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Random Access Channel with Retransmission Gain

Junmin Shi[1], Yi Sun[2], Xiaochen Zhang[2], and Jizhong Xiao[2]

[1] Department of Managerial Sciences, Georgia State University / Atlanta, GA 30303, USA shigsu@gmail.com
[2] Department of Electrical Engineering, The City College of City University of New York / New York, NY 10031, USA
ysun@ccny.cuny.edu; wyywufan@hotmail.com; jxiao@ccny.cuny.edu

* Corresponding Author: Yi Sun

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Abstract: An analysis of the throughput and stability region of random access systems is currently of interest in research and industry. This study evaluated the performance of a multiuser random access channel with a retransmission gain. The channel was composed of a media access control (MAC) determined by the transmission probabilities and a multiuser communication channel characterized by the packet reception probabilities as functions of the number of packet transmissions and the collision status. The analysis began with an illustrative two-user channel, and was extended to a general multiuser channel. For the two-user channel, a sufficient condition was derived, under which the maximum throughput was achieved with a control-free MAC. For the channel with retransmission gain, the maximum steady throughput was obtained in a closed form. The condition under which the random access channel can acquire retransmission gain was also obtained. The stability region of the general random access channel was derived. These results include those of the well-known orthogonal channel, collision channel and slotted Aloha channel with packet reception as a special instance. The analytical and numerical results showed that exploiting the retransmission gain can increase the throughput significantly and expand the stability region of the random access channel. The analytical results predicted the performance in the simulations quite well.

Keywords: Random access; ALOHA; MAC; Retransmission gain; Throughput; Stability region

1. Introduction

In multiuser communication networks, random access is one of the typical medium access control (MAC) protocols. The capacity and stability region of such systems are currently of interest in research [1-11], and there are an increasing number of reports in the literature [12-33]. Random access can be implemented with the collision channels [18-26]. The stability region of a finite-user slotted random access system [19-21] is identical to the information-theoretical capacity region of the collision channel without feedback [18]. An infinite-user random access system over the collision channel is unstable [22, 23]. Random access has also been implemented in wireless channels that are capable of multipacket reception [1-16, 27-33]. The multipacket reception capability can stabilize an infinite-user Aloha system [17]. The throughputs of symmetric wireless multi-access systems were analyzed in [32]. Multiple transmissions of a packet through a wireless channel can acquire retransmission diversity gain and improve the throughput substantially [13-15].

A number of theoretical studies on random access systems have recently been reported. The theoretical capacity and throughput of the random access channels, where each user encodes a subset of data, were analyzed [1]. A random access game for contention control was proposed and analyzed using the Nash equilibria [2]. In a game-theoretical approach, a slotted Aloha system with selfish nodes was analyzed [3] to minimize the average transmission probability (or power investment) while meeting the average throughput demand. In the random access game incorporating channel state information into slotted Aloha, the asymptotic total throughput was shown to increase with increasing number of users [4]. Contention resolution strategies for selfish random access wireless systems with multipacket reception were analyzed using non-cooperative game theory [5]. Exploiting the channel state information in slotted Aloha with a capture of the signal to interference ratio was studied [6, 7]. The channel state information might not be helpful in increasing the throughput in certain channels [7]. The optimal transmission strategy achieving the maximum achievable sum rate and steady throughput of random access over a deterministic channel was developed [8]. The information capacity of a simple on-off random access channel was derived in the framework of compressed sensing [10]. The asymptotic stability region of multiuser slotted collision channel was analyzed [11].

Although there have been a number of studies on random access channels, to the best of the authors' knowledge, there are no reports of the throughput and stability region of a general random access channel with retransmission gain. This study attempts to bridge this gap. In this paper, the communication channel is defined by the probability of packet reception when a packet has been transmitted a number of times and collided with interfering packets a number of times. The analyses begin with an illustrative two-user channel. The state distribution and steady maximum throughput are then obtained, which include the results of the orthogonal channel and slotted Aloha channel with multipacket reception [16] as special cases. For a general K-user random access channel with retransmission gain, the steady throughput is obtained in a closed form, and the stability region is then presented.

The remainder of this paper is organized as follows. Section 2 defines the channel model. Section 3 examines a two-user random access channel using a two-dimensional Markov chain approach. Section 4 treats the multiuser MAC symmetric channels followed by an investigation for asymmetric multiuser channels. The stability region is studied in Section 5. As a practical example, Section 6 examines a random access CDMA channel exploiting retransmission gain. Section 7 reports the results of numerical and simulation studies. Finally, Section 8 concludes the paper. For continuity, the proofs are presented in the 9. Appendix.

2. Channel Model

The random access channel, as depicted in Fig. 1, is composed of a MAC layer and a communication channel, where K users access a common base station randomly and independently with each other. In such a channel, each user has a queue of packets in its buffer and the packets are transmitted in a first-in-first-out (FIFO) mechanism. Among the packets (if any) stored in the buffer, the one at the front end is referred to as the current packet and the remainder are called waiting packets. Therefore, a user is always transmitting its current packet (if any). User $i = 1, 2,..., K$ independently transmits its current packet with a given probability, $\theta_i \in (0, 1]$, henceforth called transmission probability. That is, a user in each slot is either active in transmitting its current packet with a probability θ_i, or is idle with a probability $1-\theta_i$. Here, the transmission probabilities θ_i, $i = 1, 2,..., K$, for all users characterize the MAC layer of the channel. Without a loss of generality, user 1 shall be referred to as the default user. For user 1, the state of the current packet in its buffer at the beginning of slot j is denoted as $(m; c_2, c_3,..., c_K)_j$, which indicates that the current packet has previously been transmitted m times and collided $c_i \in \{0, 1,..., m\}$ times with user i. The dynamics of the packet transmission states is described as follows. (1) If user 1 is idle in slot j, then its state at the beginning of the next slot j+1 will remain the same as $(m; c_2, c_3,..., c_K)_{j+1}$. (2) If user 1 is active in slot j but its current packet is not detected successfully, then at the beginning of slot j+1 the transmission number will be m+1 and the collision number will be c_i+1 if user i is active in slot j, otherwise it will remain unchanged as c_i if user i is idle. (3) In the case that user 1 successfully transmits its current packet, the current packet will be removed from the buffer and the immediate waiting packet (if any) will become the new current packet. In this case, at the beginning of slot j+1, the state renews as $(0; 0, 0...0)_{j+1}$, which means that the new current packet has never been transmitted.

