

# Median lifting optimization for lossy edge-dominant image compression

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**Abstract:** In JPEG2000, the Cohen-Daubechies-Feauveau (CDF) 9/7-tap wavelet filter is implemented using the conventional lifting scheme. On the other hand, this wavelet filter has two problems: the filter coefficients remain complex, and the conventional lifting scheme does not consider the image edges in the coding process. This paper proposes an effective lifting scheme to solve these problems. For this purpose, optimal 9/7-tap wavelet filters were designed in two steps. In the first step, the appropriate filter coefficients were selected. In the second step, a median operator was employed to consider the image edges. The experimental results with the median lifting scheme and the combination of filter optimization with the median lifting show that the proposed methods outperform the well-known CDF 9/7-tap wavelet filter of JPEG2000 on the edge-dominant images.

**Keywords:** Wavelet transforms, Image coding, Lifting scheme, Filter design, JPEG2000

## 1. Introduction

Grossmann and Morlet [1] proposed wavelets, which are known as the first-generation wavelets. These were implemented using the convolution scheme. A review of the literature showed that the lifting scheme can be used to implement wavelets, known as second-generation wavelets [2, 3]. The lifting scheme outperforms the convolution scheme in a variety of ways: it requires less computation and less memory space, can produce the integer-to-integer wavelet transform easily, and is both invertible and reversible.

In JPEG2000, the CDF 9/7-tap wavelet filter was implemented for lossy image compression using the lifting scheme. Several wavelet filters with different coefficients have been proposed based on the CDF 9/7-tap wavelet filter and its lifting scheme. In the design process, scientists use various parameters to determine these coefficients.

Several 9/7-tap wavelet filters with different coefficients exist. Daubechies et al. [3] used the lifting scheme and factoring method with the irrational number,

$\alpha = -1.586134342\dots$ , to determine the coefficients of the CDF 9/7-tap wavelet filter. Similarly, Guangjun et al. [4] proposed a simple 9/7-tap wavelet filter using the lifting scheme and another parameter  $\alpha = -1.5$ . Compared to the CDF 9/7-tap wavelet filter, their filter provides similar performance in terms of the peak signal-to-noise ratio (PSNR), but  $\alpha$  was simplified to a rational number. Liang et al. [5] used a temporary parameter,  $t = 1.25$ , to generate a filter (for the CDF 9/7-tap wavelet filter,  $t = 1.230174\dots$ ). This filter performs at the same level, but its parameter  $t$  is less complex than that of a CDF 9/7-tap wavelet filter.

Although the wavelet filters proposed by Guangjun et al. and Liang et al. perform equally well or are slightly better than the CDF 9/7-tap wavelet filter [4, 5], their performances are not optimal. The main concern is whether there is any other  $\alpha$  value and whether its corresponding coefficients of wavelet filters can provide better performance with fewer complex coefficients than those of the CDF 9/7-tap wavelet filter.

A range of methods have been proposed to enhance the performance of the conventional lifting scheme. These methods include several adaptive lifting schemes [6-10]. In the following studies, the authors used "directional" and "orientation" terms in their methods. Velisavljevic et al. [11] proposed novel anisotropic transforms for images that

use separable filtering in many directions, including horizontal and vertical. In JPEG2000, Chang et al. [12] applied a directional quincunx lifting scheme to obtain better coding performance. Furthermore, Ding et al. [13] and Wu et al. [14] proposed a directional lifting scheme, which combines the conventional lifting scheme of JPEG2000 with a concept of H.264 intra prediction to obtain doubled coding efficiency on images with rich orientation features. After dividing an image into blocks, Ding et al. and Wu et al. first used the energy function to determine the direction of each block, and subsequently used intra-prediction to encode each block. Besides, Liu et al. [15] used weighted functions to predict different directions and code images with rich orientation features in the proposed weighted adaptive lifting scheme. Gerek et al. [16] used an orientation adaptive lifting scheme based on the 2D orientation-adaptive prediction filter. Moreover, Chang et al. [17] proposed a direction-adaptive discrete wavelet transform (DWT) implemented by the lifting scheme for the anisotropic image model.

In JPEG2000, unfortunately, both the first-generation and second-generation wavelets disregard the image edges in the image coding operation. In addition, the image edges are affected by the ringing artifacts during the coding process. To solve these problems, new ideas were developed to enhance the coding efficiency and eliminate the ringing artifacts considering the image edges.

This paper proposes a new coding algorithm for edge-dominant images. The optimal 9/7-tap wavelet filters were designed by choosing an  $\alpha$  value that can result in the optimal performance from the entire possible range of  $\alpha$ . Because median lifting, being a nonlinear operation, yields much better image quality for edge-dominant images [18, 19], this paper proposes a new lifting scheme that employs the median operator. An efficient lifting scheme was then obtained based on filter optimization and median operation.

The remainder of this paper is organized as follows. Section 2 describes the numerical analysis and the conventional lifting scheme. Section 3 reports both methods to design the optimal wavelet filters and the proposed lifting scheme. Section 4 presents the experimental results and analysis. Finally, Section 5 concludes this paper.

## 2. Numerical Analysis and Conventional Lifting Scheme

This section presents numerical analysis to construct a 9/7-tap wavelet filter. The conventional lifting scheme used in JPEG2000 is described briefly.

### 2.1 Numerical Analysis

In wavelets, the forward transform uses a low-pass filter,  $h$ , and a high-pass filter,  $g$ , and subsampling. The inverse transform takes upsampling and then uses a low-pass filter,  $\tilde{h}$ , and a high-pass filter,  $\tilde{g}$ .

