seen from Fig. 5 that the dispersion curves of the stiffened plate are well approximated by these three equivalent structures for each frequency regions. Fig. 6 illustrates 3D deformation shapes of the second symmetric wave in Fig. 5 at two different frequencies. As shown in the upper plot in Fig. 6,

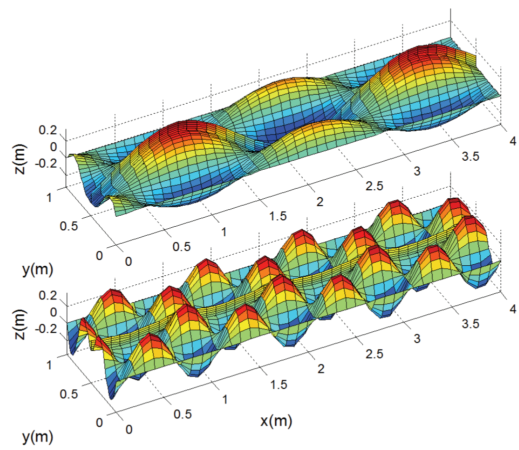

Fig. 6 Deformation shape of second symmetric modes at 204 and 350 Hz

displacement of the stiffened plate at 204 Hz, which belongs to the second region, has a similar deformation shape to that of an unstiffened plate, but the bending stiffness is predominantly governed by the stiffener. On the contrary, at a higher frequency of 350 Hz, displacement of the stiffener becomes nearly zero, as shown in the lower plot in Fig. 6, which belongs to the third region of the dispersion curve. In this third region, dispersion curves of the stiffened plate converge to those of the half width base plate, which is simply supported at one end and clamped at the other end.

For the case of anti-symmetric boundary conditions in the middle of the plate, the dispersion relations are shown in Fig. 7. The dispersion curves in Fig. 7 are quite different from those in Fig. 5. The first curve in Fig. 7 is nearly the same as that of the unstiffened plate, while the second one, of the unstiffened plate, is split into two dispersion curves in the stiffened plate case. This splitting takes place with the vibration of the stiffener, either in-phase or anti-phase with respect to the base plate. For this anti-symmetric wave, the stiffener works like a dynamic absorber.

For the plate with three stiffeners shown in Fig. 2, the dispersion curves are illustrated in Fig. 8. As shown in Fig. 8, the dispersion curves are

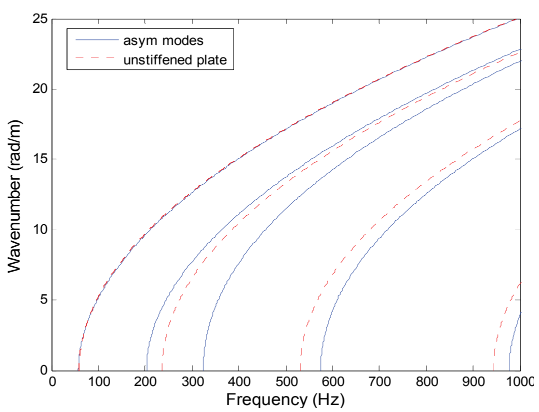

Fig. 7 Dispersion diagrams of asymmetric modes of a stiffened plate with a single stiffener

