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Analytical Study of Polarization Spectroscopy for the J_g =0 → J_e =1 Transition

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This work presents a theoretical study on the analytical calculation of the lineshape of polarization spectroscopy (PS) for the transition line $5s^2\ ^1S_0 \rightarrow 5s5p\ ^1P_1$ of ^{88}Sr. From the obtained analytical form of the PS spectrum, we were able to identify how the saturation affected the lineshape of the PS spectrum. The results obtained will be useful for polarization spectroscopy experiments using the alkaline-earth atoms such as Sr or Yb.

Keywords : Polarization spectroscopy, Strontium, Saturation effect
OCIS codes : (300.6210) Spectroscopy, atomic; (300.6460) Spectroscopy, saturation

I. INTRODUCTION

Due to the ability to provide a dispersive spectroscopic lineshape, polarization spectroscopy (PS) [1] has been widely used and studied in particular for laser frequency stabilization. In PS, a circular birefringence is established by a circularly polarized pump beam. This is detected by measuring the rotation angle of a linearly polarized probe beam propagating in the opposite direction to the pump beam. The sub-Doppler feature originates from the fact that only the atoms belonging to certain velocity classes can experience the pump and probe beams simultaneously. The atoms with zero velocity contribute to the resonance signals, whereas the crossover signals result from the contribution from the atoms satisfying the condition that the frequency spacing of the excited state is equal to twice the Doppler shift.

PS has been realized for many kinds of atoms such as Li [2], Rb [3-6], Cs [6, 7], K [8], He [9], and Sr [10]. In the case of all the atoms except for Sr, the PS spectra result from three operating mechanisms such as Zeeman and hyperfine optical pumping and the saturation effect. In contrast, the PS for Sr results from only the saturation effect because there are no degenerate sublevels in the ground state. Since the isotopes ^{84}Sr, ^{86}Sr, and ^{88}Sr possess zero nuclear spin ($I=0$), whereas ^{87}Sr possesses $I=9/2$, the $J_g=0 \rightarrow J_e=1$ transition exists in the isotopes ^{84}Sr, ^{86}Sr, and ^{88}Sr. In this paper, we will consider ^{88}Sr. In addition, Yb isotopes with mass numbers, 168, 170, 172, 174, and 176, also possess zero nuclear spin. Because the energy level structure of Sr (and Yb) is very simple, it is possible to obtain exact analytical solutions for the PS spectra. The analytical solution of saturated absorption spectroscopy (SAS) for the ideal two-level atoms was presented in the textbook [11]. Also, SAS [12] and PS [13] for Rb atoms were analytically studied in the low intensity limit. In this paper, we present analytical solutions of PS for the transition J_g=0→J_e =1 of Sr (or Yb) atoms where the intensity of the pump beam is arbitrary. This paper is organized as follows. Section II describes the theory of calculating analytical lineshape in PS. Results and discussion are presented in Sec. III. The final section summarizes the results of the paper.

II. THEORY

The energy level diagram for the transition J_g=0→J_e =1 of an atom (^{88}Sr or Yb) is shown in Fig. 1. The ground state is $|g\rangle$, while three degenerate excited states are $|e_-\rangle$, $|e_0\rangle$, and $|e_+\rangle$ where magnetic quantum numbers are -1, 0, and 1, respectively. The pump beam of σ^+ polarization excites the transition from $|g\rangle$ to $|e_+\rangle$, whereas the linearly polarized probe beam excites both the transitions from $|g\rangle$ to $|e_\pm\rangle$. ω_0 is the resonance frequency, λ is the wavelength, $k=2\pi/\lambda$ is the wave vector, $\Omega_{1(2)}$ are the Rabi frequencies for the pump (probe) beam, Γ is the

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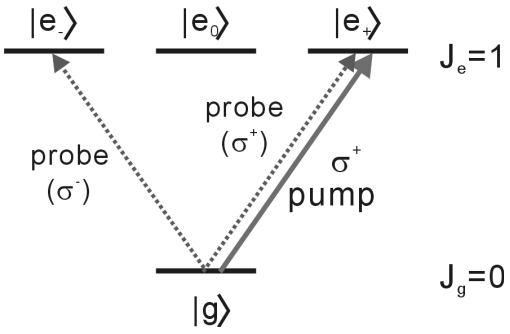


FIG. 1. Energy level diagram for the transition $5s^2\ {}^1S_0 \rightarrow 5s5p\ {}^1P_1$ in the absence of an external magnetic field.

decay rate of the excited state, and γ_t is the decay rate of the optical coherence which is equal to $\Gamma/2$ if there is no dephasing mechanism. The laser frequencies of the pump and probe beams felt by an atom moving at velocity, v, are $\omega_1 = \omega + kv$ and $\omega_2 = \omega - kv$, respectively.