In the  $z$ -domain, based on [3],  $h$  can be expressed:

$$h(z) = \sum_{k=k_{\min}}^{k_{\max}} h_k z^{-k}, \quad (1)$$

where  $k_{\min}$  and  $k_{\max}$  are the lowest and highest integer numbers, respectively, for which  $h_k$  is a nonzero coefficient.

The functions of  $h$  and  $g$  in the  $\omega$ -domain are defined as reported in [20]:

$$H(\omega) = h_0 + 2 \sum_{n=1}^{L_1} h_n \cos n\omega, \quad (2)$$

$$G(\omega) = g_0 + 2 \sum_{n=1}^{L_2} g_n \cos n\omega, \quad (3)$$

where  $L_1 = 4$  and  $L_2 = 3$  in the CDF 9/7-tap wavelet filter.

If  $H(\omega)$  and  $G(\omega)$  construct a bi-orthogonal wavelet, they must satisfy the normalized conditions:

$$H(0) = \sqrt{2} \quad \text{and} \quad G(0) = \sqrt{2}. \quad (4)$$

Combining Eq. (4) with Eq. (2) and Eq. (3), yields

$$\begin{cases} h_0 + 2(h_1 + h_2 + h_3 + h_4) = \sqrt{2} \\ g_0 + 2(g_1 + g_2 + g_3) = \sqrt{2} \end{cases}. \quad (5)$$

Substituting  $\omega = \pi$  into  $H(\omega)$  of Eq. (2) and into  $G(\omega)$  of Eq. (3), gives

$$h_0 + 2(-h_1 + h_2 - h_3 + h_4) = 0, \quad (6)$$

$$g_0 + 2(-g_1 + g_2 - g_3) = 0. \quad (7)$$

Taking the second derivative of Eq. (2), yields

$$2(-g_1 + 4g_2 - 9g_3) = 0. \quad (8)$$

In the proposed methods, the relationship of the filter coefficients was determined to identify the coefficients of the 9/7-tap wavelet filter. Eq. (1) to Eq. (8) were used to construct the CDF 9/7-tap wavelet filter.

Because the 9/7-tap wavelet filter is symmetric,  $h$  and  $g$  can be presented in the  $z$ -domain. Therefore, a poly-phase matrix  $P_a(z)$  is presented by

$$\begin{aligned} P_a(z) &= \begin{bmatrix} h_e(z) & g_o(z) \\ h_o(z) & g_e(z) \end{bmatrix} \\ &= \begin{bmatrix} h_0 + h_2(z+z^{-1}) + h_4(z^2+z^{-2}) & g_1(1+z^{-1}) + g_3(z+z^{-2}) \\ h_1(z+1) + h_3(z^2+z^{-1}) & -g_0 - g_2(z+z^{-1}) \end{bmatrix}, \end{aligned} \quad (9)$$

where  $h_e$  and  $g_e$  are even coefficients, and  $h_o$  and  $g_o$  denote the odd coefficients.

The poly-phase matrix  $P_a(z)$  can be factorized into five elementary matrices, as in [3]:

$$P_a(z) = \begin{bmatrix} 1 & \alpha(1+z^{-1}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \beta(1+z) & 1 \end{bmatrix} \begin{bmatrix} 1 & \gamma(1+z^{-1}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \delta(1+z) & 1 \end{bmatrix} \begin{bmatrix} \xi & 0 \\ 0 & \frac{1}{\xi} \end{bmatrix} \quad (10)$$

Using Eqs. (5) to (8) and making an equivalence of the matrix  $P_a(z)$  parameters in Eqs. (9) and (10), gives

$$\begin{cases} \beta = -\frac{1}{4(1+2\alpha)^2} \\ \gamma = -\frac{(1+2\alpha)^2}{1+4\alpha} \\ \delta = \frac{1}{16} \left( 4 + \frac{1-8\alpha}{(1+2\alpha)^2} - \frac{2}{(1+2\alpha)^3} \right) \\ \xi = \frac{2\sqrt{2}(1+2\alpha)}{1+4\alpha} \end{cases} \quad (11)$$

In the CDF 9/7-tap wavelet filter, the prediction and updating, and normalized parameters are represented by  $\alpha=-1.586134\dots$ ,  $\beta=-0.05298\dots$ ,  $\gamma=0.882911\dots$ ,  $\delta=0.443506\dots$ , and  $\xi=1.230174\dots$ [21].

In the proposed methods,  $\alpha$  was examined in the  $(-\infty, +\infty)$  range to find the optimal value for the 9/7-tap wavelet filters in terms of the complexity of the filter coefficients and in terms of the PSNR values. In practice, the range of  $\alpha$  values is limited by calculating the statistical distribution of the optimal  $\alpha$ . For each  $\alpha$ , the  $(\beta, \gamma, \delta, \xi)$  set was determined using Eq. (11) and calculate the coefficients of  $h$  and  $g$  using Eqs. (9) and (10). Finally, the optimal  $\alpha$  was obtained based on the maximum PSNR value of the corresponding 9/7-tap wavelet filter for each experimental image and compression ratio.

