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The Vaguelette-Curvelet Decomposition for Image Deblurring

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** Regular Paper*

Abstract**:** We present a vaguelette-curvelet decomposition based image deblurring algorithm. We first perform denoising based on the hard-thresholding rule by estimating unknown curvelet coefficients. The proposed algorithm then calculates vaguelette functions by deconvolving the curvelet bases by the point spread function. Vaguelette transform is finally performed to make a clearly restored image. Since the proposed algorithm uses the curvelet transform to sensitively express the edges in all directions, it is possible to restore images with more naturally preserved edges in all directions.

Keywords**:** Vaguelette-curvelet decomposition, Fast discrete curvelet transform (FDCT), Image restoration

1. Introduction

Recently, the digital image acquisition devices, such as digital cameras and smartphones, have become popular. In addition, the technological development of semiconductors and software has made it possible to provide fast algorithms and massive storage space. On the other hand, many problems can arise in the image acquisition process. Image blur and noise are the dominating factors that degrade the image quality. To address this problem, a proper image deblurring method is required.

An image deblurring method aims to estimate the original undegraded image from the observed degraded image using an appropriate degradation model and computationally realizable inverse process. Given the point spread function (PSF), which is a mathematical model of image degradation, the result of a mathematical inverse process becomes a solution to the deconvolution or statistical inverse problem (SIP). On the other hand, the image restoration problem becomes ill-posed if the degradation model is not well-defined or the observed image contain noise.

Because the ill-posedness makes it impossible to find a unique solution, a priori information should be used to select the optimal solution in the given range, which is referred to as regularization. Various regularization based image restoration methods include parametric Wiener filter, constrained least-squares (CLS) filter, and iterative regularized image restoration [1].

The parametric Wiener and CLS filters perform deblurring in the frequency domain using the fast Fourier transform (FFT), under the assumption that the PSF is space-invariant. On the other hand iterative methods can deal with space-variant PSFs at the cost of an indefinite processing time [2].

In this context vaguelette-wavelet decomposition can easily seek a solution to a statistical linear inverse problem (SLIP). Given the PSF, it is possible to find the vaguelette function by deconvolving the PSF on the wavelet basis [3-5].

Although the vaguelette-wavelet decomposition can restore the directional edges in the vertical, horizontal, and diagonal directions, it cannot reconstruct line singularities such as ridges and curves.

This paper proposes, an image deblurring method using curvelet based vaguelette inverse transform. Fig. 1 compares the wavelet and curvelet tilings. The proposed

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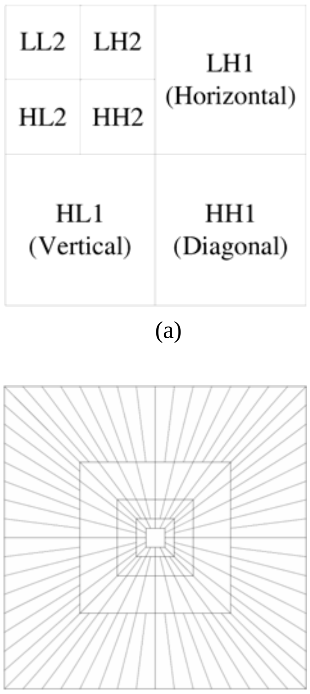


Fig. 1. (a) Wavelet tiling, (b) discrete curvelet tiling with parabolic pseudopolar supports in the frequency plane.

curvelet tiling-based restoration method can accurately express the edges in all directions. In the proposed method, the PSF is convolved in the tiling basis function of each subband to determine the vaguelette function, which expresses the degraded image as curvelet coefficients. The restored image is obtained using the inverse curvelet transform.

The remainder of this paper is organized as follows: In Sec. 2 we describes the vaguelette-wavelet decomposition and the curvelet transform. Sec. 3 presents the proposed image deblurring algorithm. The results and conclusion are reported in Secs. 4 and 5, respectively.

2. Background

This section describes the vaguelette-wavelet decomposition and the curvelet transform.

