

An Application of Hilbert-Huang Transform on the Non-Stationary Astronomical Time Series: The Superorbital Modulation of SMC X-1

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We present the Hilbert-Huang transform (HHT) analysis on the quasi-periodic modulation of SMC X-1. SMC X-1, consisting of a neutron star and a massive companion, exhibits superorbital modulation with a period varying between ~ 40 d and ~ 65 d. We applied the HHT on the light curve observed by the All-Sky Monitor onboard *Rossi X-ray Timing Explorer (RXTE)* to obtain the instantaneous frequency of the superorbital modulation of SMC X-1. The resultant Hilbert spectrum is consistent with the dynamic power spectrum while it shows more detailed information in both the time and frequency domains. According to the instantaneous frequency, we found a correlation between the superorbital period and the modulation amplitude. Combining the spectral observation made by the Proportional Counter Array onboard *RXTE* and the superorbital phase derived in the HHT, we performed a superorbital phase-resolved spectral analysis of SMC X-1. An analysis of the spectral parameters versus the orbital phase for different superorbital states revealed that the diversity of n_H has an orbital dependence. Furthermore, we obtained the variation in the eclipse profiles by folding the All Sky Monitor light curve with orbital period for different superorbital states. A dip feature, similar to the pre-eclipse dip of Her X-1, can be observed only in the superorbital ascending and descending states, while the width is anti-correlated with the X-ray flux.

Keywords: X-ray binaries, superorbital modulations, Hilbert-Huang transform, SMC X-1

1. INTRODUCTION

The high-mass X-ray binary SMC X-1 consists of a $1.06 M_\odot$ neutron star (van der Meer et al. 2007) and a $17.2 M_\odot$ supergiant (Reynolds et al. 1993). The orbital period of this system is ~ 3.89 d obtained from the X-ray eclipse. Gruber & Rothschild (1984) found that the SMC X-1 exhibits an X-ray periodicity with a rough time scale of 60 d. This periodicity was then confirmed by Wojdowski et al. (1998) and the cycle lengths were reported to vary with time. The behavior of the period variation was investigated after several time-frequency analysis techniques, like the dynamic power spectrum (Clarkson et al. 2003) and the slide Lomb-Scargle periodogram (Trowbridge et al. 2007), were applied on the All-Sky Monitor (ASM) light curve. We present our analysis

by using of a recently developed time-frequency analysis technique, the Hilbert-Huang transform (HHT), to explore the timing properties of the superorbital modulation. Furthermore, based on the phase defined in the HHT, the superorbital phase-resolved analysis on both the spectral behaviors and the variation of orbital profiles are also presented.

2. OBSERVATIONS

The dwell data, collected by the ASM onboard *Rossi X-ray Timing Explorer (RXTE)* with ~ 90 minute timing resolution, were used to explore the detailed variation of orbital profile of SMC X-1. The data were further binned into one-day light

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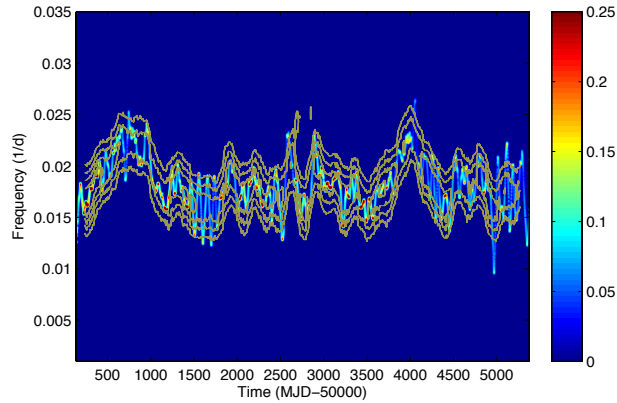


Fig. 1. The Hilbert spectrum and the dynamic power spectrum of SMC X-1.

curve to investigate the variation of superorbital period. Furthermore, the data collected by the Proportional Counter Array (PCA) onboard *RXTE* were also included to obtain the variation of spectral behavior in the different superorbital states.

3. THE HILBERT-HUANG TRANSFORM

The HHT, which is proposed by Huang et al. (1998), is designed to analyze the non-linear and non-stationary time series. The normalized Hilbert transform was used to obtain the instantaneous frequency of the superorbital modulation. However, the Hilbert transform can only be applied on the intrinsic mode functions (IMFs) to get meaningful instantaneous frequency. We used the ensemble empirical mode decomposition, proposed by Wu & Huang (2009), to decompose the ASM light curve into several IMFs.