### 2.2 Conventional Lifting Scheme

This subsection briefly presents the conventional lifting scheme because it has been used in many designs of wavelet filters, including the CDF 9/7-tap wavelet filter adopted in JPEG2000 and the wavelet filter of Guangjun’s method. The conventional lifting scheme, which is a space-domain construction of bi-orthogonal wavelets proposed by Sweldens [2], consists of four steps: splitting, prediction, updating and normalization. To apply the lifting scheme to image processing, the splitting step divides an image  $s$  into odd  $s_o$  and even  $s_e$  columns or rows [18]. In the prediction step, the detailed components were obtained using the following formula

$$h(m, n) = s_o(m, n) - \sum_i p_i s_e(m, n + i), \quad (12)$$

where  $p_i$  is the prediction parameter. In the updating step, the approximation components were calculated as follows:

$$l(m, n) = s_e(m, n) + \sum_j u_j h(m, n + j), \quad (13)$$

where  $u_j$  is the updating parameter. These steps can be reiterated to create a complete set of DWT scaling and

wavelet coefficients. The final step is normalization. From the above analysis,  $p_i$ ,  $u_j$ , and the normalized parameters were found to be equivalent to the  $(\alpha, \beta, \gamma, \delta, \xi)$  set, as mentioned in the previous subsection.

Based on Eq. (12) and Eq. (13), the conventional lifting scheme used the splitting data directly. This means that the scheme considers neither the neighboring elements nor image edges, even though the image edges are affected by ringing artifacts during the coding process. Better performance can be obtained if the image edges in the lifting scheme are considered.

### 3. Proposed Methods

This section proposes an optimization method for wavelet filters. The aim of this wavelet filter design was to simplify the wavelet coefficients. In other words, the optimal wavelet filters have rational coefficients. This is unlike the CDF 9/7-tap wavelet filter, which has irrational coefficients. A median operator was used to consider the image edges and obtain a median lifting scheme. Finally, the filter optimization and the median lifting scheme were combined to improve the coding efficiency.

#### 3.1 Optimal Wavelet Filters

From Eq. (11), because  $\beta$ ,  $\gamma$ ,  $\delta$ , and  $\xi$  depend on  $\alpha$ , different values for  $\beta$ ,  $\gamma$ ,  $\delta$ , and  $\xi$  can be obtained by changing  $\alpha$  in the  $(-\infty, +\infty)$  range. The coefficients of the 9/7-tap wavelet filters were calculated from Eqs. (9) and (10), and the corresponding PSNR value were calculated.  $\alpha$  has the optimal value when the PSNR value was a maximum, i.e.  $PSNR_{max}$ .

Fig. 1 shows a flowchart of the proposed method in

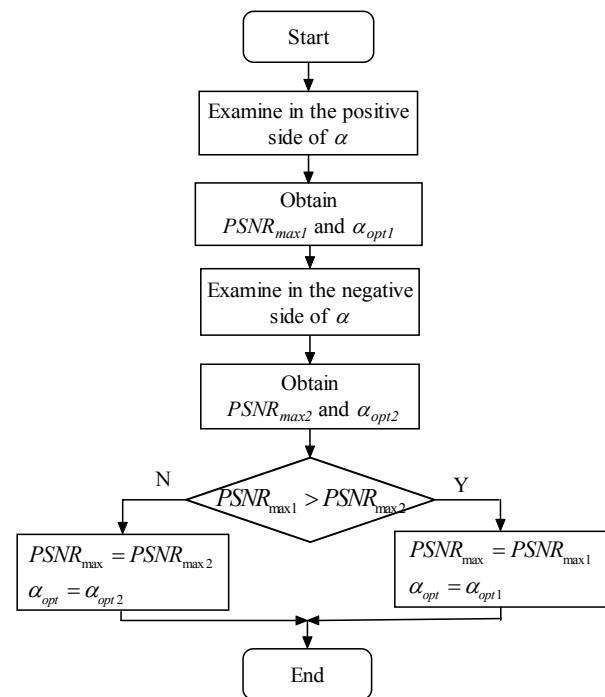


Fig. 1. Algorithm to find  $PSNR_{max}$  and the optimal  $\alpha$ .

designing the optimal wavelet filters. First,  $\alpha$  was examined in both the negative and positive sides. For each side, the local  $PSNR_{\max}$  and locally optimal  $\alpha$  could be determined. The local  $PSNR_{\max}$  values from each side were then compared. The higher  $PSNR_{\max}$  was selected as the final  $PSNR_{\max}$  and its corresponding  $\alpha$  was called the final optimal  $\alpha$ , denoted as  $\alpha_{opt}$ . The filter coefficients of the optimal 9/7-tap wavelet filter could be obtained using  $\alpha_{opt}$ . The shape of the PSNR curve for each 9/7-tap wavelet filter is a function of  $\alpha$ , as shown in Fig. 2. The experiments in Section 4 will confirm this shape.

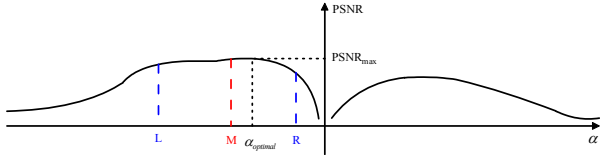


Fig. 2. Shape of PSNR as a function of  $\alpha$ .

A bi-section algorithm was used to reduce the time spent in searching the optimal  $\alpha$ , as shown in Fig. 3.  $\alpha$  at two initial positions, Left (L) and Right (R), were obtained. The PSNR value in the Middle (M) position was compared with the PSNR values of (L) and (R). Finally, (M) was used to update (R) or (L). The process was reiterated until the PSNR value reached its maximum.

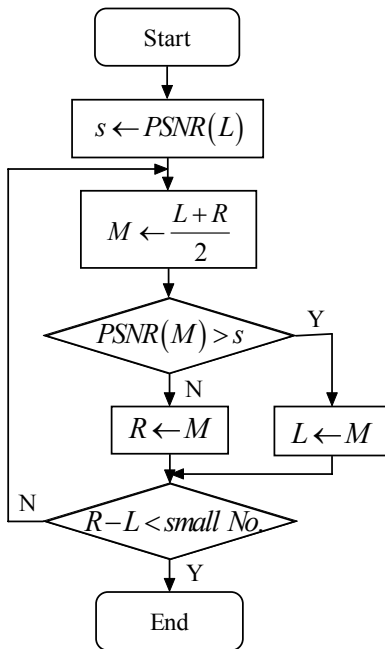


Fig. 3. Bi-section algorithm.

