IEEK Transactions on Smart Processing and Computing

# Feasibility Confirmation of Angular Velocity Stall Control for Small-Scaled Wind Turbine System by Phase Plane Method

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Received May 13, 2013; Revised May 28, 2013; Accepted June 12, 2013; Published August 31, 2013

\* Extended from a Conference: Preliminary results of this paper were presented at the ITC-CSCC 2013. This present paper has been accepted by the editorial board through the regular reviewing process that confirms the original contribution.

Abstract: The main aim of this study was to suppress the angular velocity against strong winds during storms and analyze the stability and performance of the phase plane method. The utilization of small-scale wind turbine system has become common in agriculture, houses, etc. Therefore, it is considered to be a scheme for preserving the natural energy or avoiding the use of fossil fuels. Moreover, settling small-scaled wind turbines is simpler and more acceptable compared to ordinary huge wind turbines. In addition, after converting the energy there is no requirement for distribution. Therefore, a much lower cost can be expected for small-scaled wind turbines. On the other hand, this system cannot be operated continuously because the small-scaled wind turbine consists of a small blade that has low inertia momentum. Therefore, it may exceed the boundary of angular velocity, which may cause a fault in the system due to the centrifugal force. The aim of this study was to reduce the angular velocity by controlling the stall factor. Stall factor control consists of two control methods. One is a shock absorber that is loaded in the junction of the axis of the blade of the wind turbine gear wheel and the other is pitch angle control. Basically, the stall factor itself exhibits nonlinear behavior. Therefore, this paper confirmed the feasibility of stall factor control in producing desirable performance whilst maintaining stability.

**Keywords:** Small-scaled wind turbine system, Shock absorber, Pitch angle control, Adaptive control and Phase plane method

## 1. Introduction

Small-scaled wind turbines are used widely for pumping, electricity generation, etc.. Despite of this, the importation of small scaled wind turbines for electronic power generators is low in these days. The important problem of small-scaled wind turbine is the establishment of stall control against strong winds. As a result, excessive revolutions may cause damage to the turbine. If this problem can be solved, these turbines can produce large amounts of energy. Therefore, the innovation of small-

scaled wind turbines is a very important part of energy generation. In this study, the wind turbine system was located in a gale area and the angular velocity of blade was controlled by the axis friction and pitch angle. The wind turbine rotates clockwise from the front perspective. In general, the blades of wind turbines descend 3 to 5° to the front and lay on the center axis with a slant of 2 to 4°. The axis is propped up by 2 springs that cross to the right angle of the center axis. In this moment, the pitch angle is optimized. As a matter of fact, the blade fluctuates the spring when the angular velocity reaches the boundary

speed. This study considered the improvement in the mechanism of spring installation inside turbines by repeated experiments and numerical analysis. The stall control mechanism is as follows. Under windless or calm conditions, the attitude of the blade is on the axis. When the wind turbine receives a strong wind to the area swept by the blade, the blades are pushed strongly backward. As a result, the blades reach the stall angle and the angular velocity is reduced. When the turbine is exposed to strong winds, the centrifugal force will increase and as a result, the axis of the blades will shift to the central axis line and at the same time, the slope angle will become the reverse of the pitch angle, which reduces the angular velocity. These stall control dynamics can be realized by the movement of the blade backward or forward and independent axis rotation. Finally, stall control is realized and the angular velocity of the blade is reduced. Using this procedure, the centrifugal force disappears and the mechanism of the blades returns to its origin position. Afterward, it will begin to rotate again. In other words, by passive control of the spring installed in the blades, mechanism of stall control is realizable. The next chapter introduces the mechanism of small scaled wind turbines.

# 2. Mechanism of small-scaled wind turbine

The small-scaled wind turbine system can be expressed by the dynamics of the angular velocity and pitch angle. As mentioned previously, the angular velocity of these small-scaled wind turbines can be controlled by the pitch angle of the blade. For example, if the angular velocity increases, the pitch angle moves to the stall angle, resulting in a decrease in the angular velocity of the blade. In addition, the angular velocity can be decreased by friction control. The friction stall factor is increased in proportion to the angular velocity. These procedures can be expressed mathematically. Eqs. (1) and (2) express the dynamics of the angular velocity of the bald and pitch angle, respectively.