According to the description above, the state space of user 1's current packet can be denoted as

$$\Omega = \{(m; c_2,...,c_K) : m = 0,1,...; c_i \in \{0,1,...,m\}; i = 2,...,K\} .$$

The packet reception probability at state $(m; c_2, c_3, ..., c_K)$ is denoted as

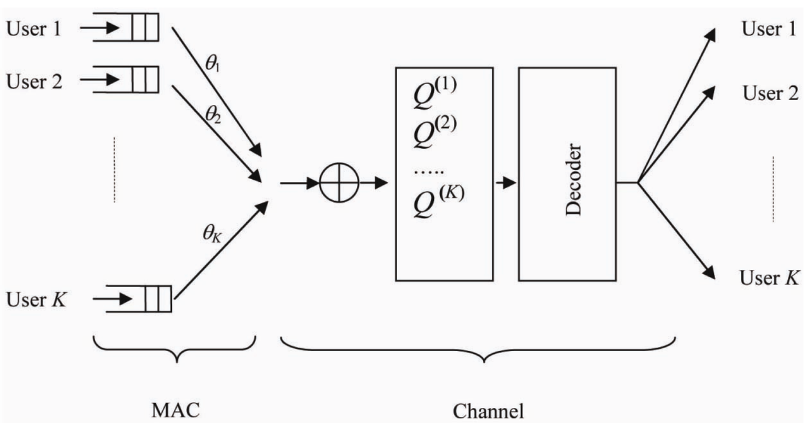

Fig. 1. A general multiuser random access channel $\aleph(K,\{\theta_i\},\{Q^{(i)}\})$ **.**

$$q_{(m;c_2,\ldots,c_K)} \equiv \Pr\{\text{The packet is successfully detected at state } (m;c_2,\ldots,c_K)\}, \tag{1}$$

and at the initial state (0; 0, …, 0) by

$$q_{(0;0,\ldots,0)} = 0 . \tag{2}$$

The communication channel defined above represents a general class of random access channels, which includes many channels that have been studied in the random access literature. For example, the orthogonal channel and slotted Aloha channel are special examples of the above channel. The reception probability is determined only by the state (m; c_2, …, c_K), indicating the transmissions of the current packet and its collision status with other users. In random access channels that exploit retransmission diversity, the reception probability increases in the transmission number m, which is the so called retransmission gain. On the other hand, the reception probability generally decreases with increasing collision number c_i.

As discussed above, the random access channel is composed of a MAC layer and a communication channel. The former is defined by the transmission probability $\theta_i \in [0, 1]$ and latter is characterized by the reception probability $q_{(m;c_2,\ldots,c_K)}$. Throughout this paper, the following notational convention is taken: an interfering user will be indexed by a subscript for a scalar variable, whereas a superscript will be used for a matrix or vector and for the default user; and the indices are omitted when it does not cause any confusion. For interfering user i, its reception probabilities over its state space is denoted as $Q^{(i)} \equiv \{q^{(i)}_{(m;c^{(i)})} : (m;c^{(i)}) \in \Omega^{(i)}\}$, where $c^{(i)} = (c_1, \ldots, c_{i-1}, c_{i+1}, \ldots, c_K)$ is the collision status of user i with other users. A multiuser random access channel can then be denoted in general as $\aleph(K,\{\theta_i\},\{Q^{(i)}\})$. Fig. 1 shows the framework of the general channel, $\aleph(K,\{\theta_i\},\{Q^{(i)}\})$, where there are K users sending packets to a base station with a probability θ_i, and the packet sent from user i can be detected successfully with probability $Q^{(i)}$.

Consider user 1 in the general channel, $\aleph(K,\{\theta_i\},\{Q^{(i)}\})$, for any state ($m$; c_2, …, c_K) with $m > 0$, its preceding states are referred to as those states that can come to (m; c_2, …, c_K) immediately after a one step transition, and are denoted as $(m-1;c'_2,\ldots,c'_K)$, where $c'_i = c_i - 1$ if user i also transmitted its packet in the transition; otherwise $c'_i = c_i$. The set of all the preceding states of state (m; c_2, …, c_K) is denoted as

$$\Omega_{(m;c_2,\ldots,c_K)} = \{(m-1;c'_2,\ldots,c'_K) : c'_i = c_i - 1 \text{ or } c_i, c'_i \ge 0\} . \tag{3}$$

The visiting probability to state (m; c_2, …, c_K) is defined as the probability of the current packet reaching the state (m; c_2, …, c_K), which is given recursively by

$$r_{(m;c_2,\ldots,c_K)} = \sum_{(m-1;c'_2,\ldots,c'_K)\in\Omega_{(m;c_2,\ldots,c_K)}} r_{(m-1;c'_2,\ldots,c'_K)}(1-q_{(m;c_2,\ldots,c_K)}) , \tag{4}$$

where $r_{(0;0,\ldots,0)} = 1 - q_{(0;0,\ldots,0)} = 1$ by convention. The visiting probabilities and reception probabilities can be obtained from each other. Therefore, they can be used interchangeably to define the channel. If the visiting probabilities for user i is denoted as $R^{(i)} \equiv \{r^{(i)}_{(m;c^{(i)})} : (m;c^{(i)}) \in \Omega^{(i)}\}$, the multiuser random access channel can be denoted equivalently by $\aleph(K,\{\theta_i\},\{R^{(i)}\})$.

Clearly, the state process (m; $c^{(i)}$) for each user is a Markov chain process over the slotted times. Under general conditions, the process can be shown to be ergodic (i.e. positive recurrent and aperiodic). In this sense, the stationary distribution can be derived, thereby allowing the throughput to be obtained. To this end, the packet arrival rate to the buffer is assumed to be sufficiently high so that the buffers are not empty. To present the analysis in a smooth manner, Section 3 analyzes a simple but illustrative channel with two users, and Section 4 examines a generic multi-user channel.

3. Two-User Channel

In a two-user channel, the state space of user 1 is

$$\Omega = \{(m;c) : c, m = 0,1,\ldots; c \le m\} ,$$

and its state transition diagram is depicted in Fig. 2. In view of (4), the visiting probability can be expressed iteratively as

$$r_{(m;c)} = \bar{q}_{(m;c)} \sum_{\max(0,c-1)\le j\le \min(c,m-1)} r_{(m-1;j)} \tag{5}$$

which can be expressed further as

$$r_{(m;c)} = \underbrace{\bar{q}_{(1;1)}\bar{q}_{(2;2)}\cdots\bar{q}_{(c;c)}\bar{q}_{(c+1;c)}\cdots\bar{q}_{(m;c)}}_{m} + \cdots + \underbrace{\bar{q}_{(1;0)}\bar{q}_{(2;0)}\cdots\bar{q}_{(m-c;0)}\bar{q}_{(m-c+1;1)}\cdots\bar{q}_{(m;c)}}_{C(m;c)} \tag{6}$$

where $\bar{q}_{(m;c)} = 1 - q_{(m;c)}$. The visiting probability is a summation of the products of the unsuccessful reception probabilities of states along any path from the initial state (0; 0) to state (m; c). A path from state (0; 0) to (m; c) is composed of a set of consequentially connected transitions from (0; 0) to (m; c). As shown in Fig. 2, there are $C(m; c)$ distinct paths from state (0; 0) to (m; c), in each of which there are m steps, where $C(m; c)$ is the binomial coefficient. For example, from state (0; 0) to (4; 2), one path is (0; 0) → (1; 0) → (2; 1) → (3; 1) → (4; 2). There are $C(4; 2) = 6$ distinct paths, each of which has 4 steps.