After selecting the optimal  $\alpha$ , this  $\alpha$  was substituted into Eq. (11), and the proposed algorithm was used to obtain the other parameters, such as  $\beta$ ,  $\gamma$ ,  $\delta$ , and  $\xi$ . The coefficients of the 9/7-tap wavelet filters were obtained from Eqs. (9) and (10). The optimal  $(\alpha, \beta, \gamma, \delta, \xi)$  set was then applied to the median lifting scheme, as described in the next subsection.

### 3.2 Median Lifting Scheme

In image coding, the conventional lifting scheme does not consider the image edges. In this paper, the conventional lifting scheme was replaced with the median lifting scheme and the optimal  $\alpha$  was used. This subsection describes the median lifting scheme in detail.

In general, the  $N^{\text{th}}$ -order median filter is defined by

$$\text{median}_N(s(i, k)) = \text{median}\left\{\left\{s\left(i, k - \left\lfloor \frac{N}{2} \right\rfloor\right\}, \dots, s\left(i, k + \left\lfloor \frac{N-1}{2} \right\rfloor\right)\right\}\right\}. \quad (14)$$

To show the efficiency of median filtering for handling image edges, nine pixel values were extracted from the chin of the GRANDMA image (row 313, columns 130-138). The median and mean operations were then taken individually. While the mean filtering matches the wrong edge, the precise real edge was obtained using the median operation. Fig. 4 shows the results in detail.

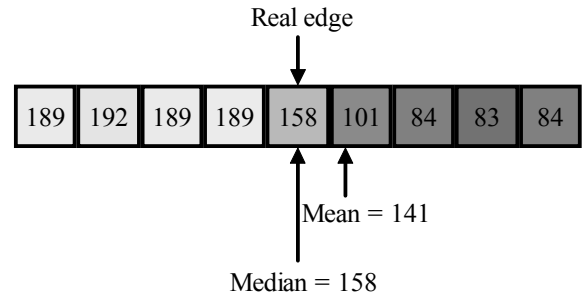


Fig. 4. Median operation for handling edge.

In proving the efficiency of median filtering for handling the image edges mathematically, some mathematical statisticians have applied various models of median filters successfully for images with edges [19]. The median filter is nonlinear. Other studies showed the advantages of nonlinear filters [18, 19].

The median lifting scheme consists of four steps: splitting, prediction, updating and normalization. The splitting step is similar to that of the conventional lifting scheme. The prediction step can be represented by

$$h(m, n) = s_o(m, n) - \sum_i p_i \text{median}_N(s_e(m, n+i)), \quad (15)$$

where  $p_i$  is the prediction parameter. The median of  $s_e(m, n+i)$  and its  $N-1$  neighboring even elements are calculated. Next, the updating step is represented by

$$l(m, n) = s_e(m, n) + \sum_j u_j \text{median}_M(h(m, n+j)), \quad (16)$$

where  $u_j$  is the updating parameter, and  $M$  denotes the order of the median updating filter. The above steps can be reiterated to create the complete set of DWT scaling and that of the wavelet coefficients. The  $(p_i, u_j)$  set is

equivalent to the optimal  $(\alpha, \beta, \gamma, \delta)$  set when the optimal  $(\alpha, \beta, \gamma, \delta)$  set is applied to a combination of filter optimization and median lifting. In the conventional lifting scheme, the  $(p_i, u_i)$  set is equivalent to the  $(\alpha, \beta, \gamma, \delta)$  set, which is used in the CDF 9/7-tap wavelet filter of JPEG2000. The final step is the normalization step that uses the  $\xi$  parameter.

A comparison of Eqs. (15) and (16) of the median lifting scheme with Eqs. (12) and (13) of the conventional lifting scheme showed that the median lifting scheme uses the median operator as an efficient additional operator. This is the main difference between the two schemes. Although other authors proposed different functions to present the direction for each image block [7, 13-17], a median operator was used to consider the image edges.

Prediction, updating, and normalization parameters were similarly used as they are in the conventional lifting scheme. These parameters were determined when the optimal wavelet filters were designed in the decoding process as a combination of filter optimization and median lifting. Therefore, images were encoded and decoded using the identical lifting scheme.

The median filtering is a smoothing operation [22]; it achieves some fixation in any types of problems due to the distortion introduced in the transformation of the original information to the wavelet domain. Some mathematicians and statisticians showed that the median filtering handles image edges nicely [19]. In addition, the median filter reduces flickering properly [23]. Compared to the conventional lifting scheme, the combination of the median operator and lifting scheme was more robust because it is a nonlinear lifting scheme that considers the neighboring elements. This filter is also robust for image de-noising. Moreover, using both primal and dual lifting steps, it produces much better image quality when the target image contains complex edges. Therefore, improved results for edge-dominant images can be obtained using the median operator and a combination of filter optimization and median lifting. Both the mathematical formulations in [19] and the current experiments confirmed this.

## 4. Experiment Results and Analysis

This section reports the experiment results for edge-dominant images.

### 4.1 Statistics of Optimal $\alpha$ Values

In the first experiment, the distribution of the optimal  $\alpha$  for PSNR<sub>max</sub> was obtained. Fig. 5 shows the statistics of the optimal  $\alpha$ . As explained in Section 2,  $\alpha$  was examined in the  $(-\infty, +\infty)$  range to find the optimal wavelet filter. To limit the examination range of  $\alpha$ , however, the statistics of the optimal  $\alpha$  with step size 0.1 were determined for the database. In this experiment, the PSNR values were calculated at four different rates: 0.125 bits per pixel (bpp), 0.25 bpp, 0.5 bpp, and 1 bpp.