2.1 Vaguelette-Wavelet Decomposition

Vaguelette-wavelet decomposition has been proposed by Donoho for image restoration in [3]. The degraded image is represented as a linear combination of wavelets, as

$$g = Hf = \sum_j \sum_k d_{jk}\psi_{jk}, \tag{1}$$

where f represents the original undegraded image, H is the PSF, ψ_{jk} the kth wavelet function in the jth scale, and d_{jk} the inner product of Hf and ψ_{jk}.

The vaguelette function is defined as

$$v_{jk} = H^{-1}\psi_{jk}/\beta_{jk}, \tag{2}$$

assuming that there are constants, β_{jk}, such that the set of scaling functions are properly scaled. Note that wavelets ψ_{jk} in Eq. (1) and vaguelettes v_{jk} in Eq. (2) are different from those used in wavelet-vaguelette decomposition.

The directional wavelet bases have properties to reduce the remaining noise in each block. The resulting directional wavelet bases are shown in Fig. 2.

The vaguelette-wavelet decomposition algorithm for 2D deconvolution can be summarized as Algorithm 1.

To obtain the vaguelettes, four 2D wavelet function coefficients are deconvolved by the PSF. The vaguelettes are generated by deconvolving the estimated PSF from directional wavelet bases as follows:

$$\begin{aligned} v^A(x,y) &= h(x,y)^{-1} * \varphi_V(x,y) \\ v^H(x,y) &= h(x,y)^{-1} * \psi_V^H(x,y) \\ v^V(x,y) &= h(x,y)^{-1} * \psi_V^V(x,y) \\ v^D(x,y) &= h(x,y)^{-1} * \psi_V^D(x,y), \end{aligned} \tag{3}$$

where $h(x,y)^{-1}$ represents the estimated PSF.

The inverse vaguelette transform reconstructs the deconvolved image by adding four results as follows:

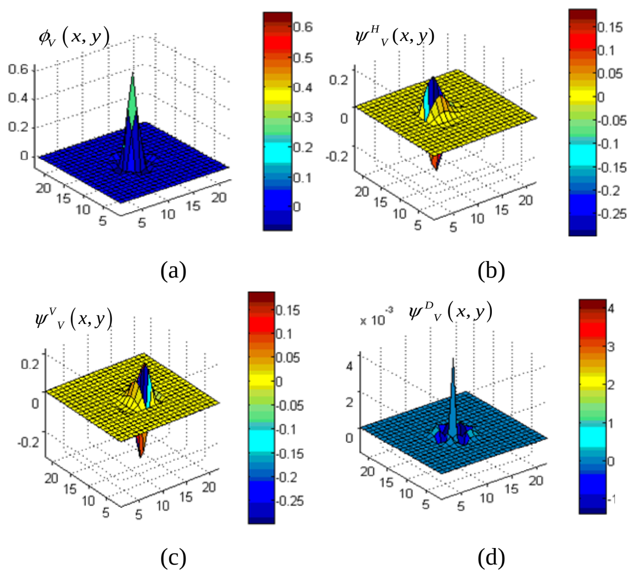


Fig. 2. Directional wavelet bases using symlet filters.

Algorithm 1. Vaguelette-Wavelet Decomposition.

step 1 :	Given a blurred image, choose the proper 2D wavelet bases ψ_{jk}.
step 2 :	Compute the wavelet transform coefficients d_{jk} using the 2D discrete wavelet transform.
step 3 :	Perform the shrinking wavelet coefficients.
step 4 :	Compute the vaguelettes function by deconvolving the wavelet basis by the PSF and obtain an estimate of the restored using the inverse vaguelette transform.

$$\begin{aligned}\widehat{f} = W^A(J+1,m,n) = \\ v^A(m,n) * W^{A,UP}(J-1,m,n)+ \\ v^H(m,n) * W^{H,UP}(J-1,m,n)+ \\ v^V(m,n) * W^{V,UP}(J-1,m,n)+ \\ v^D(m,n) * W^{D,UP}(J-1,m,n),\end{aligned} \tag{4}$$

where $W^{i,UP}$ represents the up-sampled wavelet coefficients, and W^A is the result of inverse vaguelette transformation expressed as

$$\begin{aligned}W^A(J,m,n) = \\ \psi^A(m,n) * W^{A,UP}(j-1,m,n)+ \\ \psi^H(m,n) * W^{H,UP}(j-1,m,n)+ \\ \psi^V(m,n) * W^{V,UP}(j-1,m,n)+ \\ \psi^D(m,n) * W^{D,UP}(j-1,m,n).\end{aligned} \tag{5}$$