Let $p_{(m;c)(n;d)}$ denote the one-step transition probability from state (m; c) to (n; d). Then for $m \ge 1$,

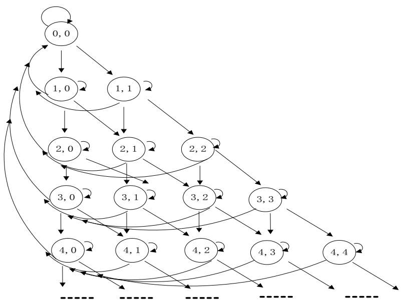

Fig. 2. State transition diagram of user 1 in a two-user channel.

$$p_{(m;c)(n;d)} = \begin{cases} 1-\theta_1, & (n;d)=(m;c), \\ \theta_1(1-\theta_2)(1-q_{(m+1;c)}), & (n;d)=(m+1;c), \\ \theta_1\theta_2(1-q_{(m+1;c+1)}), & (n;d)=(m+1;c+1), \\ \theta_1(\theta_2 q_{(m+1;c+1)}+(1-\theta_2)q_{(m+1;c)}), & (n;d)=(0;0), \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Here, the first equality in (7) holds when user 1 is idle with a probability $1-\theta_1$; the second and third equalities mean that user 1 transmits but not successfully given that user 2 is idle and active, respectively; and the fourth one means user 1 successfully transmits the current packet given that user 2 is either idle or active. For other cases, the one-step transition probability is zero.

Therefore, the distribution probability $P_{(m;c)}$ of $(m;\ c)$ can be obtained recursively by (7). Accordingly, the following holds:

$$P_{(m;c)} = P_{(0;0)}\theta_2^c(1-\theta_2)^{m-c} r_{(m;c)}. \tag{8}$$

Note that $\sum_{(m;c)\in\Omega} P_{(m;c)} = 1$. In view of (8),

$$P_{(0;0)} = \left(\sum_{(m;c)\in\Omega} r_{(m;c)}\theta_2^c(1-\theta_2)^{m-c}\right)^{-1}. \tag{9}$$

Next, substituting (9) into (8) results in the stationary distribution probability for state $(m;\ c)$, which is given as follows:

$$P_{(m;c)} = r_{(m;c)}\theta_2^c(1-\theta_2)^{m-c}\left(\sum_{(n;d)\in\Omega} r_{(n;d)}\theta_2^d(1-\theta_2)^{n-d}\right)^{-1}. \tag{10}$$

As illustrated in (10), the stationary state distribution for any state $(m;\ c)$ of user 1 is jointly identified by $r_{(m;c)}$ and θ_2, and has nothing to do with its own transmission probability θ_1.

3.1 Matrix Form of the Channel

The channel can be expressed in matrix form in terms of $q_{(m;\ c)}$, called the reception matrix,

$$Q = (q_{(m;c)})_{(m;c)\in\Omega}. \tag{11}$$

The channel can also be expressed in a matrix form in terms of $r_{(m;\ c)}$, which is referred to as the visiting matrix,

$$R = \begin{bmatrix} r_{(0;0)} & 0 & 0 & \cdots \\ r_{(1;0)} & r_{(1;1)} & 0 & \cdots \\ r_{(2;0)} & r_{(2;1)} & r_{(2;2)} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{12}$$

The transmission matrix is defined as

$$\Gamma(\theta) = \begin{bmatrix} 1 & 1-\theta & (1-\theta)^2 & (1-\theta)^3 & \cdots \\ 0 & \theta & \theta(1-\theta) & \theta(1-\theta)^2 & \cdots \\ 0 & 0 & \theta^2 & \theta^2(1-\theta) & \cdots \\ 0 & 0 & 0 & \theta^3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{13}$$

Appendix proves that

$$\sum_{(m;c)\in\Omega} r_{(m;c)}\theta_2^c(1-\theta_2)^{m-c} = \mathrm{tr}(R\Gamma(\theta_2)), \tag{14}$$

where $\mathrm{tr}(\mathbf{A})$ denotes the trace of matrix $\mathbf{A}$, the stationary distribution probability of the initial state given in (9) can be rewritten alternatively as

$$P_{(0;0)} = \frac{1}{\mathrm{tr}(R\Gamma(\theta_2))}. \tag{15}$$

Note that the denominator $\mathrm{tr}(R\Gamma(\theta_2))$ is the expected number of transmissions of a packet.

3.2 Steady Throughput

In view of Theorem 3 in [13], the steady throughput for user 1 equals

$$\begin{aligned} T_1 &= \theta_1 P_{(0;0)}^{(1)} \\ &= \frac{\theta_1}{\mathrm{tr}(R^{(1)}\Gamma(\theta_2))}. \end{aligned} \tag{16}$$

Similarly, the steady throughput for user 2 equals

$$T_2 = \frac{\theta_2}{\mathrm{tr}(R^{(2)}\Gamma(\theta_1))}. \tag{17}$$

Therefore, the throughput of the two-user channel defined by $T = T_1 + T_2$ can be expressed as

$$\begin{aligned} T = {} & \theta_1\left(\sum_{(m;c)\in\Omega} r_{(m;c)}^{(1)}\theta_2^c(1-\theta_2)^{m-c}\right)^{-1} \\ & + \theta_2\left(\sum_{(m;c)\in\Omega} r_{(m;c)}^{(2)}\theta_1^c(1-\theta_1)^{m-c}\right)^{-1}, \end{aligned} \tag{18}$$

which in matrix form is

$$T = \frac{\theta_1}{\mathrm{tr}(R^{(1)}\Gamma(\theta_2))} + \frac{\theta_2}{\mathrm{tr}(R^{(2)}\Gamma(\theta_1))}. \quad (19)$$

3.3 Throughput Maximization

In a control-free MAC channel, user i transmits its packet without control of the transmission, i.e. $\theta_i = 1$. The conditions under which each user can achieve the maximum throughput with a control-free MAC need to be derived. The symmetric and asymmetric channels are considered.