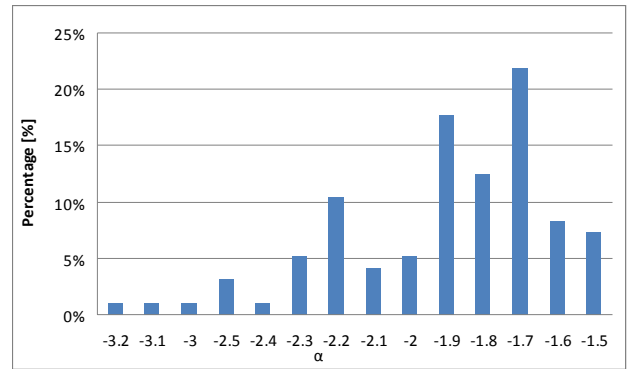


Fig. 5. Distribution of the optimal  $\alpha$  for PSNR<sub>max</sub>.

### 4.2 Performance at Different Rates

In the second experiment, PSNR was confirmed to be a function of  $\alpha$  that is used under the assumption in Section 3 that the shape of the PSNR curve for each 9/7-tap wavelet filter is a function of  $\alpha$ . Fig. 6 shows PSNR curves at different bit rates, i.e., 0.125 bpp, 0.25 bpp, 0.5 bpp, and 1 bpp, for two test images (LYNDA and GRANDMA) as a function of  $\alpha$ . These two images show higher PSNR values

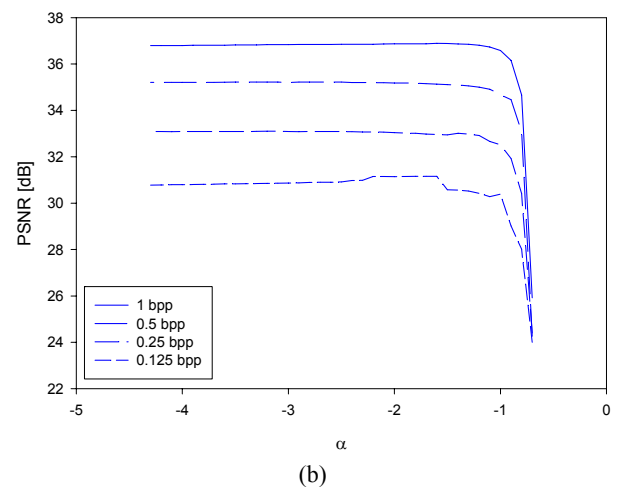
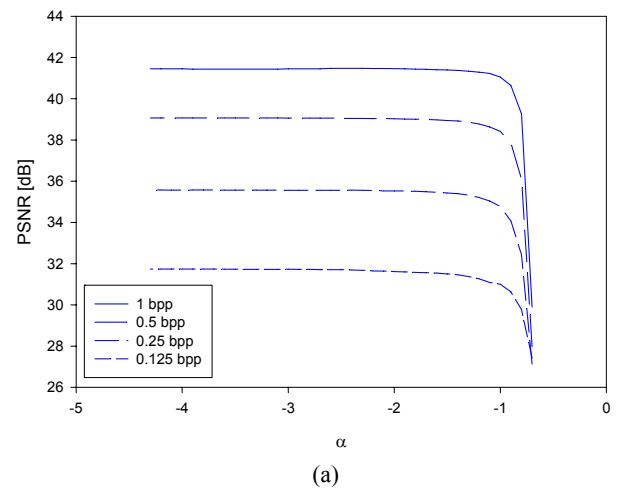


Fig. 6. PSNR survey at different rates (a) LYNDA image, (b) GRANDMA image.

Table 1. Comparison of the PSNR values.