Dynamics of angular velocity

$$J_{\omega} \frac{d\omega(t)}{dt} = \frac{1}{2} C_T \rho A R \int_0^t V^2(t) e^{-(\tau - t)} d\tau \Theta(\theta(t)) - \frac{1}{2} I_G \omega^2(t) - f_r \omega(t)$$

$$\tag{1}$$

Dynamics of pitch angle

$$J_b \frac{d^2 \theta(t)}{dt^2} = MR^2 \omega^2(t)\theta(t) - L\theta(t)k - c\frac{d\theta(t)}{dt}$$
 (2)

where, 
$$\Theta(\theta) = \frac{\tilde{\theta}(t)}{\theta_{\text{max}}}$$
 and  $\tilde{\theta}(t) = \theta_{\text{max}} - \theta(t)$ .

As shown in Eq. (1), the squared wind velocity is convoluted by the first order filter. This is because of the inertia moment that works in the blade. Therefore, there would be a slight lag for rotation initially for a massive blade.

Table 1. Identification of the parameters.

$J_{\omega}$	Inertia moment of blade	
$\omega(t)$	Angular velocity	
$C_{\mathrm{T}}$	Torque factor	
ρ	Density of atmosphere	
A	Area swept by blade	
R	Length of blade	
V(t)	Speed of wind	
$\Theta(\theta)$	Wind turbine stall factor	
$I_{\mathrm{G}}$	Power Generation Inertia Moment Factor	
$f_{\rm r}$	The axis friction factor	
$J_b$	Inertia moment of a plate of blade	
M	Mass of a blade	
L	The distance from spring to the edge of blade	
k	Factor of spring	
С	damper	
$ heta_{ ext{max}}$	Initial values of pitch angle	

Table 1 lists the parameters of Eqs. (1) and (2).

As shown in Eq. (1), the input wind includes the input delay. From Eq. (2) the wind turbine stall factor is fed back to Eq. (1) and by this, the angular velocity is controlled.

## 3. Stability and performance Analysis

As mentioned in the previous chapter, wind turbines contain nonlinear parameters, which are time variables. Moreover, the nonlinearity of the wind turbine dynamics can affect the performance of the angular velocity. Although the pitch angle controller is set to avoid fluctuations, however, the angular velocity itself has rapid alterations compared to large scale of wind turbine systems due to the low factor of the inertia moment. Furthermore. considering the internal stability of the wind turbine system, if the angular velocity exceeds the limited value it might cause a malfunction of the entire system. Therefore, the priority procedure aims to guarantee the stability of the wind turbine system, which can refer to the angular velocity and for next step, focus on the performance. An improvement in performance constantly sustains the angular velocity at a specified value, even in stormy days. Therefore, the energy can be produced efficiently and be distributed to the consumer.

To evaluate the internal stability, the phase shape was considered. The phase shape is to plot the displacement as a function of the derivative of the displacement. If the curve diverges to infinity or is a large scale value, it is an unstable system. If the curve converges to a specific or constant value, it is internally stable and if the steady value fluctuates between the specified value then it is semi-stable. For example, in this case of a wind turbine system, the

angular velocity in the vertical axis and pitch angle in the horizontal axis were taken. To evaluate the performance using the phase plane method, if the system is stable or semi-stable, then the performance of the system can be evaluated by measuring the average of the curve radius. If the radius is large, the system performance is degraded. In addition, if the radius contains a small value or is less than the specified values, it represents enhanced performance. Assume a nonlinear asymptotic-stable system,

$$\begin{cases} \dot{x} = f(x) \\ y = g(x) \end{cases}$$
 (3)

where x is a state variable, y is the observer and f(.) and g(.) are nonlinear function, respectively. f(.) is the dynamics of the system and g(.) is the observer function of the system. For example, the phase-plane trajectory of a system can be indicated as follows.