2.2 Curvelet Transform

Candes and Donoho developed a new geometric multiscale transformation, called the curvelet transform [6-8], which allows an optimal sparse representation of objects with C^2-singularities. The needle-shape elements of this transform has very high directional sensitivity and anisotropy. For a smooth object f with discontinuities along the smooth curves, the best m-term approximation by curvelet thresholding obeys $\|f - \widetilde{f}_m\|_2^2 \ll Cm^{-2}(\log m)^3$, whereas the decaying rate of the wavelet is only m^{-1}. The new transform represents the edges and singularities along the curves much more efficiently than the wavelet transforms.

The curvelet transform enables parabolic (anisotropic) scaling as shown in Fig. 3.

In this paper, the fast discrete curvelet transform (FDCT) with a Cartesian data structure was used [6]. This transform uses shearing in place of rotation and a pseudo-polar grid in place of the regular polar grid. The transformation process requires the $O(N^2 \log(N))$ flops of operations, for an $N \times N$ image.

The FDCT is expressed as

$$c(j,l.k) = \int \widehat{f}(\omega)\widetilde{U}_j(S_{\theta_l}^{-1}\omega)e^{i<b,\omega>}d\omega, \tag{6}$$

where $\widetilde{U}_j$ is a Cartesian window and S_θ is the shear matrix.

Equipped with this definition, the architecture of the FDCT via wrapping is expressed as Algorithm 2.

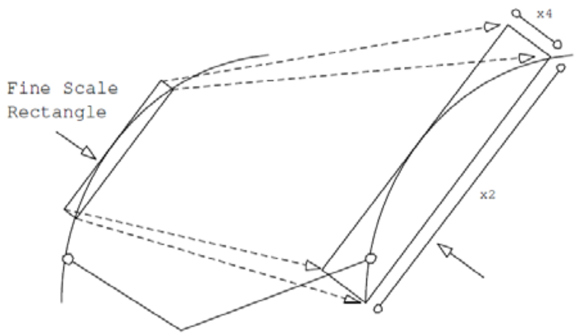


Fig. 3. Two invariant curves under parabolic scaling.

Algorithm 2. Fast Discrete Curvelet Transform.

Input :	Image
Output :	Curvelet Coefficients
step 1 :	Apply the 2D FFT and obtain Fourier samples $\widehat{f}[n_1,n_2], -n/2 \le n_1, n_2 \le n/2$.
step 2 :	For each scale and angle form the product $\widetilde{U}_{j,l}[n_1,n_2]\widehat{f}[n_1,n_2]$.
step 3 :	Wrap this product around the origin and obtain $v_{jk} = H^{-1}\psi_{jk}/\beta_{jk}$, where the range for n_1 and n_2 is now $0 \le n_1 \le L_{1,j}$ and $0 \le n_2 \le L_{2,j}$ (for θ in the range $(-\pi/4, \pi/4)$).
step 4 :	Apply the inverse 2D FFT to each $\widetilde{f}_{j,l}$, hence collecting the discrete coefficients $c^D(j,l,k)$.

3. Image Restoration Algorithm Using the Curvelet Transform

In this section, we present the curvelet-based image restoration algorithm. Fig. 4 is the flowchart of the proposed algorithm.

A degraded image is transformed using the curvelet transform, and then curvelet coefficients are shrunk to remove noise. The vaguelette function is generated by deconvolving the PSF from the curvelet basis. The corresponding inverse vaguelette transform results in the restored image. In this paper, we assume that the PSF is known.

3.1 Shrinkage of Curvelet Coefficients for Denoising

Suppose that noisy data is given as follows:

$$x_{i,j} = f(i,j) + \sigma z_{i,j}, \tag{7}$$

where f is the image to be restored, and $z_{i,j} \sim N(0,1)$ is the white Gaussian noise. Unlike the FFT or the fast wavelet transform (FWT), the discrete curvelet transform is not norm-preserving, so the variance of the noisy curvelet coefficients will depend on the curvelet index λ. For example, by letting F denote the discrete curvelet transform matrix, $Fz_{i,j} \sim N(0, FF^T)$. Because the computation of FF^T is prohibitively expensive, an approximate value σ_λ^2 of the individual variances was calculated using Monte-Carlo simulations where the diagonal elements of FF^T are simply estimated by evaluating the curvelet transforms of a few standard white noise images.