Then, the susceptibilities of the $\sigma^{\pm}$ components of the probe beam for an atom moving at velocity, v, are given by [14]

$$\chi_{\pm}^{(v)} = -N_{\text{at}} \frac{3\lambda^3}{8\pi^2} \Gamma \frac{p - q_{\pm}}{\omega_2 - \omega_0 + i\gamma_t}, \tag{1}$$

where p denotes the population of the ground state, $q_{\pm}$ denotes the populations of the excited states, $|e_{\pm}\rangle$, and N_{at} represents the atomic density. The populations in Eq. (1) are calculated analytically by considering the effect of only the pump beam. Since the σ^+ polarized pump laser field couples only the ground state and the excited state ($|e_{\pm}\rangle$), we can use the results for the case of a two-level atom. Thus, we refer to our previous result [14] or a textbook [15], and the results are given by

$$p = \frac{2A + B}{2(A + B)}, \quad q_+ = \frac{B}{2(A + B)},$$

and $q_0 = q_- = 0$, where $A = (\omega_1 - \omega_0)^2 + \gamma_t^2$ and $B = \Omega_1^2 \gamma_t / \Gamma$.

Therefore, the susceptibilities in Eq. (1) become the following equations:

$$\chi_+^{(v)} = -N_{\text{at}} \frac{3\lambda^3}{8\pi^2} \Gamma \frac{1}{\omega_2 - \omega_0 + i\gamma_t} \frac{A}{A + B}, \tag{2}$$

$$\chi_-^{(v)} = -N_{\text{at}} \frac{3\lambda^3}{8\pi^2} \Gamma \frac{1}{\omega_2 - \omega_0 + i\gamma_t} \frac{A + (B/2)}{A + B}. \tag{3}$$

Equations (2) and (3) are then averaged over the Maxwell-Boltzmann velocity distribution as

$$\chi_{\pm} = \frac{1}{\sqrt{\pi} u} \int_{-\infty}^{\infty} e^{-(v/u)^2} \chi_{\pm}^{(v)} dv, \tag{4}$$

where u is the most probable speed of the atom. Equation (4) is further simplified by changing the integration variable as

$$\chi_{\pm} = -\frac{C_0}{\pi} \int_{-\infty}^{\infty} e^{-\left[\frac{\gamma_t}{ku}\left(\frac{w}{2} - y\right)\right]^2} \left(\frac{1 - \eta_{\pm} L(w - y)}{y + i} \right) dy, \tag{5}$$

where

$$C_0 = \frac{3\lambda^3}{8\pi^{3/2}} \frac{N_{\text{at}} \Gamma}{ku}, \quad w = \frac{2\delta}{\gamma_t},$$
$$\eta_+ = 1, \quad \eta_- = 1/2,$$
$$L(y) = \frac{s_0}{y^2 + 1 + s_0},$$

and $\delta (= \omega - \omega_0)$ is the detuning and $s_0 \left(= \Omega_1^2 / (\gamma_t \Gamma) = I_1 / I_s\right)$ is the on-resonance saturation parameter where I_1 is the pump beam intensity and $I_s = \pi h c \Gamma / (3\lambda^3)$ is the saturation intensity with c being the speed of light in vacuum. The integration in Eq. (5) can be easily performed using a convolution theorem. When $\gamma_t \ll ku$, the real and imaginary parts of the susceptibilities are given by

$$\chi_{\pm}^r = -C_0 e^{-\delta^2/(ku)^2} \text{Erfi}\left(\frac{\delta}{ku}\right) + C_0 e^{-\delta^2/(ku)^2} \frac{s_0 \eta_{\pm}}{\sqrt{1 + s_0}} \frac{w}{w^2 + \left(1 + \sqrt{1 + s_0}\right)^2},$$
$$\chi_{\pm}^i = C_0 e^{-\delta^2/(ku)^2} - C_0 e^{-\delta^2/(ku)^2} \frac{s_0 \eta_{\pm}}{\sqrt{1 + s_0}} \frac{1 + \sqrt{1 + s_0}}{w^2 + \left(1 + \sqrt{1 + s_0}\right)^2},$$

respectively.

In PS, a linearly polarized probe beam (intensity is I_0 and polarization vector is $\hat{x}\cos\theta + \hat{y}\sin\theta$) is incident on an atomic cell of length l along the z axis. After traversing the cell, the polarization of the probe beam changes due to the circular anisotropy from the pump beam. The electric field of the probe beam is then given by [5, 14]

$$E = \frac{E_0}{\sqrt{2}} \left[-\hat{\varepsilon}_+ a_+ e^{i(kn_+ l - \theta)} - \hat{\varepsilon}_- a_- e^{i(kn_- l + \theta)} \right], \tag{6}$$

where E_0 is the amplitude of the incident probe beam's electric field, $a_{\pm} = e^{-(k/2)\chi_{\pm}^i l}$, and $n_{\pm} \cong 1 + (\chi_{\pm}/2)$ are the refractive indices of the $\sigma^{\pm}$ components of the probe beam.