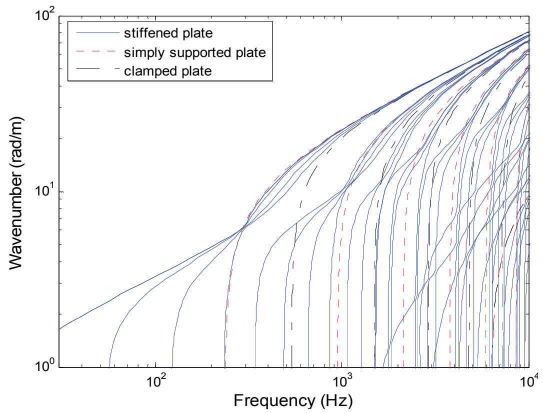

Fig. 8 Dispersion diagram of a stiffened plate with three stiffeners

bound into several groups as the wavenumber increases. Each group contains four waves in general, which correspond to the number of bays of the stiffened plate separated by three stiffeners. To approximate the dispersion curves of the stiffened plate, dispersion curves of the single-bay structure are compared for simply supported-simply supported and clamped-clamped boundary conditions at each end. Dispersion curves are of the single bay of the stiffened plate. It can be seen in Fig. 8 that four waves in each group are bound by those of the single bay. Note that there is an exceptional group (the third group) in Fig. 8 that is not bound by the waves of the single-bay structure. This group contains three curves, which represent bending waves propagating along the stiffeners. The deformation shape of the stiffened plate in the y direction in this group at 3922 Hz is shown in Fig. 9. The three stiffeners in Fig. 9 have a quite large deformation in the y direction, which does not appear in the other groups. The stiffened region of the base plate also has a similar y-directional deformation, as shown in Fig. 9.

Point mobilities were calculated for the unstiffened plate, the stiffened plate with a single stiffener and three stiffeners at two different excitation points, as shown in Fig. 10 and Fig. 11. The single stiffener has the same properties and dimension as the three stiffeners. When a point force is applied in the middle of the plate, the responses of these stiffened plates generally decrease compared to that of the unstiffened one, due to the presence of the stiffeners, as shown in Fig. 10. If a point force is off the center of the plate, the point mobilities of these stiffened plates become lower than that of the unstiffened plate because of the stiffeners at low frequencies, but at high frequencies, these stiffened plates have a level of response similar to that of the unstiffened plate, despite the added stiffeners.

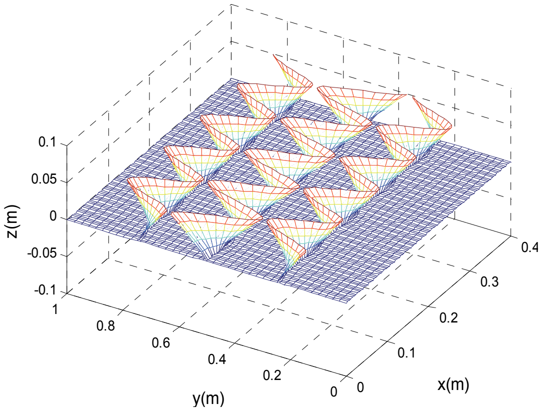

Fig. 9 Deformation shape in the y direction of a stiffened plate with three stiffeners

3.5 Results for Stiffened Double Plate

Wave propagation in a stiffened double plate is investigated by adding an upper plate on top of