# 3.1 Evaluation of the stability

General, an evaluation of non-linear systems stability is performed using the direct Lyapunov theorem. Consider a system with the following form:

$$\dot{x} = f(x)$$

where  $x \in \Re^n$ , and f(x) is a smooth function, with f(0)=0. If there is a positive definite and proper smooth function V(x) such that

$$\frac{\partial V}{\partial x}f(x) < 0$$

for all nonzero x, then the equilibrium x=0 of the system is globally asymptotically stable.

The above theorem can be applied to the phase plane analysis method.

From Fig. 1, the radius of the non-linear system trajectory becomes:

$$r = \sqrt{\dot{x}^2(t) + x^2(t)}$$

The square radius then becomes:

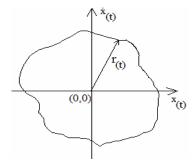


Fig. 1. Phase plane diagram.

$$r^2 = \dot{x}^2(t) + x^2(t)$$

If and only if the square radius satisfies the following condition is the system asymptotically stable.

$$0 \le r^2(t) \le \gamma \tag{4}$$

where  $\gamma = \min(\dot{x}(t), x(t))$ .

If the angular velocity converges to a constant value,  $\gamma$  becomes equivalent to the angular velocities value, which is the boundary of angular velocity.

#### 3.2 Evaluation of Performance

The performance a nonlinear system can also be assessed using the phase plane method. The advantage of performance analysis in the phase plane method is that a long time frame can be expressed in the phase plane because the time axis is not considered. Moreover, the trajectory of a system can express the behavior of its dynamics. Therefore, the phase plane method is one of the suitable performance evaluations of nonlinear systems. Again, the system indicated in Fig. 1 was considered. The square of the radius of the system trajectory becomes

$$r^2 = \dot{x}^2(t) + x^2(t)$$
.

If and only if the average of the square of the radius satisfies the following condition is the system well performed.

$$\tan^{-1}(\frac{\dot{x}(t)}{x(t)}) = cons. \tag{5}$$

$$r(t) \approx \langle r(t) \rangle$$
 (6)

where 
$$\langle r(t) \rangle = \frac{1}{T} \int_{0}^{T} r(t)dt$$
.

The performance of a system is dependent on the shape of the curve on the phase plane. If the curve is a circle, then the performance is good. Eq. (5) represents the ratio of the angular displacement and the velocity should be constant to draw a circle. In addition, Eq. (6) shows that the radius of curve should be approximately the same as the average value of the curve radius. In this case, the angular velocity should maintain a constant value to produce more efficient power. The next chapter introduces the design of the controller.

# 4. Design of an Adaptive Nonlinear Controller

As discussed previously, the resistance control is considered to avoid exceeding the angular velocity from the limitation. Therefore, by rewriting Eq. (1), the dynamics of a wind turbine system in terms of the angular velocity is indicated as follows. If the angular velocity exceeds the limitation, the adaptive friction  $\gamma(\omega(t))$  works

and decreases the velocity [1]. Otherwise,  $\gamma(\omega(t))$  has no influence.

$$\gamma(\omega(t)) = 1 + e^{\omega(t) - \omega_{stall}} \tag{7}$$

(8)

 $\omega_{stall}$  stands for the boundary of angular velocity. Dynamics of angular velocity with adaptive friction

$$\begin{split} J_{\omega} \frac{d\omega(t)}{dt} &= \frac{1}{2} C_{T} \rho A R \int_{0}^{t} V^{2}(t) e^{-(\tau - t)} d\tau \Theta(\theta) - \frac{1}{2} I_{G} \omega^{2}(t) \\ &- f_{r} \gamma(\omega(t)) \omega(t) \end{split}$$

To have better performance, pitch angle control [2] is required. Therefore, we assume a stability factor, such as  $\alpha$ , is assumed, and annexed to the pitch angle dynamics. Subsequently, Eq. (8) is linearized [3-5]. The process of linearization is to fix the angular velocity and neglect the multiplication between the pitch angle and squared angular velocity [6]. Eq. (8) can be rewritten as follows:

Dynamics of pitch angle

$$J_b \frac{d^2 \theta(t)}{dt^2} = MR^2 \omega^2 \theta(t) - \alpha L \theta(t) k - c \frac{d \theta(t)}{dt}$$

The Laplace transform of above equation is

Laplace transform  $J_b(s^2\theta(s) - s\theta(0) - \dot{\theta}(0))$   $= (MR^2\omega^2 - \alpha Lk)\theta(s) + c(s\theta(s) - \theta(0))$ 

where  $\alpha$  is the stability factor and is correlated with the pitch angle and spring.

$$G_{\theta}(s) = \frac{(1-s)\theta(0)}{s^2 + \frac{c}{J_b}s + \frac{Lk\alpha - MR\omega^2}{J_b}}$$
(9)

 $\dot{\theta}(0)$  =0 and  $\theta(0) = \theta_{\rm max}$ . Here, if  $\alpha = \frac{J_b + MR\omega^2}{Lk}$  and is substitute into Eq. (8), then

$$G_{\theta} = \frac{(1-s)\theta(0)}{s^2 + \frac{c}{J_L}s + 1}$$
 (10)

According to the final value theorem the steady step response is:  $\theta_{steady} = \theta(\infty) = \lim_{s \to 0} sG_{\theta} \frac{1}{s} = \theta(0)$ 

Therefore, the performance of the turbine can be improved by choosing  $\alpha$  as  $\alpha(\omega^2(t)) = \frac{J_b + MR\omega^2(t)}{Lk}$ .

## 5. Simulation and Results

To confirm the stability of the small-scaled wind turbine system and its performance, the turbines were simulated using actual model specifications (Fig. 3) and actual wind velocities with an average 25 [m/s] (Fig. 4). Subsequently, the wind turbines behavior, which is the pitch angle, angular velocity and its angular accelerations sequences, were analyzed using the phase plane method [7, 8]. The following table and equation indicate the simulation conditions with an adaptive angular velocity and pitch angle control, respectively.

Table 2. Identification of the parameters.

Parameters	Values	Units
$J_{\omega}$	16	kg⋅m²
$\omega(t)$	Variable	rad/s
$C_{T}$	0.2	N·m
ρ	1.2	Kg/m <sup>3</sup>
A	7.0686	m <sup>2</sup>
R	1.5	m
V(t)	Actual wind	m/s
$\Theta(\theta)$	Variable	
$I_G$	4.5185	kg⋅m²
$f_r$	0.001	kg ⋅ m <sup>2</sup> /s
$J_b$	8	kg⋅m²
M	14	kg
L	0.2	m
k	1000	N/m
c	200	kg/s
$\theta_{ m max}$	0.0873	rad
$\omega_{stall}$	15	rad/s

$$\begin{cases} J_{\omega} \frac{d\omega(t)}{dt} = \frac{1}{2} C_{T} \rho A R_{J}^{t} V^{2}(t) e^{-(\tau - t)} d\tau \Theta(\theta) \\ -\frac{1}{2} I_{G} \omega^{2}(t) - f_{r} \gamma(\omega(t)) \omega(t) \end{cases}$$

$$\begin{cases} J_{b} \frac{d^{2} \theta(t)}{dt^{2}} = M R^{2} \omega^{2} \theta(t) - \alpha L \theta(t) k - c \frac{d\theta(t)}{dt} \\ \Theta(\theta) = \frac{\theta_{\text{max}} - \theta(t)}{\theta_{\text{max}}} \end{cases}$$

$$(11)$$

Eventually, the  $\gamma$  stall factor is correlated with the angular velocity and  $\alpha$  is correlated with the pitch angle of blade

The operation factor in Fig. 2 isindicated as follows:

$$f(\omega) = 1 + e^{\omega(t) - \omega_{stall}} \tag{12}$$

$$z(\theta) = \frac{\theta_{\text{max}} - \theta(t)}{\theta_{\text{max}}}$$
 (13)

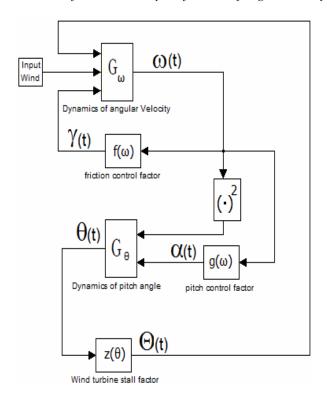


Fig. 2. Block diagram of friction and pitch angle control.