$\hat{\varepsilon}_{\pm} = \mp 2^{-1/2}(\hat{x} \pm i\hat{y})$ are the spherical bases where $\hat{x}$ and $\hat{y}$ are the unit vectors for x and y axes, respectively. The inclination angle of the electric field in Eq. (6) with respect to the x axis is given by $\theta + \varsigma$ where θ is the inclination angle of the incident probe beam's polarization and $\varsigma = (kl/2)(n_- - n_+) \cong (kl/4)(\chi_-^r - \chi_+^r)$ is the rotation angle of the probe beam's polarization after traversing the atomic cell [16]. Then, the difference in the intensities along the x and y axes, $\Delta I = I_x - I_y$, is given by

$$\Delta I = I_0 a_+ a_- \cos[2\theta + 2\varsigma],$$

and becomes further

$$\Delta I = -I_0 a_+ a_- \sin 2\varsigma,$$

when $\theta = \pi/4$. Since the rotation angle, ζ, is very small, the PS signal is given by

$$\Delta I = -\frac{I_0}{2} kl e^{-\bar{\alpha} l} \Delta\chi^r, \tag{7}$$

where $\bar{\alpha} = k(\chi_+^i + \chi_-^i)/2$ is the average of the absorption coefficients. The difference in the real parts of the susceptibilities ($\Delta\chi^r \equiv \chi_-^r - \chi_+^r$) is given by

$$\Delta\chi^r = -C_0 \frac{s_0}{2\left(1 + s_0 + \sqrt{1+s_0}\right)} \frac{x}{x^2+1}, \tag{8}$$

where

$$x = \frac{2\delta}{\gamma_t\left(1 + \sqrt{1+s_0}\right)}.$$

We neglect $e^{-\delta^2/(ku)^2}$ in Eq. (8) because $|\delta| \ll ku$.

III. RESULTS AND DISCUSSION

The typical PS spectra ($\Delta\chi^r$) for several pump beam intensities are presented in Fig. 2(a). In Fig. 2(a), the saturation parameters were s_0 = 1, 5, 10, 50, and 100. The saturation parameter of s_0 = 100 corresponds to the intensities of 4.27 W/cm^2 because the saturation intensity of Sr atom is about 4.27×10^{-2} W/cm^2. The amplitude of the spectrum, defined as $|\Delta\chi^r|$ at x=±1, and accordingly at the detunings of $\pm\left(1+\sqrt{1+s_0}\right)\gamma_t/2$, is given by

$$\frac{C_0 s_0}{4\left(1 + s_0 + \sqrt{1+s_0}\right)}, \tag{9}$$

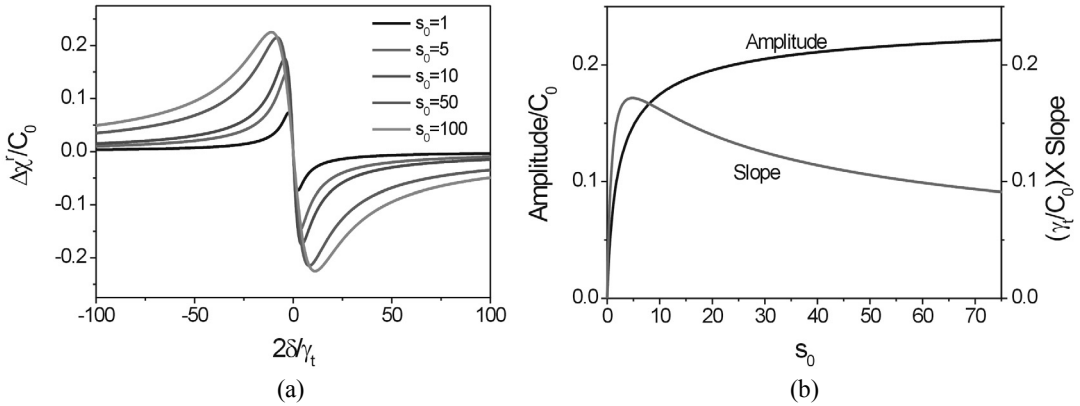
and the magnitude of the slope of the PS spectrum at the resonance condition is given by

$$-\left.\frac{d\Delta\chi^r}{d\delta}\right|_{\delta=0} = \frac{C_0 s_0}{\gamma_t\sqrt{1+s_0}\left(1+\sqrt{1+s_0}\right)^2}. \tag{10}$$

The calculated amplitude and slope as functions of s_0 are presented in Fig. 2(b). In Fig. 2(b), the amplitude increases and is then saturated at the value of $C_0/4$. In Fig. 2(b), the slope is maximum when $s_0 = 2\left(1+\sqrt{2}\right) \cong 4.8$. This value corresponds to the intensity of 0.206 W/cm^2. This is in excellent agreement with the experimental results in Fig. 4(b) in Ref. [10].

IV. CONCLUSION

In this paper we have presented a theoretical study of



(a) (b)

FIG. 2. (a) Typical calculated PS spectra for several pump beam intensities. (b) Dependence of the amplitude and the magnitude of the slope on the on-resonance saturation parameter.

lineshape in PS for the transition J_g=0→J_e =1 of Sr atoms. Equations (7) and (8) are the main result of the paper. The amplitude and the slope of the spectrum are presented in Eq. (9) and Eq. (10), respectively. The theoretical results were compared with experimental results presented in Ref. [10], and excellent agreement between them was found. Since the obtained results in this paper are very concise, these can be applied to study of PS for other atoms such as Yb and to study of other spectroscopy such as sub-Doppler dichroic atomic vapor laser lock (DAVLL) [10].

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