Test Image	Rate [bpp]	PSNR Values [dB]				PSNR Gains [dB]			
		JPEG2000 Method	Guangjun's Method	Proposed Method I*	Proposed Method II**	$\Delta$ PSNR <sub>PJII</sub>	$\Delta$ PSNR <sub>PJI</sub>	$\Delta$ PSNR <sub>PJII</sub>	$\Delta$ PSNR <sub>PJII</sub>
BIRD $\alpha_{opt}=-1.65$	0.125	26.56	26.53	27.69	27.71	+1.13	+1.15	+1.16	+1.18
	0.25	29.49	29.46	30.79	30.87	+1.3	+1.38	+1.33	+1.41
	0.5	32.6	32.6	34.17	34.19	+1.57	+1.59	+1.57	+1.59
	1	36.27	36.27	37.76	37.77	+1.49	+1.5	+1.49	+1.5
CLOWN $\alpha_{opt}=-1.55$	0.125	25.56	25.54	25.53	25.53	-0.03	-0.03	-0.01	-0.01
	0.25	27.56	27.55	27.63	27.63	+0.07	+0.07	+0.08	+0.08
	0.5	29.58	29.55	30.42	30.47	+0.84	+0.89	+0.87	+0.92
	1	32.5	32.48	33.73	33.72	+1.23	+1.22	+1.25	+1.24
GRANDMA $\alpha_{opt}=-1.68$	0.125	29.12	29.11	30.58	31.16	+1.46	+2.04	+1.47	+2.05
	0.25	32.2	32.19	32.96	32.98	+0.76	+0.78	+0.77	+0.79
	0.5	34.45	34.42	35.13	35.15	+0.68	+0.7	+0.71	+0.73
	1	36.35	36.32	36.89	36.89	+0.54	+0.54	+0.57	+0.57
LYNDA $\alpha_{opt}=-2.6$	0.125	30.42	30.4	31.54	31.71	+1.12	+1.29	+1.14	+1.31
	0.25	33.8	33.79	35.46	35.56	+1.66	+1.76	+1.67	+1.77
	0.5	37.38	37.35	38.98	39.06	+1.6	+1.68	+1.63	+1.71
	1	40.38	40.36	41.42	41.48	+1.04	+1.1	+1.06	+1.12
EINSTEIN $\alpha_{opt}=-3.2$	0.125	23.77	23.74	23.64	23.73	-0.13	-0.04	-0.1	-0.01
	0.25	25.49	25.46	25.85	26.23	+0.36	+0.74	+0.39	+0.77
	0.5	28.7	28.7	28.63	28.85	-0.07	+0.15	-0.07	+0.15
	1	30.6	30.6	31.17	31.18	+0.57	+0.58	+0.57	+0.58
ELAINE $\alpha_{opt}=-1.59$	0.125	28.62	28.61	28.32	28.32	-0.3	-0.3	-0.29	-0.29
	0.25	29.87	29.84	30.14	30.14	+0.27	+0.27	+0.3	+0.3
	0.5	31.44	31.41	31.66	31.66	+0.22	+0.22	+0.25	+0.25
	1	32.47	32.43	32.78	32.78	+0.31	+0.31	+0.35	+0.35
MRI $\alpha_{opt}=-1.52$	0.125	24.01	23.98	24.08	24.19	+0.07	+0.18	+0.1	+0.21
	0.25	26.61	26.59	26.74	26.76	+0.13	+0.15	+0.15	+0.17
	0.5	28.91	28.9	29	29.02	+0.09	+0.11	+0.1	+0.12
	1	31.46	31.45	32.35	32.37	+0.89	+0.91	+0.9	+0.92
AVERAGE					+0.67	+0.75	+0.69	+0.77	

\* Proposed Method I – Proposed method using median lifting scheme.

\*\* Proposed Method II – Proposed method using combination of filter optimization and median lifting.

and result in better visual quality. The other test images revealed similar improvements. When the bi-section algorithm was used to find  $PSNR_{max}$  and  $\alpha_{opt}$ , it was assumed that the PSNR values can be changed depending on  $\alpha$ . From Fig. 6, the  $PSNR_{max}$  and  $\alpha_{opt}$  can be found using the bi-section algorithm, as discussed in Section 3.

### 4.3 Performance of Different Methods

In the third experiment, the optimal  $\alpha$  was determined and PSNR gains were calculated from two reference methods: the conventional lifting scheme in the CDF9/7 of JPEG2000 and Guangjun's method. To compare the coding performance of the proposed methods with that of the two above-mentioned methods, the PSNR gain was defined as

$$\Delta PSNR_{PR} = PSNR_{Proposed\ method} - PSNR_{Reference\ method} \quad (17)$$

The conventional lifting scheme was used in the CDF 9/7-tap wavelet filter of JPEG2000. Guangjun *et al.* designed a wavelet filter and obtained a simpler  $\alpha$ , but the performance was similar to that of the conventional lifting

scheme in the CDF 9/7-tap wavelet filter of JPEG2000 [4]. Table 1 lists the performance. As discussed in Subsection 4.1, the range of  $\alpha$  was first examined to obtain the optimal  $\alpha$ , called  $\alpha_{opt}$ . This  $\alpha_{opt}$  was then used for the median lifting scheme. As a result, the median lifting scheme and a combination of filter optimization with median lifting was obtained.

As shown in Table 1, the optimal  $\alpha$  lies between -3.2 and -1.52. On the other hand, it lies between -1.7 and -1.5 with high probability, as shown in Fig. 5. This confirms the statistics discussed in Subsection 4.1. In the CDF 9/7-tap wavelet filter of JPEG2000,  $\alpha$  is an irrational number,  $-1.586134342\dots$  [3, 20]. In the proposed method, the optimal  $\alpha$ , which is simply a rational number, can be found for each image. For example,  $\alpha_{opt}$  of the LYNDA and GRANDMA images were -2.6 and -1.68, respectively. Moreover, slight PSNR improvement was obtained with the optimal  $\alpha$  for each image. For example, using filter optimization in a previous study, up to 0.11 dB of PSNR gain was obtained for the FAXBALLS image, which is one of JPEG2000 standard images [24]. Therefore, these methods result in improved efficiency.