Let $y\lambda$ be the noisy curvelet coefficients $(y = Fx)$. The following hard-thresholding rule was used to estimate the unknown curvelet coefficients [9, 10]

$$\widehat{y}\lambda = \begin{cases} y\lambda, & |y\lambda|/\sigma \ge k\widetilde{\sigma}\lambda \\ 0, & |y\lambda|/\sigma < k\widetilde{\sigma}\lambda. \end{cases} \tag{8}$$

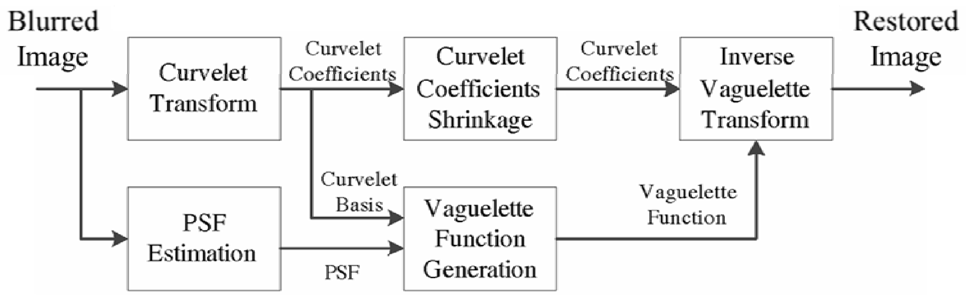
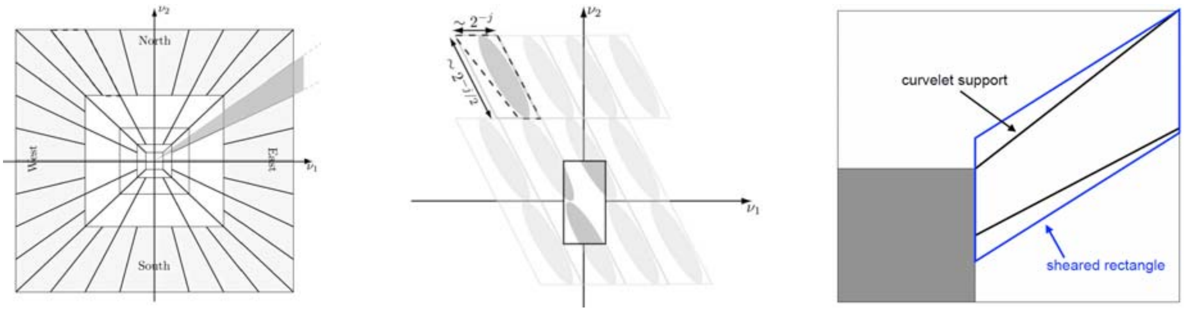


Fig. 4. Flowchart of the proposed algorithm.



(a) (b) (c)

Fig. 5. (a) Discrete curvelet frequency tiling, (b) its support adapted to the wrapping transformation, (c) sheared rectangle.

3.2 Vaguelette-Curvelet Decomposition

The curvelet-based image deblurring algorithm is presented.

The vaguelette is a new basis function that can be considered as the restored basis. Step 5 in Algorithm 3 is called the inverse vaguelette transform.

Figs. 5(a) and 5(b) show the tiled basis function of each sub-band. The sheared rectangle in Fig. 5(c) is a single basis function.

Assume that window $W_{j0}(n_1, n_2)$ is supported within a sheared rectangle expressed as

$$\mathcal{P}_j = \{(n_1, n_2) : 0 \le n_1 - n_0 < L_j, -\tfrac{l_j}{2} \le n_2 < \tfrac{l_j}{2}\}. \tag{9}$$

The curvelet coefficients $C^D_{j,l,k}$ are defined as

$$\sum_{n_1, n_2 \in \mathcal{P}_j} \widehat{f}(n_1, n_2 + n_1 \tan\theta_{j,l}) W_{j0} e^{-i2\pi \frac{n_1 k_1}{L_j} + \frac{n_2 k_2}{l_j}}, \tag{10}$$

where $\widehat{f}$ needs to be evaluated inside the sheared rectangle, and W_{j0} is the curvelet basis function. The vaguelette function can be obtained by deconvolving W_{j0} by the PSF, such as

$$v_{j0} = H^{-1} W_{j0}. \tag{11}$$

The inverse FDCT can be obtained by performing the FDCT of Algorithm 3 in reverse order. Substituting the vaguelette function into the inverse FDCT, resulting in Eq. (12).