4. TIME-FREQUENCY ANALYSIS

After decomposing the light curve into several IMFs, we applied the normalized Hilbert transform on them to obtain the Hilbert spectrum. Fig. 1 shows the Hilbert spectrum that contains information of frequency and amplitude variation in a color map, which is stacking with the dynamic power spectrum in contour for comparison. We noticed that the Hilbert spectrum is basically consistent with the dynamic power spectrum, but the resolutions are better in both the time and frequency domains. Since the HHT provided us a well-defined phase of the superorbital modulation, we folded the light curve according to the superorbital phase to obtain the asymmetric superorbital profile, which is shown in Fig. 2. Furthermore, we obtained a significant correlation

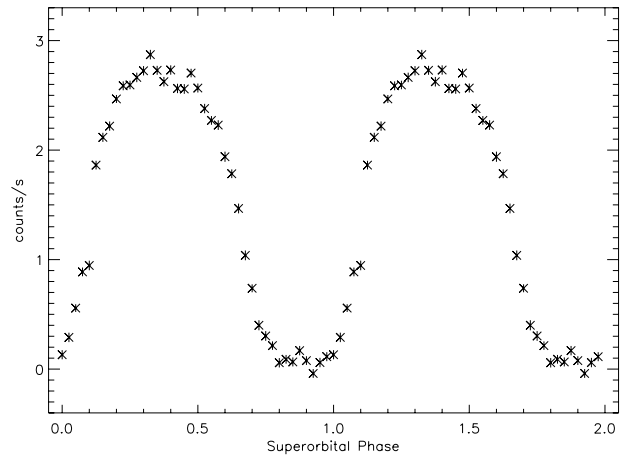


Fig. 2. Folded light curve of superorbital modulation of SMC X-1.

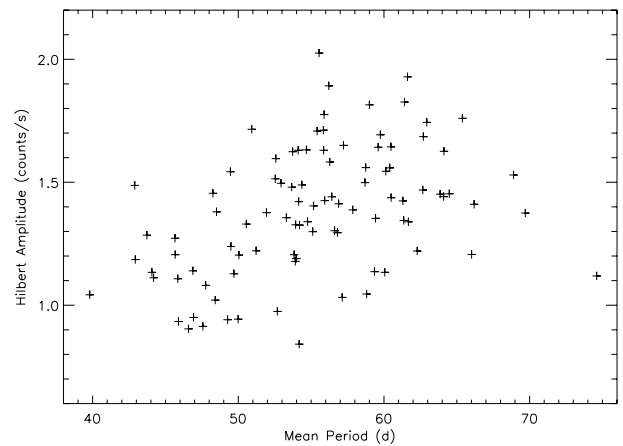


Fig. 3. The correlation between the superorbital modulation period and the amplitude. The rank correlation coefficient is $r = 0.46$ with a null hypothesis probability value of 3.2×10^{-6} .

between the modulation period and the amplitude shown in Fig. 3. This correlation is similar to that between the main-on flux and the superorbital period of Her X-1 (Still & Boyd 2004). Both of the examples are inconsistent with the prediction of radiation-induced warp disk (Wijers & Pringle 1999).

5. SUPERORBITAL PHASE-RESOLVED ANALYSIS

We further studied the superorbital phase-resolved spectroscopic properties of SMC X-1 by using all the PCA observations during 1996 and 2004. The standard-2 mode spectrum can be fitted with a cutoff power law continuum and a Gaussian emission line centered at 6.4 keV. The