Table 1 lists the maximum PSNR gain. The maximum

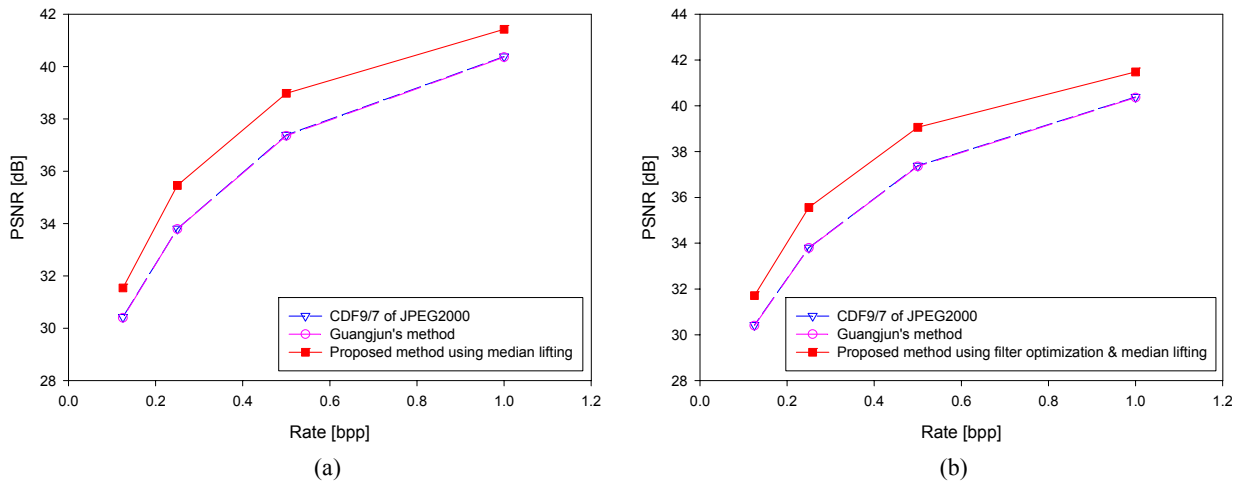


Fig. 7. Rate-distortion (RD) curves of (a) median lifting, (b) combination of filter optimization and median lifting for LYNDA image.

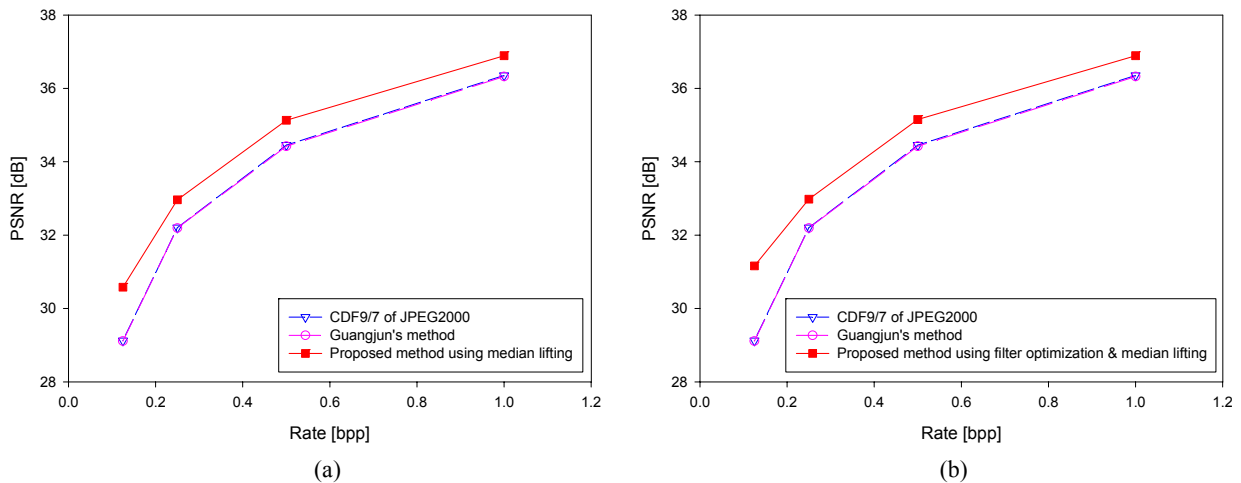


Fig. 8. RD curves of (a) median lifting, (b) combination of filter optimization and median lifting for GRANDMA image.

PSNR gains were 1.36 dB and 1.48 dB on average for the LYNDA image using the median lifting scheme and the combination of filter optimization with median lifting, respectively. The PSNR improvements were also obtained by 1.37 dB and 1.42 dB on average for the BIRD image. In addition, up to 2.05 dB was obtained on the maximum PSNR gain at 0.125 bpp for the GRANDMA image.

This study compared the maximum and average PSNR gains achieved by the proposed methods with those obtained using the methods reported by Ding *et al.* and Liu *et al.* The proposed methods provided several advantages for edge-dominant images while their methods provided good performance for images with rich orientation features, such as BARBARA and WOMEN images [13]. Comparing the experimental results of different databases is not appropriate; hence, this paper reports the results of the proposed methods and two previous studies.

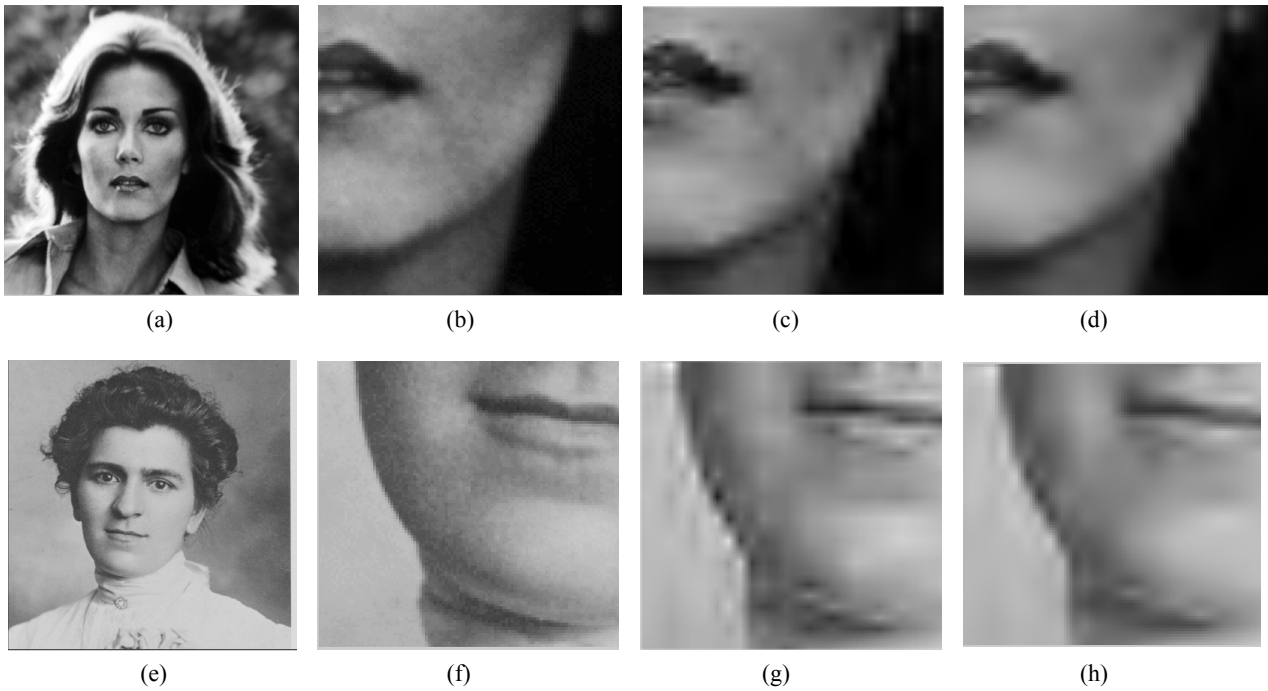
Compared to the CDF 9/7-tap wavelet filter of JPEG2000, the maximum PSNR gain of the Ding *et al.* method [13] and Liu *et al.* method [15] were 1.51 dB and 1.79 dB, respectively. The proposed methods achieved 1.66 dB for the median lifting scheme and 2.04 dB for the