Algorithm 3. Vaguelette-Curvelet Decomposition.

Input	: Degraded Image
Output	: Restored Image
step 1 :	Obtain curvelet coefficients by implementing the FDCT of a degraded image.
step 2 :	Shrink curvelet coefficients for removing noise.
step 3 :	Estimate the PSF if it is not given a priori.
Step 4 :	Obtain the vaguelette function by deconvolving the curvelet basis by the PSF.
step 5 :	Perform the inverse FDCT of the vaguelette function using the new curvelet basis.

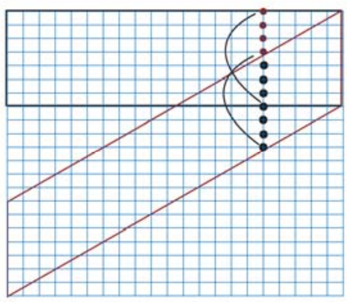


Fig. 6. Periodic wrap around.

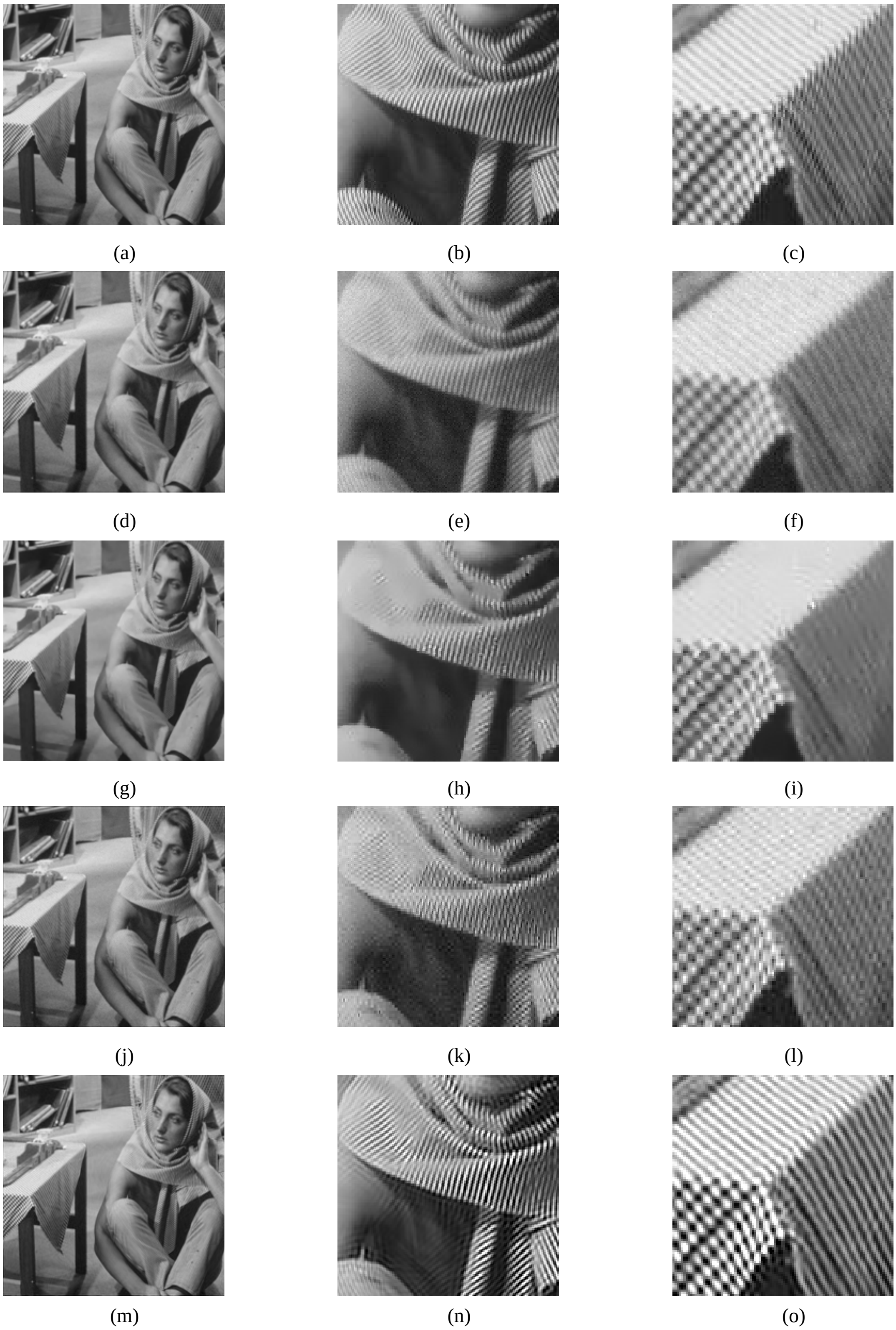
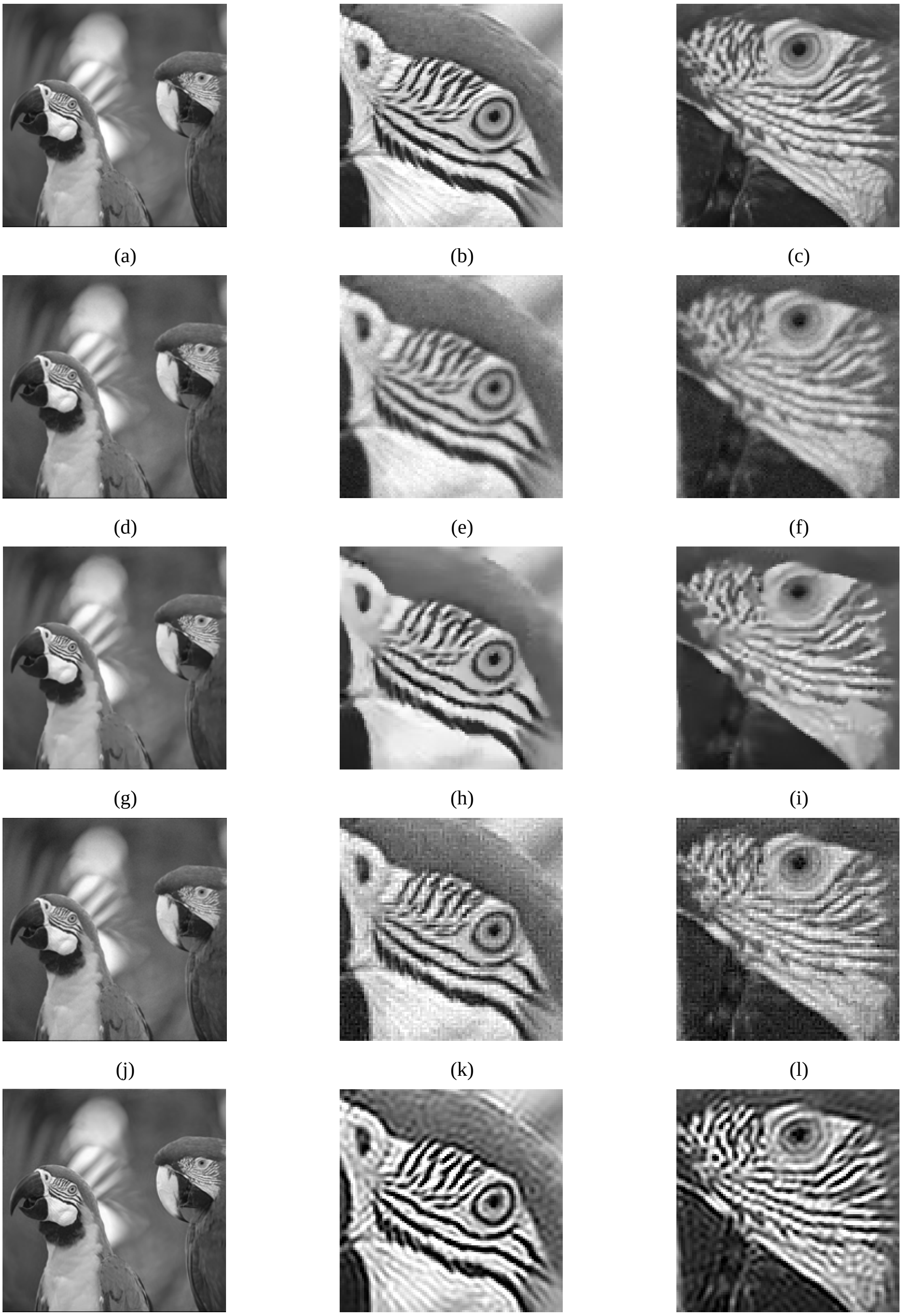