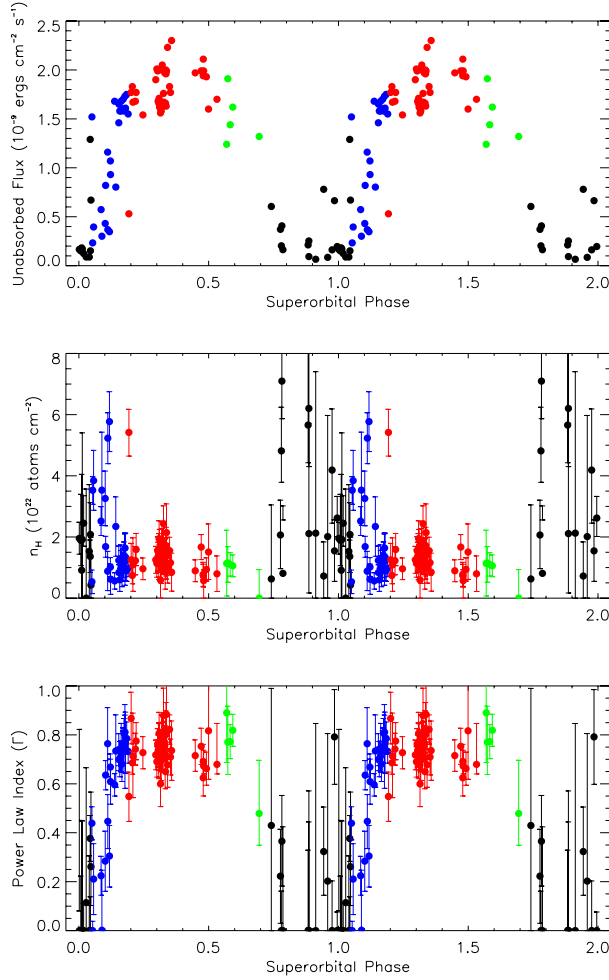


Fig. 4. The unabsorbed flux (top panel), n_H (middle panel) and power-law index (bottom panel) verse the superorbital phase. Black, blue, red, and green symbols represent the spectral parameters in the low, ascending, high, and descending states, respectively.

variation of the uneclipsed hydrogen column density (n_H) versus the superorbital phase is shown in the middle panel of Fig. 4. We noticed that the n_H values show great diversity in the ascending state but remain stable in the descending state. By further checking the orbital distribution of n_H in the different superorbital states, we obtained that the diversity might be caused by the dip feature occurred in orbital phase ~ 0.7 .

The dip feature could be further examined by the folded light curve. Since the appearance of dip may have superorbital dependence, a two-dimensional folded light curve is a good way to investigate the dip properties. We perform this analysis method on the dwell light curve collected by the ASM. We first folded the X-ray photons in the data window of superorbital phase 0.0-0.05 according to the orbital ephemeris proposed by Wojdowski et al. (1998)

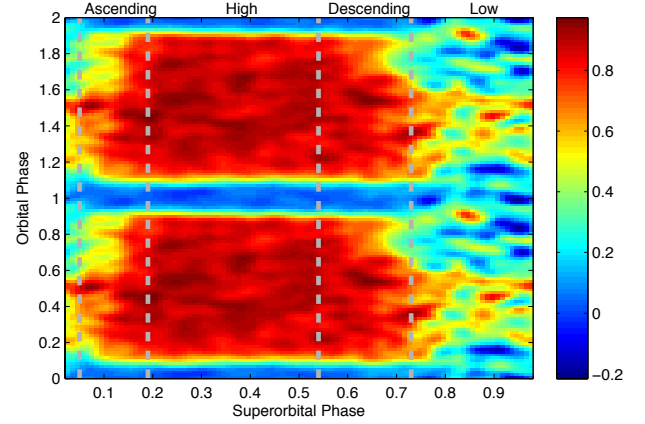


Fig. 5. Two-dimensional dynamic folded light curve of SMC X-1.

and normalized the uneclipsed count rate to 1. Then, we move the window 0.01 superorbital cycles forward to obtain the next orbital profile. This process was repeated until the end of the data set. The combined folded light curve, drawn in a three-dimensional color map, is shown in Fig. 5. We can easily obtain the dip feature in the dynamic folded light curve, especially that occurred in superorbital descending state. From this figure, we also noticed that the dip width has an anti-correlation with the X-ray count rate. This dip feature could be associated with the pre-eclipse dip of Her X-1 (Moon & Eikenberry 2001).

6. CONCLUSIONS

This research demonstrated a time-frequency analysis of the non-stationary superorbital modulation of SMC X-1 by the HHT. The Hilbert spectrum shows great detail in both the time and frequency domains. Furthermore, we found a correlation between the superorbital period and amplitude, which is inconsistent with the outer warp model. Based on the phase derived in the HHT, we studied the phase-resolved spectra and the variation of orbital profile. The variation in both the n_H and the orbital profile indicate that the system contains a pre-eclipse dip. The width variation of the dip can be interpreted by the warp and tilted disk model.

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