Symmetric channel: Two users are identical. That is, both of them have the same transmission probability, $\theta_1 = \theta_1 = \theta$ and the visiting probability, **R**. The following theorem presents a sufficient condition.

Theorem 1 . In a two-user symmetric channel $\aleph(2,\theta,R)$, if

$$\sum_{m=2}^{\infty}(m-1)r_{(m,m)} \le 1, \quad (20)$$

then both users reach their maximum throughputs under the control-free MAC, and the maximum throughput of the channel is $T = 2/\mathrm{tr}(R)$. □

Asymmetric channel: Analogous to Theorem 1, the following theorem provides a sufficient condition for an asymmetric channel.

Theorem 2. In a two-user channel, $\aleph(2,\{\theta_i\},\{R^{(i)}\})$, for a given $0 \le \theta_2 \le 1$, if

$$\left(\sum_{n=0}^{\infty} r^{(2)}_{(n;n)}\theta_1^n\right)^2 \ge \left(\sum_{n=1}^{\infty} n r^{(2)}_{(n;n)}\theta_1^{n-1}\right)\left(\sum_{m=0}^{\infty} r^{(1)}_{(m;m)}\right), \quad (21)$$

for any $0 \le \theta_1 \le 1$, the throughput of user 1 is optimized at $\theta_1^* = 1$. □

3.4 Two Examples

Two degenerated examples, the orthogonal channel and the slotted Aloha channel, were considered.

Orthogonal channel: In an orthogonal channel, each user sends a packet independently without interfering with each other. Therefore, the packet reception probability is state-independent and has a constant value $q_{(m;c)} = q_1$. Accordingly, the visiting probability in (6) is expressed as

$$r_{(m;c)} = C(m;c)(1-q_1)^m. \quad (22)$$

By substituting (22) into (9), the distribution probability of the initial state can be obtained as follows:

$$P_{(0;0)} = \left(\sum\nolimits_{(m;c)\in\Omega} C(m;c)(1-q_1)^m\theta_2^c(1-\theta_2)^{m-c}\right)^{-1}$$
$$= \left(\sum\nolimits_{m\ge 0}(1-q_1)^m\right)^{-1} = q_1. \quad (23)$$

Therefore, according to Theorem 3 in [13], the throughput of user 1 is given by

$$T_1 = \theta_1 q_1. \quad (24)$$

Because the two users do not interfere with each other in an orthogonal channel, the performance of an individual user is determined exclusively according to its own reception and transmission probabilities. Therefore, each user can maximize its throughput at $\theta_i^* = 1$, i = 1, 2, which also maximizes the throughput of the orthogonal channel.

Aloha Channel: The conventional Aloha channel is built upon a collision channel, where the packet reception probability is zero provided there is a collision of multiple packets. The Aloha channel considered in this paper is built upon a channel with multipacket reception (MPR). Such a channel has been studied in [13-16, 29-33].

Let $0 \le q_i \le 1$ and $0 \le \rho_i \le 1$ denote the packet reception probability of user i with and without collision, respectively. The visiting probability of user 1 is

$$r^{(1)}_{(m;c)} = C(m;c)(1-q_1)^{m-c}(1-\rho_1)^c. \quad (25)$$

Substituting the above equation into (9) yields the distribution probability of the initial state as follows:

$$P^{(1)}_{(0;0)} = \left(\sum\nolimits_{(m;c)\in\Omega} C(m;c)[(1-\rho_1)\theta_2]^c[(1-q_1)(1-\theta_2)]^{m-c}\right)^{-1}$$
$$= \left(\sum\nolimits_{m\ge 0}[(1-\rho_1)\theta_2 + (1-q_1)(1-\theta_2)]^m\right)^{-1}$$
$$= q_1(1-\theta_2) + \rho_1\theta_2. \quad (26)$$

Accordingly, by Theorem 3 in [13], the throughput of user 1 is

$$T_1 = (q_1 - q_1\theta_2 + \rho_1\theta_2)\theta_1. \quad (27)$$

Therefore, by the symmetry of users, the channel throughput can be expressed as

$$T = (q_1 - q_1\theta_2 + \rho_1\theta_2)\theta_1 + (q_2 - q_2\theta_1 + \rho_2\theta_1)\theta_2. \quad (28)$$

In the design of a communication channel, there are basic choices on (i) exploiting the retransmission gain or not and (ii) controlling the MAC or not. For channel $\aleph(2,\{\theta_i\},\{R^{(i)}\})$, the throughput with retransmission gain denoted as T^+ is given by (19). Let $q_i = 1 - r^{(i)}_{(1;0)}$ and $\rho_i = 1 - r^{(i)}_{(1;1)}$. The throughput without retransmission gain denoted by T^- can then be given by (28). Therefore, a comparison of T^+ with T^- leads to a decision as to whether or not exploiting the retransmission gain can maximize the channel throughput.

In what follows, the condition under which the MAC is optimally control-free was analyzed. To this end, the first order conditions of T with respect to θ_1 and θ_2 were taken as

$$\frac{\partial T}{\partial \theta_1} = q_1 - q_1\theta_2 + \rho_1\theta_2 - q_2\theta_2 + \rho_2\theta_2 = 0\,, \tag{29}$$

$$\frac{\partial T}{\partial \theta_2} = q_2 - q_2\theta_1 + \rho_2\theta_1 - q_1\theta_1 + \rho_1\theta_1 = 0\,. \tag{30}$$

Accordingly, the following result was obtained.

Theorem 3. Given q_1, q_2, ρ_1, and ρ_2, let $\rho = \rho_1 + \rho_2$. If $\rho \geq q_2$ or $\rho \geq q_1$, the optimal transmission probability is $\theta_1^* = 1$ or $\theta_2^* = 1$, respectively. Furthermore, if $\rho \geq \max\{q_1, q_2\}$, then the maximum channel throughput is $T^* = \rho$ if and only if $\theta_1^* = \theta_2^* = 1$. Otherwise, the optimal transmission probabilities are

$$\theta_1^* = \frac{q_2}{q_1 + q_2 - \rho}\,, \tag{31}$$

$$\theta_2^* = \frac{q_1}{q_1 + q_2 - \rho}\,. \tag{32}$$

□

According to *Theorem 3*, if $\rho \leq \min\{q_1, q_2\}$, the total optimal transmission probability is

$$\theta_1^* + \theta_2^* = 1 + \frac{\rho}{q_1 + q_2 - \rho}\,. \tag{33}$$

Accordingly, the following corollary outlines the optimal transmission probabilities.