Fig. 3. Small-scale wind turbine system.

$$g(\omega) = \frac{J_b + MR\omega^2}{Lk} \tag{14}$$

In the following graph, " $\omega$ " and " $\theta$ " denote the angular velocity and pitch angle, respectively. In addition, in the vertical axis of the figures, there is "d/dt $\omega$ ", which is the derivative of  $\omega$  (angular acceleration).

Figs. 5 to 7 show the angular velocity, acceleration and pitch angle for the case without a loading shock absorber, respectively. The amplitudes of each graph fluctuate and

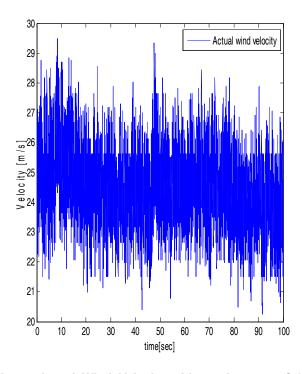


Fig. 4. Actual Wind Velocity with an Average of 25 [m/s].

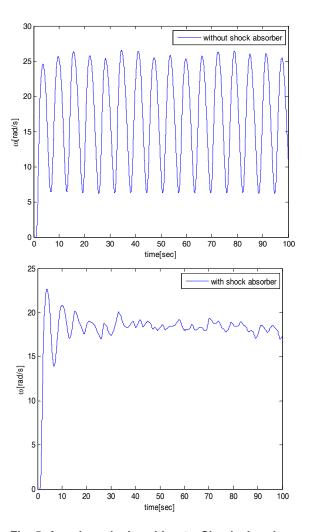


Fig. 5. Angular velocity without a Shock absorber.

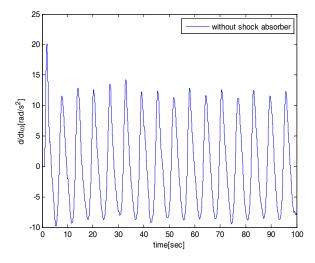


Fig. 6. Angular acceleration without a Shock absorber.

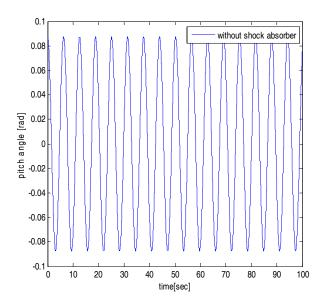


Fig. 7. Pitch angle without a Shock absorber.

do not converge. The performance can also be observed in the phase plane from Figs. 11 to 13. Figs. 11 to 13 show the trajectory of  $\omega$ -d $\omega$ /dt,  $\theta$ - $\omega$  and  $\theta$ -d $\omega$ /dt, respectively. From these three trajectories, the performance and stability can be determined because the main object is to suppress the angular velocity fluctuation. On the other hand, the phase planes for the three cases do not satisfy the desirable performance conditions. Therefore, a shock absorber was loaded to overcome the problem. Figs. 8 to 10 show the angular velocity, acceleration and pitch angle for the case with a loading shock absorber, respectively. The entire curve shows that the system is not globally asymptotically stable but is asymptotically stable. Stable and desirable performance can be confirmed from Figs. 14 to 16. Figs. 14 to 16 shows the trajectory of  $\omega$ -d $\omega$ /dt,  $\theta$ - $\omega$  and  $\theta$ -d $\omega$ /dt, respectively. The entire trajectory satisfies the desirable performance and asymptotical stability conditions. Therefore, phase plane method can easily indicate the system performance and behavior.