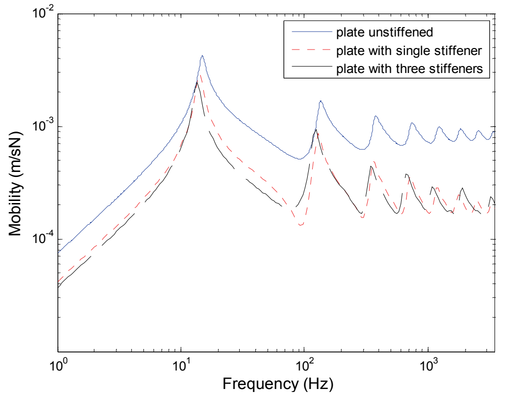

Fig. 10 Point mobilities of plates excited at y=0.5 m

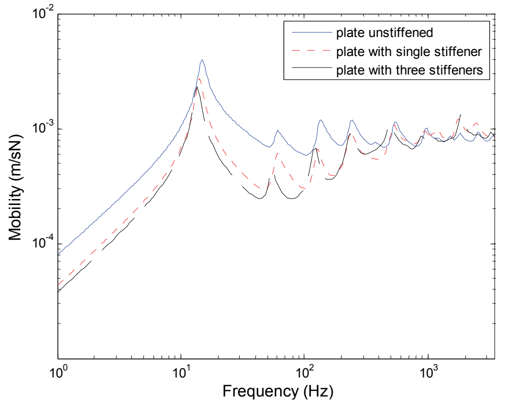

Fig. 11 Point mobilities of plates excited at y=0.425 m

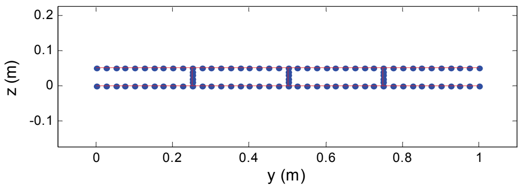

Fig. 12 Model of a cross-section of a stiffened double plate

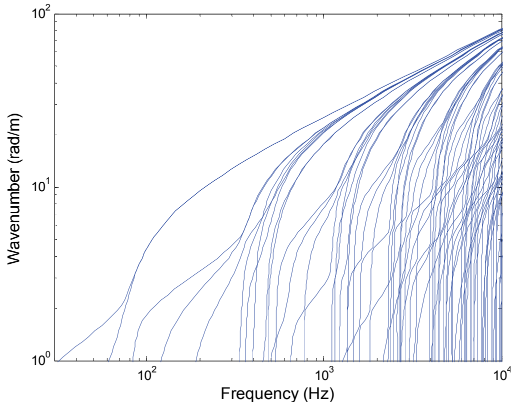

Fig. 13 Dispersion diagram of a stiffened double plate

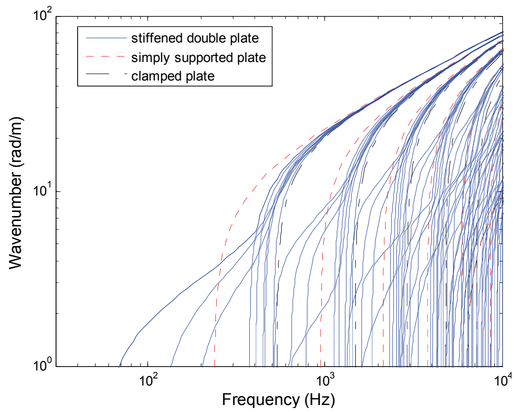

Fig. 14 Dispersion diagram of a stiffened double plate blocked by stiffeners

the stiffeners, as shown in Fig. 12. The upper plate and the lower base plate have the same properties and dimensions. Dispersion curves of the stiffened double plate are shown in Fig. 13. The curves in Fig. 13 may be classified into three types. The first two waves had flapping modes of the unconstrained bays in the upper plate. The next six waves corresponded to those propagating along the six bays (four in the lower plate and two in the upper plate). These two types of waves are shown repeatedly in Fig. 13. Because the stiffeners are restricted by the lower and upper plates, the waves propagating along the stiffeners, which can be considered the third type of curve, occur at fairly high frequencies, and hence, are not shown in Fig. 13.

When the stiffened double plate is blocked by stiffeners at the both edges of the plate, the dispersion curves of the stiffened double plate assume forms as shown in Fig. 14. Each group consists of eight waves. Flapping modes do not appear anymore because the upper plate is constrained by added stiffeners. In each group, eight waves can be classified into two types. Four waves are in-phase, and the rest are in anti-phase between the upper and lower plate. It can be seen in Fig. 14 that eight waves in each group are bound by those of a single bay, a behavior similar to that in a stiffened plate with three stiffeners.

4. Conclusion

In this study, the characteristics of wave propagation along plates with stiffeners were investigated numerically using the WFE method. For a strip plate with a single stiffener, it was found that the stiffener increased the stiffness of the base plate at low frequencies, while it acted like a fixed boundary at high frequencies. From the investigation of a plate with three stiffeners, it was observed that the wave behavior of the stiffened plate could be interpreted clearly using the bay segments. In terms of mobility, it was seen that the stiffeners reduced the responses of the plates at low frequencies, but at high frequencies, these stiffened plates may have had a level of response similar to that of the unstiffened plate, depending on the excitation and receiving locations. Even for the stiffened double plates, the dispersion curves were grouped with the number of bay segments and were well described by the wave behavior in the bay segment.

The forced responses will be calculated for the stiffened double plate as the next step. Based on the work presented in this paper, the sound radiation from stiffened plates will be calculated by coupling boundary elements to the WFEs in further studies on the same topic.

Acknowledgements

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