Corollary 1. The optimal packet transmission probabilities satisfy $1 \leq \theta_1^* + \theta_2^* \leq 2$. The equality of $\theta_1^* + \theta_2^* = 1$ holds if and only if $\rho = 0$, representing a collision channel; and the equality of $\theta_1^* + \theta_2^* = 2$ holds if and only if $\rho_1 + \rho_2 \geq \max\{q_1, q_2\}$. □

4. Multiuser Channel

4.1 MAC Symmetric Channel

In a K-user MAC symmetric channel, $\aleph(K, \theta, \{R^{(i)}\})$, every user sends a packet with the same transmission probability $\theta \in (0, 1]$ but may have distinct reception probabilities $Q^{(i)}$. The collision effects to the reception probability for all users are assumed to be the same. All the collision numbers to user 1 can be encapsulated as a single factor

$$c = \sum_{i=2}^{K} c_i \tag{34}$$

and $c = 1, 2, \ldots, (K-1)m$ since $c_i = 1, 2, \ldots, m$. Accordingly, the state space is

$$\Omega = \{(m; c) : m = 0, 1, \ldots; 0 \leq c \leq (K-1)m\}\,.$$

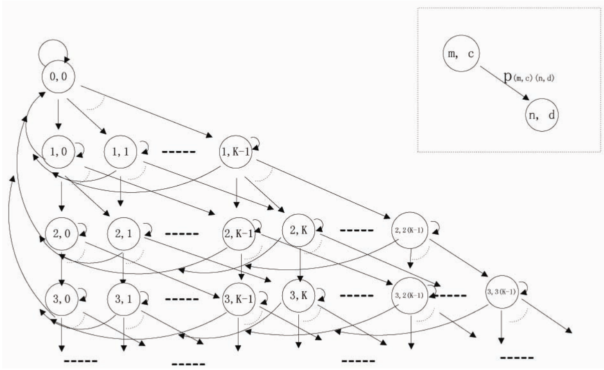

Fig. 3. State transition diagram of user 1 in a *K*-user channel.

Fig. 3 shows the one-step state transition. Following a similar argument in the preceding discussion for the two-user channel, the stationary distribution probability is obtained as

$$P_{(m;c)} = P_{(0;0)} r_{(m;c)} \theta^c (1-\theta)^{m(K-1)-c}\,, \tag{35}$$

where the visiting probability $r_{(m;c)}$ is given by

$$r_{(m;c)} = \bar{q}_{(m;c)} \sum_{\max(0, c-K) \leq j \leq \min(c, mK-K-m)} r_{(m-1;j)}\,. \tag{36}$$

Because $\sum_{(m;c)\in\Omega} P_{(m;c)} = 1$ and (35),

$$P_{(0;0)} = \left(\sum_{(m;c)\in\Omega} r_{(m;c)} \theta^c (1-\theta)^{m(K-1)-c} \right)^{-1}. \tag{37}$$

The following theorem summarizes the main result for channel, $\aleph(K, \theta, \{R^{(i)}\})$.

Theorem 4. For a K-user MAC symmetric channel $\aleph(K, \theta, \{R^{(i)}\})$, the throughput of user i is

$$T_i = \theta \left(\sum_{(m;c)\in\Omega} r_{(m;c)}^{(i)} \theta^c (1-\theta)^{m(K-1)-c} \right)^{-1}. \tag{38}$$

□

4.2 Channel with an Arbitrary MAC

Consider the general channel $\aleph(K, \{\theta_i\}, \{R^{(i)}\})$ with an arbitrary MAC. Following a similar logic applied in previous analysis, the stationary distribution probability of user 1 for a K-user random access channel can be obtained as

$$P_{(m;c_2,\ldots,c_K)} = P_{(0;0,\ldots,0)} r_{(m;c_2,\ldots,c_K)} \prod_{i=2}^{K} \theta_i^{c_i} (1-\theta_i)^{m-c_i}\,. \tag{39}$$

Because

$$\sum_{(m;c_2,\ldots,c_K)\in\Omega} P_{(m;c_2,\ldots,c_K)} = 1\,,$$

$$P_{(0;0,\ldots,0)}=\left(\sum\nolimits_{(m;c_2,\ldots,c_K)\in\Omega}r_{(m;c_2,\ldots,c_K)}\prod\nolimits_{j=2}^{K}\theta_j^{c_j}(1-\theta_j)^{m-c_j}\right)^{-1}. \quad (40)$$

By substituting (40) into (39), the distribution probability is obtained as

$$P_{(m;c_2,\ldots,c_K)}=\frac{r_{(m;c_2,\ldots,c_K)}\prod\nolimits_{i=2}^{K}\theta_i^{c_i}(1-\theta_i)^{m-c_i}}{\sum\nolimits_{(m;c_2,\ldots,c_K)\in\Omega}r_{(m;c_2,\ldots,c_K)}\prod\nolimits_{j=2}^{K}\theta_j^{c_j}(1-\theta_j)^{m-c_j}}. \quad (41)$$

The following theorem summarizes the throughput of the general random access channel.

Theorem 5. In a K-user random access channel, $\aleph(K,\{\theta_i\},\{R^{(i)}\})$, the throughput of user i is

$$T_i=\theta_i\left(\sum\nolimits_{(m;c^{(i)})\in\Omega^{(i)}}r^{(i)}_{(m;c^{(i)})}\prod\nolimits_{j=1,j\neq i}^{K}\theta_j^{c_j}(1-\theta_j)^{m-c_j}\right)^{-1}. \quad (42)$$

□

Note that a MAC symmetric channel $\aleph(K,\theta,\{R^{(i)}\})$ is a special case of the asymmetric channel $\aleph(K,\{\theta_i\},\{R^{(i)}\})$. In (40) and (42), set $\theta_j=\theta$ and let $r_{(m;c)}$ be the summation of $r_{(m;c_2,\ldots,c_K)}$ over the states such that $c=\sum\nolimits_{i=2}^{K}c_i$. Eqs. (40) and (42) are simplified as (37) and (38), respectively.

5. Stability Region

This section examines the stability and stability region of the K-user random access channel $\aleph(K,\theta,\{R^{(i)}\})$ in Fig. 1. The stability and stability region are defined as follows. The reader should also refer to [34] for further details.

Definition 1: A K-dimensional stochastic process $L^t=(L_1^t,\ldots,L_K^t)$ is stable if for any vector $x\in\mathbb{N}^K$, the following holds: $\lim_{t\to\infty}\Pr(L^t<x)=F(x)$, and $\lim_{x\to\infty}F(x)=1$. □

For a queuing channel, the stability can be interpreted as the existence of a limiting distribution.