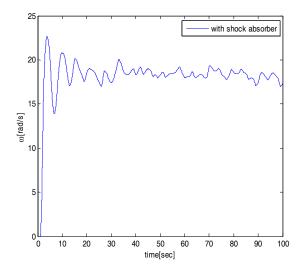


Fig. 8. Angular velocity with a Shock absorber.

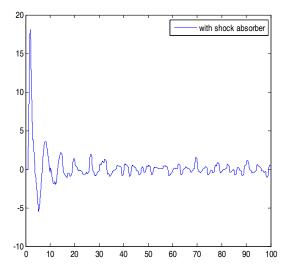


Fig. 9. Angular acceleration with a Shock absorber.

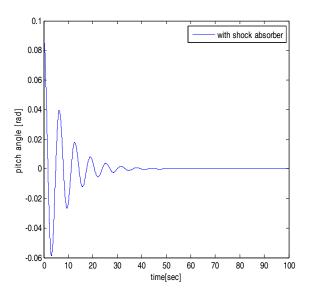


Fig. 10. Pitch angle with a Shock absorber.

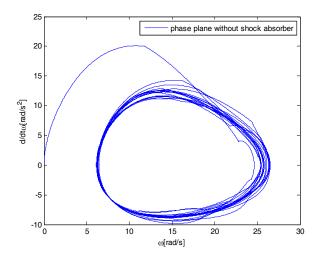


Fig. 11. Trajectory of thw phase plane for  $\omega\text{-}d\omega\text{/}dt$  without a shock absorber.

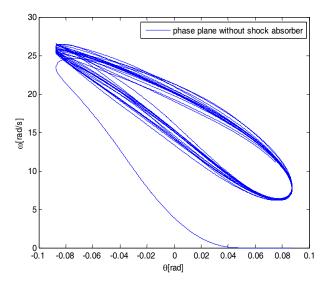


Fig. 12. Trajectory of the phase plane for  $\theta\text{-}\omega$  without a shock absorber.

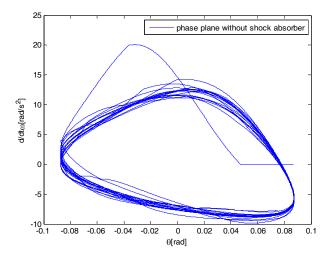


Fig. 13. Trajectory of the phase plane for  $\theta\text{-}d\omega/dt$  without a shock absorber.

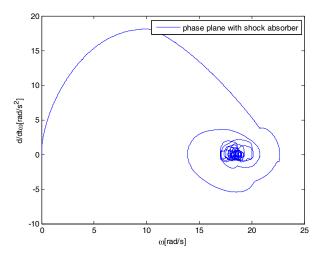


Fig. 14. Trajectory of the phase plane for  $\omega\text{-}d\omega\text{/}dt$  with a shock absorber.

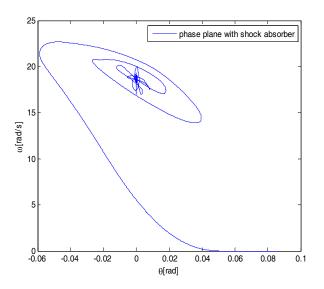


Fig. 15. Trajectory of the phase plane for  $\theta\text{-}\omega$  with a shock absorber.

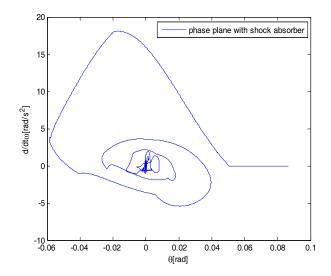


Fig. 16. Trajectory of the phase plane for  $\theta\text{-}d\omega/dt$  with a shock absorber.

#### 6. Conclusion

This study examined the small-scaled wind turbine system stability and performance with the phase plane method. Moreover, a shock absorber was loaded and pitch angle control was installed to improve the angular velocity performance. As a consequence, the feasibility of the pitch angle control and shock absorber could be confirmed using phase plane method by observing the trajectory of the system.

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