combination of filter optimization with median lifting, as shown in Table 1.

Compared to the CDF 9/7-tap wavelet filter of JPEG2000, the average PSNR gain obtained using the Ding *et al.* method [13] was 0.57 dB, whereas that obtained using the Liu *et al.* method [15] was 0.69 dB. In these methods, as shown in Table 1, the gain of the median lifting scheme was 0.67 dB and that of the combination of filter optimization with median lifting was 0.75 dB.

Figs. 7 and 8 show the rate-distortion (RD) curves that clearly indicate the significant PSNR improvements in these methods over the two other methods. As discussed in Section 1, Guangjun's method simplified the filter coefficients using the fixed value  $\alpha = -1.5$ . On the other hand, the performance of Guangjun's method was similar to that of the CDF-9/7 method of JPEG2000 [4]. As shown in Figs. 7 and 8, the proposed methods outperformed the two reference methods: the CDF-9/7 method of JPEG2000 and Guangjun's method.

Compared to the RD curves in Figs. 7 and 8, the combination of filter optimization and median lifting performed slightly better than the median lifting scheme.



**Fig. 9. Visual quality comparison for the LYNDA image at 0.25 bpp (top row) and GRANDMA image at 0.125 bpp (bottom row) (a) and (d) original images, (b) and (f) zoomed original image, (c) CDF9/7 of JPEG2000 for LYNDA image (33.8 dB), (d) Proposed lifting scheme for the LYNDA image (35.56 dB), (g) CDF9/7 of JPEG2000 for the GRANDMA image (29.12 dB), (h) Proposed lifting scheme for the GRANDMA image (31.16 dB).**

On the other hand, both proposed methods outperformed the two reference methods.

#### 4.4 Performance of Visual Quality and Complexity

In the final experiment, the output image quality of different methods was examined. Fig. 9 represents a comparison of the visual quality. Because PSNR is not the ultimate judge in image quality [5], the best method can only be decided after demonstrating the improvement of PSNR and visual quality. The visual quality of the decoded images that result from the proposed lifting scheme and the conventional lifting scheme in the CDF-9/7 of JPEG2000 was compared. Fig. 9 compares the visual quality in detail. Fig. 9 indicates the output image quality from the different methods. The first column shows the original images. For a better visual comparison, some parts of the LYNDA and GRANDMA images were magnified, which are represented in the second column. Fig. 9 shows that these methods resulted in significant visual quality improvement over the conventional scheme.

Because both the median lifting scheme and proposed lifting scheme employ the median operator, which have several advantages for edge-dominant images [18], the proposed lifting scheme provided much better ability of preserving the edges compared to the conventional lifting scheme. Therefore, the proposed methods were tested with edge-dominant images. Indeed, from Fig. 9, the quality of the fourth column images is much better than that of the third column images. Moreover, ringing artifacts can be seen in the face, chin, and boundaries of objects in the

images. The experiment results showed that the ringing artifacts were less severe in the output images of the proposed lifting scheme than in those of the conventional lifting scheme. Therefore, the proposed lifting scheme obviously outperforms the conventional lifting scheme in terms of the visual quality.

Because the lifting scheme is a combination of the filter optimization and median lifting, it has greater computational complexity than both the CDF-9/7 method of JPEG2000 and Guangjun's method. On the other hand, the lifting scheme reduces the complexity of the wavelet filter coefficients because their irrational coefficients were replaced by rational coefficients, compared to the CDF-9/7 method of JPEG2000, and it has similar complexity of filter coefficients to the Guangjun's method.

## 5. Conclusion

This paper proposes an efficient lifting scheme as a combination of filter optimization and median lifting for edge-dominant images. The proposed method reduced the complexity of the wavelet filter coefficients because the irrational coefficients were replaced with rational coefficients. In addition, the best PSNR value was determined as a function of  $\alpha$ . The experimental results showed that the proposed method performed better than the well-known CDF-9/7 method of JPEG2000 for edge-dominant images, and improved the visual quality significantly. Furthermore, a comparison of the average and maximum PSNR gains confirmed that the proposed method outperformed the previous methods.



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