Definition 2: For a K-user multiple-access channel, the stability region is defined as the closure of a set of arrival rates $\lambda=(\lambda_1,\ldots,\lambda_K)$, at which the queues in the channel are stable. □

In channel $\aleph(K,\{\theta_i\},\{R^{(i)}\})$, the arrival rates of K users are denoted by $\lambda=(\lambda_1,\ldots,\lambda_K)$, where $\lambda_i\geq 0$ is the packet arrival rate to the buffer of user i. $L^t=(L_1^t,\ldots,L_K^t)$, the channel status, where L_i^t is the queue length in the buffer of user i in slot t. Without a loss of generality, assume that the channel status $L^t=(L_1^t,\ldots,L_K^t)$ is an irreducible and aperiodic Markov chain process. Furthermore, if the Markov chain is a positive recurrence, it has a positive probability distribution when time approaches infinity. The stability of the queue is equivalent to the ergodicity of the process. Equivalently, the channel is stable if and only if each user has a positive probability at the initial state (0; 0, …, 0), i.e. $P_{(0;0,\ldots,0)}>0$.

The Loynes theorem [35] was applied to derive the stability region. The Loynes theorem states that if the arrival process and service process of a queue are all stationary and ergodic, the queue is stable if and only if the average arrival rate is less than the average service rate. The stable throughput region is defined as the set of all arrival rates $\lambda=(\lambda_1,\ldots,\lambda_K)$, at which the process $L^t=(L_1^t,\ldots,L_K^t)$ is stable for some transmission probabilities $(\theta_1,\ldots,\theta_K)$. Note that in Theorem 5, the service rate given by (42) is obtained assuming that the arrival rate is sufficiently large. Therefore, the throughput obtained in (42) is the maximum steady throughput, i.e. the maximum service rate pertaining to the queue of each user. Applying the Loynes theorem, the following theorem can be obtained.

Theorem 6. In the K-user random access channel $\aleph(K,\{\theta_i\},\{R^{(i)}\})$, the stability region is the closure of a set of arrival rates such that the arrival rate λ_i for each user is no greater than its maximum steady throughput T_i in (42), i.e.

$$L(\theta_1,\ldots,\theta_K)=\left\{(\lambda_1,\ldots,\lambda_K)\middle|\lambda_i\leq T_i\right\}. \quad (43)$$

The stable throughput region is

$$L=\bigcup_{(\theta_1,\ldots,\theta_K)\in(0,1]^K}L(\theta_1,\ldots,\theta_K). \quad (44)$$

□

For the two-user channel, $\aleph(2,\{\theta_i\},\{R^{(i)}\})$, the stability region is

$$L(\theta_1,\theta_2)=\left\{(\lambda_1,\lambda_2)\middle|\begin{array}{l}\lambda_1\leq\theta_1/\mathrm{tr}(R^{(1)}\Gamma(\theta_2))\\ \lambda_2\leq\theta_2/\mathrm{tr}(R^{(2)}\Gamma(\theta_1))\end{array}\right\}, \quad (45)$$

and the stable throughput region is the union of the above sets with respect to $(\theta_1,\theta_2)\in(0,1]^2$. Note that for a two-user random access channel without retransmission gain, the stable region is

$$L(\theta_1,\theta_2)=\left\{(\lambda_1,\lambda_2)\middle|\begin{array}{l}\lambda_1\leq(q_1-q_1\theta_2+\rho_1\theta_2)\theta_1\\ \lambda_2\leq(q_2-q_2\theta_1+\rho_2\theta_1)\theta_2\end{array}\right\}, \quad (46)$$

which can be rewritten equivalently as

$$L(\theta_1,\theta_2)=\left\{(\lambda_1,\lambda_2)\middle|\begin{array}{l}\lambda_1\leq q_1\theta_1+\lambda_2\theta_1\dfrac{\rho_1-q_1}{q_2-q_2\theta_1+\rho_2\theta_1}\\ \lambda_2\leq q_2\theta_2+\lambda_1\theta_2\dfrac{\rho_2-q_2}{q_1-q_1\theta_2+\rho_1\theta_2}\end{array}\right\}. \quad (47)$$

This is the stability region of the slotted Aloha with a general packet reception model, which is also obtained in [16].

Specifying $q_1 = q_2 = 1$, (47) becomes the stability region of the Aloha channel, which allows multipacket reception

$$L(\theta_1,\theta_2)=\left\{(\lambda_1,\lambda_2)\left|\begin{array}{l}\lambda_1 \le \theta_1+\lambda_2\theta_1\dfrac{\rho_1-1}{1-\theta_1+\rho_2\theta_1}\\ \lambda_2 \le \theta_2+\lambda_1\theta_2\dfrac{\rho_2-1}{1-\theta_2+\rho_1\theta_2}\end{array}\right.\right\}. \quad (48)$$

Further setting $\rho_1 = \rho_2 = 0$, (48) becomes the stability region of a collision channel

$$L(\theta_1,\theta_2)=\left\{(\lambda_1,\lambda_2)\left|\begin{array}{l}\lambda_1 \le \theta_1-\lambda_2\theta_1/(1-\theta_1)\\ \lambda_2 \le \theta_2-\lambda_1\theta_2/(1-\theta_2)\end{array}\right.\right\}, \quad (49)$$

also given in [16].

Application to Random Access CDMA with Retransmission

In this section, as an example, the analytical result is applied to a two-user random access code-division multiple access (CDMA) channel, which exploits the retransmission gain [13-15]. Consider that user 1 transmits the same packet m times and with c collisions with user 2. The equal-weight combiner of user 1 outputs [13-15] is then

$$y(m;c)=mb_1A_1+\sum_{i=1}^{c}b_2A_2g(i)+\sum_{j=1}^{m}z(j) \quad (50)$$

where A_1 and A_2 are the amplitudes of users 1 and 2, respectively, $z(j) \sim N(0, \sigma^2)$ for $1 \le j \le m$ denotes the white Gaussian noise, and $g(i)$ is the correlation between the spreading codes of the two users at collision slot $i = 1, \ldots, c$.