Fig. 7. Experimental results using the Barbara image (a) original image, (b) and (c) two enlarged parts of the original image, (d) degraded image, (e) and (f) two enlarged parts of the degraded image, (g) restored image using the shape-adaptive DCT [11], (h) and (i) two enlarged parts of the restored image, (j) restored image using the vaguelette-wavelet decomposition [5], (k) and (l) two enlarged parts of the restored image, (m) restored image using the proposed vaguelette-curvelet decomposition, (n) and (o) two enlarged parts of the restored image using the proposed algorithm.



(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

(j) (k) (l)

(m) (n) (o)

Fig. 8. Experimental results using the Parrot image (a) original image, (b) and (c) two enlarged parts of the original image, (d) degraded image, (e) and (f) two enlarged parts of the degraded image, (g) restored image using the shape-adaptive DCT [11], (h) and (i) two enlarged parts of the restored image, (j) restored image using the vaguelette-wavelet decomposition [5], (k) and (l) two enlarged parts of the restored image, (m) restored image using the proposed vaguelette-curvelet decomposition, (n) and (o) two enlarged parts of the restored image using the proposed algorithm.

Table 1. Summary of the PSNR and SSIM results of Figs. 7 and 8.

	Method	PSNR	SSIM
Fig. 7 (Barbara)	Blurred image	24.3117	0.7084
	Shape-adaptive DCT [11]	25.1194	0.7563
	Vaguelette-wavelet [5]	24.8203	0.7176
	Proposed method	26.5421	0.8221
Fig. 8 (Parrot)	Method	PSNR	SSIM
	Blurred image	29.1036	0.7662
	Shape-adaptive DCT [11]	31.8135	0.9030
	Vaguelette-wavelet [5]	30.4104	0.8473
	Proposed method	31.7224	0.9106

$$\sum_{n_1,n_2\in\mathcal{P}_j} \widehat{f}(n_1, n_2 + n_1\tan\theta_{j,l})\, v_{j0}\, e^{-i2\pi\left(\frac{n_1 k_1}{L_j}+\frac{n_2 k_2}{l_j}\right)}. \quad (12)$$

The resulting restored image can be obtained by performing the inverse vaguelette transform using the vaguelette function as the basis function.

Fig. 6 shows the samples inside each parallelogram tile by periodic wrap-around. This can be calculated by taking the FFTs on rectangular tiles.

4. Experimental Results

The experiments were conducted to evaluate the performance of the proposed algorithm. In this study, a 7×7 Gaussian PSF and 20 dB additive white Gaussian noise were used to simulate the image degradation. Fig. 7 compares the results obtained from the proposed algorithm with the result from the vaguelette-wavelet decomposition [5] and shape-adaptive DCT (SADCT) [11]. Although the edges in Figs. 7(k) and 7(l) are smoothed, in Figs. 7(n) and 7(o) are well restored in all directions. Fig. 8 summarizes the same experiment using a different test image.

Table 1 lists the measure of each image of the experimental results of Figs. 7 and 8.

5. Conclusion

This paper proposed a novel vaguelette-curvelet decomposition restoration algorithm to remove noise by shrinking the curvelet coefficient, and restoring the image details in all directions using the vaguelette inverse transformation of an individual tiling basis.

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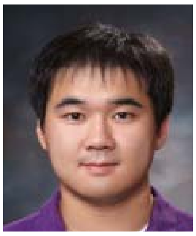

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