Following simple algebra, the bit error rate (BER) of user 1 was obtained as

$$BER(m;c)=\frac{1}{2}\Phi\left(\frac{mA_1+\sum_{i=1}^{c}A_2g(i)}{\sqrt{m}\sigma}\right)+\frac{1}{2}\Phi\left(\frac{mA_1-\sum_{i=1}^{c}A_2g(i)}{\sqrt{m}\sigma}\right) \quad (51)$$

where $\Phi(x)=\Pr(N(0,1)>x)$. In the case that $g(i) = \gamma$ is identical to all collisions, BER is

$$BER(m;c)=\frac{1}{2}\Phi\left(\frac{mA_1+cA_2\gamma}{\sqrt{m}\sigma}\right)+\frac{1}{2}\Phi\left(\frac{mA_1-cA_2\gamma}{\sqrt{m}\sigma}\right). \quad (52)$$

If the packet is composed of l-bit, the packet reception probability is

$$q(m;c)=(1-BER(m;c))^l. \quad (53)$$

The reception probability allows the throughput and stability region of the channel to be derived. Section 0 presents the results of a numerical evaluation with a simulation.

7. Numerical and Simulation Results

For ease of graphical exposition, the numerical study will focus on a two-user channel of which the analytical results are presented in Section 0. Simulated is a random access CDMA channel exploiting the retransmission gain, where the packet length is l = 32 bits, spreading factor is 48 and SNR is 6 dB. In the simulation, the spreading code is adopted randomly and every bit can be detected successfully with no more than 4 transmissions. Based on the simulation, BER(m; c) for each state (m; c) is calculated. The packet reception probability of the channel is obtained by (53) as

$$\mathbf{Q}=\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0.4748 & 0.2899 & 0 & 0 & 0\\ 0.9263 & 0.8288 & 0.6980 & 0 & 0\\ 0.9913 & 0.9780 & 0.9356 & 0.8756 & 0\\ 0.9989 & 0.9973 & 0.9913 & 0.9751 & 1\end{bmatrix}.$$

Based on the packet reception probability matrix, two numerical studies were conducted as follows.

In the first study, the analytical results of the throughput were justified by simulations, and the throughputs with and without retransmission gain were compared. Fig. 4 shows the throughput versus transmission probability of user 1. The simulation and analytical results for the channels with and without retransmission gain are presented. The upper two curves are the analytical and simulated throughputs for the channel exploiting

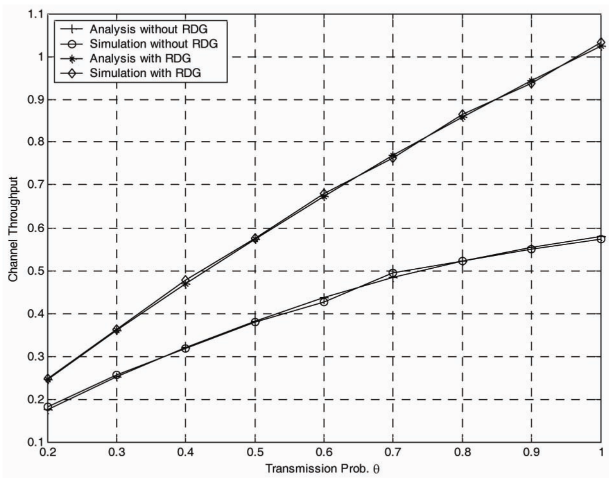

Fig. 4. CDMA channel throughput vs. transmission probability. RDG means the retransmission diversity gain.

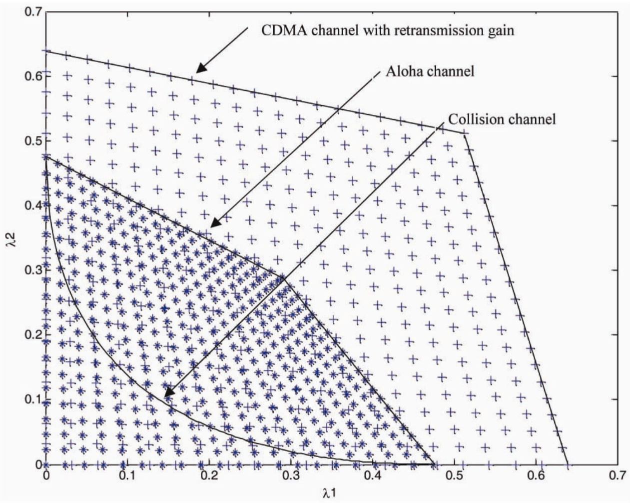

Fig. 5. Stability regions of the collision channel, ALOHA channel, and CDMA channel (with retransmission gain). The latter is much broader than the former in the sequence.

retransmission gain, respectively. In contrast, the lower two lines are the analytical and simulated throughputs, respectively, for the channel without exploiting the retransmission gain. Exploiting the retransmission gain increases the channel throughput considerably. The analytical results predict the simulation results quite well. Meanwhile, the throughput in all cases increases monotonically with increasing transmission probability. This means that the channel has been freed sufficiently to accommodate arrival users.

In the second study, the stability regions of the collision channel, Aloha channel and random access CDMA channel with a retransmission gain were compared. In the collision channel, the packet detection probability was 0.4748 without a collision and 0 with a collision. In the Aloha channel, the packet detection probability was 0.4748 without a collision and 0.2899 with a collision. In the CDMA channel exploiting the retransmission gain, the packet detection probability was **Q**. As shown in Fig. 5, the Aloha channel has a much broader stability region than the collision channel and the CDMA channel with retransmission gain has an even broader stability region. Therefore, exploiting the retransmission gain in a random access channel can expand the stability region significantly.

8. Conclusions

In this paper, a general multiuser random access channel with retransmission gain was defined, and its throughput and stability region were analyzed. First analyzed is a two-user random access channel. By analyzing a two-dimensional Markov Chain, the stationary distribution probability was derived, from which the maximum steady throughput was obtained. A sufficient condition, under which the control-free MAC achieves the maximum steady throughput, was developed. In a similar vein, the analysis was extended to a general multiuser random access channel and the throughput and stability region were obtained in closed forms. Numerical studies showed that exploiting the retransmission gain in a random access channel can enhance the throughput significantly and expand the stability region. The analytical results predicted the simulation results well.

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9. Appendix

Proof of Equation (14)

$$\sum_{(m;c)\in\Omega} \theta_2^c (1-\theta_2)^{m-c} r_{(m;c)} = \sum_{(m;c)\in\Omega} (1-\theta_2)^m \theta_2^c (1-\theta_2)^{-c} r_{(m;c)}$$

$$= \left(1, 1-\theta_2, (1-\theta_2)^2, \ldots\right) R \left(1, \theta_2(1-\theta_2)^{-1}, \theta_2^2(1-\theta_2)^{-2}, \ldots\right)^T$$

$$= \mathrm{tr}\left[\left(1, 1-\theta_2, (1-\theta_2)^2, \ldots\right) R \left(1, \theta_2(1-\theta_2)^{-1}, \theta_2^2(1-\theta_2)^{-2}, \ldots\right)^T\right]$$

$$= \mathrm{tr}\left[R \left(1, \theta_2(1-\theta_2)^{-1}, \theta_2^2(1-\theta_2)^{-2}, \ldots\right)^T \left(1, 1-\theta_2, (1-\theta_2)^2, \ldots\right)\right]$$

$$= \mathrm{tr}\left[\begin{pmatrix} r_{(0;0)} & 0 & 0 & \cdots \\ r_{(1;0)} & r_{(1;1)} & 0 & \cdots \\ r_{(2;0)} & r_{(2;1)} & r_{(2;2)} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \times \begin{pmatrix} 1 & 1-\theta_2 & (1-\theta_2)^2 & \cdots \\ \theta_2(1-\theta_2)^{-1} & \theta_2 & \theta_2(1-\theta_2) & \cdots \\ \theta_2^2(1-\theta_2)^{-2} & \theta_2^2(1-\theta_2)^{-1} & \theta_2^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}\right]$$

$$= \mathrm{tr}\left[\begin{pmatrix} r_{(0;0)} & 0 & 0 & \cdots \\ r_{(1;0)} & r_{(1;1)} & 0 & \cdots \\ r_{(2;0)} & r_{(2;1)} & r_{(2;2)} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} 1 & 1-\theta_2 & (1-\theta_2)^2 & \cdots \\ 0 & \theta_2 & \theta_2(1-\theta_2) & \cdots \\ 0 & 0 & \theta_2^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}\right]$$

$$= \mathrm{tr}(R\Gamma(\theta_2)) . \qquad \square$$

Proof of Theorem 1

According to (18), the channel throughput is

$$T = \frac{2\theta}{\sum_{(m;c)\in\Omega} r_{(m;c)} \theta^c (1-\theta)^{m-c}} .$$

To maximize T, it is equivalent to minimize

$$\frac{2}{T} = \frac{1}{\theta} \sum_{(m;c)\in\Omega} r_{(m;c)} \theta^c (1-\theta)^{m-c}$$

which is equal to

$$\frac{1}{\theta}\left(1+\sum_{m\neq c,c\neq 0} r_{(m;c)}\theta^{c}(1-\theta)^{m-c}+\sum_{m=1}^{\infty} r_{(m;m)}\theta^{m}+\sum_{m=1}^{\infty} r_{(m;0)}(1-\theta)^{m}\right)$$
$$\geq \frac{1}{\theta}\left(1+\sum_{m=1}^{\infty} r_{(m;m)}\theta^{m}\right) \tag{54}$$

where the equality holds if and only if $\theta = 1$. Denote the right-hand side of (54) by $f(\theta)$. Then

$$f'(\theta) = -\frac{1}{\theta^2}+\sum_{m=2}^{\infty}(m-1)r_{(m;m)}\theta^{m-2},$$

which is increasing in $0 < \theta \leq 1$. Therefore, $f'(\theta) \leq f'(1)$. Letting $f'(1) \leq 0$ guarantees that $f(\theta)$ decreases monotonously in θ. In other words, $f'(1) = \sum_{m=2}^{\infty}(m-1)r_{(m;m)} - 1 \leq 0$ is a sufficient condition under which MAC is control free. □

Proof of Theorem 2

In an asymmetric two-user channel, the throughput given by (18) satisfies

$$T \leq \frac{\theta_1}{\sum_{m=0}^{\infty} r^{(1)}_{(m;m)}\theta_2^{m}}+\frac{\theta_2}{\sum_{m=0}^{\infty} r^{(2)}_{(m;m)}\theta_1^{m}} \tag{55}$$

where the equality holds if and only if $\theta_1 = \theta_2 = 1$. The right-hand side of (55) is denoted by $\tilde{T}$. For any $\theta_2 \in (0, 1]$, it suffices to obtain the condition under which $\tilde{T}$ is increasing in θ_1. Because $\tilde{T}$ is continuous and differentiable with respect to θ_1, its first order derivative given below is positive,

$$\frac{\partial \tilde{T}}{\partial \theta_1} = \frac{1}{\sum_{m=0}^{\infty} r^{(1)}_{(m;m)}\theta_2^{m}} - \frac{\theta_2\sum_{n=1}^{\infty} n r^{(2)}_{(n;n)}\theta_1^{n-1}}{\left(\sum_{n=0}^{\infty} r^{(2)}_{(n;n)}\theta_1^{n}\right)^2}$$
$$= \frac{\left(\sum_{n=0}^{\infty} r^{(2)}_{(n;n)}\theta_1^{n}\right)^2 - \theta_2\left(\sum_{n=1}^{\infty} n r^{(2)}_{(n;n)}\theta_1^{n-1}\right)\left(\sum_{m=0}^{\infty} r^{(1)}_{(m;m)}\theta_2^{m}\right)}{\left(\sum_{m=0}^{\infty} r^{(1)}_{(m;m)}\theta_2^{m}\right)\left(\sum_{n=0}^{\infty} r^{(2)}_{(n;n)}\theta_1^{n}\right)^2}. \tag{56}$$

The numerator in (56) is nonnegative, which is true due to (21). □

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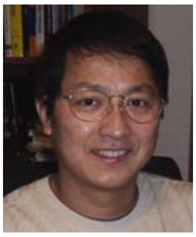
Junmin Shi, Ph.D., is a faculty member with Georgia State University, Atlanta, GA US. His research interests include wireless communication, operations management and supply chain management. Part of his current research is in the interface between operations management and finance.



Yi Sun, Ph.D., is an Assistant Professor with tenure in the Department of Electrical Engineering at the City College of City University of New York. Dr. Sun's research interests are in the areas of wireless communications and networking, biomedical optics and nanoscopy. He has published more than one hundred peer-reviewed journal and conference papers in these areas.

Xiaochen Zhang is a PhD candidate in the Department of Electrical Engineering, the City College of the City University of New York. He received his BS and MS in Computer Science from Shandong University and Wuhan University, China, in 2007 and 2009, respectively. His research interests include robotics navigation, communication systems, visual odometry and cloud computation.

Jizhong Xiao, Ph.D., is a Professor of electrical engineering with The City College of City University of New York. He started the Robotics Research Program at CCNY in 2002 as the Founding Director of CCNY Robotics Laboratory. His current research interests include robotics and control, body area network, assistive navigation, and multiagent systems. Dr. Xiao received the U.S. National Science Foundation CAREER Award in 2007 and the CCNY Outstanding Mentoring Award in 2011.